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GLOBAL SAFETY FACTOR FOR NONLINEAR DESIGN OF CONCRETE STRUCTURES

SZ. WOLIŃSKI[1]

The paper focuses on different approaches to the safety assessment of concrete structures designed using nonlinear analysis. The method based on the concept of partial factors recommended by Eurocodes, and methods proposed by M. Holicky, and by the author of this paper are presented, discussed and illustrated on a numerical example. Global safety analysis by M. Holicky needs estimation of the resistance coefficient of variation from the mean and characteristic values of resistance, and requires two separate nonlinear analyses. The reliability index value and the sensitivity factor for resistance should be also identified. In the method proposed in this paper, the resistance coefficient of variation necessary to calculate the characteristic value of resistance may be adopted from test results and the resultant partial factor for materials properties, and may be calculated according to Eurocodes. Thus, only one nonlinear analysis from mean values of reinforcing steel and concrete is required.

Key words: nonlinear design, global safety factor, concrete structures.

1. Introduction

Four types of idealization of reinforced concrete structures are recommended in Eurocode 2 [1] for structural analysis and design:
- linear elastic behaviour, that assumes perfect elasticity and uncracked cross sections;
- linear elastic behaviour with limited redistribution;
- plastic behaviour based on kinematic or static approach;
- nonlinear behaviour, that takes into account cracking of concrete and plasticization of reinforcing steel and concrete in compressed zones.

Rapid increase of new capabilities of the available software for numerical simulation of structural performance supports more and more frequent applications of non-linear analysis as a tool for design and assessment of concrete structures. It may be used for both the serviceability and ultimate limit states, provided the method satisfies the equilibrium and compatibility. The non-linear method of analysis requires a global approach and becomes useful when it is difficult to identify the sections to be chec-

[1] Assoc. Prof., Department of Building Structures, Rzeszów University of Technology, Rzeszów, Poland, e-mail: szwolkkb@prz.edu.pl

ked, for example in structures with complicated geometry, with opening, with special detailing, etc. It checks automatically all locations of sections, gives information about the structural redundancy, robustness, safety, and is often referred to as virtual testing of structures.

On the other hand, the non-linear analysis brings also some disadvantages. The force superposition is not valid anymore and a separate analysis is necessary for each combination of actions. A non-linear analysis is more demanding than a linear one, and standard models of resistance and effect of actions are not sufficient. Another important point is that in the partial factor method, the reliability check is ensured in cross sections (local level) whereas a global check of safety should be performed on higher level. However, the code provisions provide very general and vague information on how to use the results of non-linear analysis for structural safety assessment and design. On global level, the full probabilistic analysis can be applied in case of non-linear analysis of reinforced concrete structures, but at present full probabilistic analysis are used mainly in advanced research and in the design of critical structures. In today practice some working safety formats based on global safety factors and additional assumptions are used.

The paper presents a comparison of several practical methods for the assessment of global safety factors for resistance of reinforced concrete structures designed by using non-linear analysis. The concept of global resistance factor proposed by the author of this paper, is based on the idea that the global resistance factor which can be calculated as the product of two partial factors. The first one is the weighted average value of partial factors for materials properties (local level) and the second one, estimated from mean and characteristic values of the structural resistance, takes into account the resistance coefficient of variation adopted from tests results and the type of structural system consisting of interconnected components and elements (global level).

2. Partial factors format for linear and non-linear analysis

2.1. Linear analysis according to Eurocodes

The format of partial factors separates the influence of uncertainties and variables originating from causes by means of design values assigned to basic variables: actions, material properties, and geometrical quantities. The assessment procedure based on partial factor, for linear analysis, involves three following steps [2, 3]:

a) Calculation of the design values of the effects of actions in critical cross-sections of the structure, considering all possible loads combinations and using design values of actions and geometrical data:

$$E_d = \gamma_{Sd} E\{\gamma_{f,i} \times F_{rep,i}; a_d\}, \quad i \geqslant 1 \tag{2.1}$$

or

$$E_d = E\{\gamma_{F,i} \times F_{rep,i}, a_d\}, \quad i \geqslant 1 \tag{2.2}$$

where γ_{Sd} is the partial factor taking account of the uncertainties in the action model and in the action effects model, $\gamma_{f,i}$ is the partial factor taking into account the possibility of unfavourable deviation of the action values from the representative values $F_{rep,i}$, $\gamma_{F,i} = \gamma_{Sd} \times \gamma_{f,i}$ is the single partial factor for an action F_i and a_d is the design value of geometrical data.

