



Research paper

Decision support for subcontractor selection and allocation to work packages in the contractor's project portfolio

Piotr Jaśkowski¹, Sławomir Biruk², Michał Krzemiński³

Abstract: Contractors engaged in a number of concurrent projects are rarely able to complete all works relying only on in-house resources. This is due to these resources' limited availability and missing qualifications. The subcontractor selection process is time-consuming and complex, which is why it is often carried out whilst the general contractor (GC) is already executing the works. The selection of subcontractors affects the time, cost and quality of the works, and requires decision support tools that help analyse constraints of project milestones and due dates set in the main contracts. Traditional tendering procedures, even when restricted to a selected group of reliable subcontractors, do not allow for the modification of submitted bids to adapt them to the GC's needs. The paper proposes a mathematical model to support subcontracting work packages and negotiations with subcontractors. It is intended to be used after the subcontractor pre-qualification procedure. The model allows for the analysis of alternative bids and the negotiation of discounts when awarding a larger number of packages to the same subcontractor. The proposed approach focuses on minimising the total cost of delivering work packages over a predefined period of analysis and, at the same time, ensuring high utilization levels of in-house resources (a requirement set using a minimum value of works to be delivered by in-house crews). The purpose of the proposed model is to allocate resources and to schedule packages planned for a specified planning horizon.

Keywords: subcontractors selection, construction processes scheduling, resources allocation, mathematical modelling, supporting general construction decisions

¹DSc., PhD., Eng., Lublin University of Technology, Faculty of Civil Engineering and Architecture, Nadbystrzycka St. 40, 20-618 Lublin, Poland, e-mail: p.jaskowski@pollub.pl, ORCID: [0000-0003-1661-3373](https://orcid.org/0000-0003-1661-3373)

²DSc., PhD., Eng., Lublin University of Technology, Faculty of Civil Engineering and Architecture, Nadbystrzycka St. 40, 20-618 Lublin, Poland, e-mail: s.biruk@pollub.pl, ORCID: [0000-0003-4392-8426](https://orcid.org/0000-0003-4392-8426)

³DSc., PhD., Eng., Warsaw University of Technology, Faculty of Civil Engineering, Al. Armii Ludowej 16, 00-637 Warsaw, Poland, e-mail: m.krzeminski@il.pw.edu.pl, ORCID: [0000-0002-6352-5942](https://orcid.org/0000-0002-6352-5942)

1. Introduction

Any construction project is a natural generator of decision problems. Managers and engineers may devise a number of solutions to each of them. The selection of the solutions to be implemented involves finding the best trade-off between the conflicting criteria of cost, time, and quality under constraints of the project budget, project documentation, and specification.

Contractors typically run several projects at a time, concurrently bidding for new jobs as opportunities arise, answering calls for tenders from both public and private clients [1, 2]. As supply of skilled crew, expert staff, and specialized plant is limited, the commercial success (or even a mere survival in the competitive market) relies heavily on the contractor