b) Calculation of the design resistance of critical sections of the structure by using the design values of material parameters and geometrical data:

$$R_d = \frac{1}{\gamma_{Rd}} R(X_{d,i};\ a_d) = \frac{1}{\gamma_{Rd}} R(\eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d),\ i \geqslant 1 \tag{2.3}$$

or

$$R_d = R(\eta_i \frac{X_{k,i}}{\gamma_{M,i}}; a_d),\ \gamma_{M,i} = \gamma_{Rd} \times \gamma_{m,i},\ i \geqslant 1 \tag{2.4}$$

where γ_{Rd} is the partial factor taking account of the uncertainties in the resistance model, η is a conversion factor, $\gamma_{m,i}$ is the partial factor for a material property, taking into account only uncertainties in the material property, X_d and X_k are the design and the characteristic value of a material property, and γ_M is the single partial factor for a material property.

c) Reliability check of limit state is performed by the following design condition:

$$E_d \leqslant R_d \tag{2.5}$$

In the partial factor method, reliability verification is ensured in local material points and the probability of violation of the design criterion (2.5) is not known.

2.2. Non-linear analysis according to Eurocodes

The non-linear methods of analysis require a global approach and they check automatically all locations of sections, give information about the structural redundancy, robustness, safety, and can be referred to as virtual testing of structures. The force superposition is not valid anymore and a separate non-linear analysis should be performed for each combination of actions. Moreover, the design values of the basic random variables are extremely low in the case of material properties and extremely high in the case of actions, and do not represent a real material and actions. Consequently, the non-linear analysis based on the design values may result in unrealistic and unsafe redistribution of internal forces and may also change the failure mode of the structure.

According to the Eurocode 2 [1] and ISO Standard 2394 [2], non-linear methods may be used for both the ultimate and serviceability limit states of concrete structures, provided the method satisfies equilibrium and compatibility, and that the ability of local critical sections to withstand any inelastic deformations are checked. The analysis should take the appropriate account of uncertainties and the distribution of deformations, and internal forces and material properties should provide realistic stiffness of the members. Unfortunately, these requirements are very general and imprecise.

Contradictory recommendation has been inserted in the Eurocode [2]. For structures or structural members that are analysed by non-linear methods, and comprise more than one material acting in association, the following expression for design resistance can be used alternatively to (2.3) or (3.4):

$$R_d = \frac{1}{\gamma_{M,1}} R(\eta_1 X_{k,1}; \eta_i X_{k,i(i>1)} \frac{\gamma_{m,1}}{\gamma_{m,i}}; a_d) \tag{2.6}$$

In some cases, the design resistance can be expressed by applying directly γ_M partial factors to the individual resistance due to material properties. According to the Eurocode [2], in case of a single predominant action, when the relationship between actions and their effects is not linear, the following simplifications may be considered:

– when the action effect increases more than the action, the partial factor γ_F should be applied to the representative value of the action;
– when the action effect increases less than the action, the partial factor γ_F should be applied to the action effect of the representative value of the action.

3. Global factors format for non-linear analysis

3.1. ECOV method

The ECOV method (Estimate of Coefficient Of Variation) is based on the concept proposed by M. Holicky [4], that the random distribution of resistance can be estimated from mean R_m and characteristic R_k values. Under the assumption, that random distribution of resistance is according to log-normal distribution, its coefficient of variation v_R can be calculated as:

$$v_R = \frac{1}{1,645} \ln\left(\frac{R_m}{R_k}\right) \tag{3.1}$$

According to Eurocode [3], the global safety factor γ_R can be estimated as:

$$\gamma_R = \exp(\alpha_R \beta v_R) \tag{3.2}$$

where α_R is the FORM sensitivity factor for resistance reliability, which may be taken as $\alpha_R = 0,80$ or 1,0 or 0,40 for, respectively $0,16 < \sigma_E \sigma_R < 7,6$; $\sigma_E/\sigma_R \geqslant 7,6$; $0,16 \geqslant \sigma_E/\sigma_R$, σ_E and σ_R are the standard deviations of the action effect and resistance, β is the target reliability index depending on the reliability class and design working life of the structure (e.g. for the class RC2 and T = 50 years, $\beta = 3,8$).

The ECOV method requires two separate non-linear analysis; the first one based on the mean values and the second based on the characteristic values of materials parameters. The justification for the ECOV method is that the characteristic values are not so far from the mean one, thus able to change the failure mode of the structure, but well reflect their scatter.

The reliability index β is a conventional measure of reliability, and is related to the notional value of the probability of failure P_f by:

$$P_f = \Phi(-\beta) \text{ and } \beta = -\Phi^{-1}(P_f) \tag{3.3}$$

where Φ is the cumulative distribution function of the standardised normal distribution.

3.2. Alternative approach to estimation of global factors

While searching for a more suitable estimation method of the global safety factors, a new approach based on one step non-linear analysis, statistical parameters of material properties and the resistance of structural concrete members has been proposed [5]. The underlying assumption is that the resultant probability of down-crossing the design value of a structural resistance calculated by using the partial factors method and using the non-linear analysis and global factor of resistance γ_R should be the same. The γ_R – value may be calculated as the product of two partial factors $\gamma_R = \gamma_1 \times \gamma_2$, where γ_1 is the ratio of the mean to characteristic value of the resistance, and γ_2 is the ratio of characteristic to the design value of the structural resistance. In other words, the global factor γ_R may be split into two partial factors γ_1 and γ_2 that may be estimated separately.

The procedure for calculation of the design value of a concrete structure or a structural member is as follows:

1. Calculation of the mean value of structural resistance R_m by using the non-linear analysis with the suitable computer program and the mean values of input material parameters $f_{m,i}$.
2. Determination of the partial factor γ_1, which is defined as the ratio of the mean structural resistance to the characteristic one:

$$\gamma_1 = \frac{R_m}{R_k} = \frac{R_m}{R_m(1-1,645v_R)} = \frac{1}{1-1,645v_R} \tag{3.4}$$

where v_R is the coefficient of variation of structure or member resistance.

3. The value of coefficient of resistance v_R for an individual structural member should be assessed from test results. For example, the v_R – values of resistance for reinforced and prestressed concrete members may be summarized as follows [6]:

- RC one-way slabs in bending $v_R = 0,15 \div 0,16$;
- RC two-way slabs in bending $v_R \approx 0,14$;
- RC beams in bending $v_R = 0,08 \div 0,12$;
- Short RC columns $v_R \approx 0,16$;
- Slender RC columns $v_R \approx 0,17$;
- RC beams in shear $v_R = 0,17 \div 0,21$
- Plant precast pretensioned elements in bending $v_R = 0,06 \div 0,10$;
- Cast-in place posttensioned elements in bending $v_R = 0,06 \div 0,15$.

For a structural system the coefficient of resistance should be calculated taking the type of members (brittle or ductile) into account, type of the system (series, parallel, combined), and independency or different modes of correlation between members [7].

4. Determination of the partial factor γ_2, which is defined as the ratio of the characteristic to the design value of structural resistance:

$$\gamma_2 = \frac{R_k}{R_d} \tag{3.5}$$

Partial factor γ_2 may be calculated using formula (2.6). The dominant structural material

and the proportion of their influence on the resistance of the structure should be specified. As a rule, for RC structures, the reinforcing steel has dominant influence on the resistance of a structure. The partial factors in persistent and temporary design situations are: for steel reinforcement γ_S= 1.15 and for concrete γ_C = 1.4 (Polish NA to Eurocode 2). Consequently, the γ_2 – value may be calculated as follows:

$$\gamma_2 = \gamma_S \times k(\gamma_C/\gamma_S) \tag{3.6}$$

where $k(\gamma_C/\gamma_S) \cong 1,15 \div 1,32$ for different failure modes and the reinforcement ratio.

5. The design resistance of a structural member or a structure may be estimated from the following expression:

$$R_d = \gamma_R R_m = \gamma_1 \gamma_2 R_m \tag{3.7}$$

6. In case of a structural system composed of many different elements the bounds on the global factor of resistance γ_R can be derived, and the specific γ_R-value should be estimated, taking into consideration the reliability class, the design working life, as well as an experience and intuition of the designer.

4. NUMERICAL EXAMPLE

In order to illustrate how the ECOV and the proposed alternative approaches to estimation of global factors γ_R can affect its value, results of example calculations are presented. For some chosen values of coefficients of variation for the resistance of reinforced concrete elements $v_R = 0.10$; 0.15; 0.20; 0.25; 0.30; 0.40 were calculated using the formula (3.1) for the FORM sensitivity factor $\alpha_R = 0.80$, and the target reliability index $\beta = 3.8$; values of the global safety factor values γ'_R were calculated according to the formula (3.2). For the same values of variation coefficients for the resistance (considered as values estimated from test results), values of the partial factor γ_1 were calculated using the formula (3.3). Assuming that the partial factor γ_2 can be estimated by bounds for $k(\gamma_C/\gamma_S) \cong 1,15 \div 1,32$, according to the formula (3.6), values of the global factors γ_R were calculated according to the alternative approach as: $\gamma_R = \gamma_1 \times \gamma_2$. Results of these calculations are presented in Table 1.

Table 1

Values of global factors γ'_R and γ_R depending on values of variation coefficients v_R.
Wartości globalnych współczynników γ'_R i γ_R dla różnych współczynników zmienności v_R

v_R	0.10	0.15	0.20	0.25	0.30	0.40	0.50
γ'_R	1.35	1.58	1.84	2.14	2.49	3.37	4.57
γ_R	1.38÷1.58	1.53÷1.76	1.71÷1.97	1.95÷2.24	2.27÷2.60	3.36÷3.85	6.47÷7.43

The lower bound of global factor values assessed by using the approach alternative to the ECOV method corresponds to RC members whose resistance depends on the reinforcing steel only, and the upper bound to RC members whose resistance are dominated by concrete. In both cases, global factors for resistance are determined by the variation coefficient v_R of a member or structure resistance. For $v_R > (0.25 \div 0.30)$ values of global factors calculated by using both methods seems to be unrealistic.

5. CONCLUSIONS

The semi-probabilistic partial factors method is incoherent with the non-linear analysis of structures. Principles and recommendations given in Eurocode 2 [1] concerning applications of the partial factors method in non-linear analysis and design of concrete structures are very general and vague.

Among different methods for the assessment of global safety factors for resistance of reinforced concrete structures proposed by different authors, the ECOV-method proposed by M. Holicki may be mentioned as general and compatible with the partial factors safety format applied in Eurocodes. However, this method requires two separate non-linear analyses.

The concept of global resistance factor proposed by the author of this paper is based on the idea that the global factor may be assessed as the product of two partial factors. The first one is the weighted average value of partial factors for material properties, and it expresses the influence of material properties at the local level of a structure. The second partial factor estimated from mean and characteristic values of the global structural resistance takes into account the coefficient of resistance variation, adopted from test results and the type of structural system consisting of interconnected components and elements. This approach requires only one non-linear analysis and enables to make use of a prior information about resistance statistics for concrete elements and structures. The global safety factor for resistance can be estimated by bounds, and its value may be specified by the designer according to his or her experience and intuition.

Full probabilistic analysis is a general tool for reliability assessment of reinforced concrete structures. It can be used for determination of the design value of resistance function. Unfortunately, at present, full probabilistic approach is used mainly in the design of critical structures and in advanced research.

REFERENCES

1. Eurocode 2. Design of concrete structures – Part 1-1: General rules and rules for buildings. CEN, 2004.
2. International Standard ISO 2394:1998. General principles on reliability for structures.
3. Eurocode. Basis of structural design. CEN, 2002.
4. M. HOLICKY, *Global resistance factors for reinforced concrete members*. Acta Polytechnica, CTU in Prague, 2006.
5. SZ. WOLIŃSKI, *Global safety coefficient in non-linear analysis of concrete structures* [in Polish], 56 Konferencja Naukowa KILiW PAN i KN PZITB, „Problemy naukowo-badawcze budownictwa”. Referaty, str.627-633, Kielce – Krynica, 2010.
6. A. NOWAK, K. COLLINS, *Reliability of Structures*. Mc Graw-Hill, Boston, 2000.
7. O. Ditlevsen, *Structural Reliability Methods*. Wiley, New York, 1996.

Streszczenie

W artykule przedstawiono kilka metod zapewnienia bezpieczeństwa konstrukcji z betonu projektowanych z uwzględnieniem analizy nieliniowej. Omówiono metodę opartą na koncepcji współczynników częściowych zalecaną w Eurokodach, metodę ECOV opracowaną przez M. Holický'ego ego i metodę zaproponowaną przez autora niniejszego artykułu. Pół-probabilistyczna metoda częściowych współczynników jest z natury niezgodna z analizą nieliniową. Ponadto, zasady i reguły dotyczące stosowania metody częściowych współczynników, które zamieszczono w Eurokodzie 2, są bardzo ogólne i nieprecyzyjne. Globalna analiza bezpieczeństwa opracowana przez M. Holický'ego opiera się na oszacowaniu współczynnika zmienności nośności, który należy obliczyć na podstawie dwukrotnej analizy konstrukcji; dla średnich i charakterystycznych wartości zmiennych podstawowych. Należy również przyjąć wartość wskaźnika niezawodności i odpowiedniego współczynnika wrażliwości dla których można obliczyć wartość globalnego współczynnika nośności. Zaproponowana przez autora artykułu metoda globalnego współczynnika nośności opiera się na jego podziale na dwa współczynniki częściowe i wymaga jednokrotnej analizy nieliniowe konstrukcji dla średnich wartości parametrów materiałowych. Pierwszy z częściowych współczynników

nośności należy obliczyć jako ważoną średnią częściowych współczynników materiałowych (poziom lokalny), natomiast drugi można oszacować na podstawie statystyk współczynników zmienności nośności elementów konstrukcyjnych z uwzględnieniem struktury niezawodnościowej rozważanej konstrukcji (poziom globalny). Rozważania teoretyczne zilustrowano przykładem obliczeń i porównaniem wartości globalnych współczynników nośności według metody ECOV i autorskiej dla konstrukcji o różnych współczynnikach zmienności nośności.

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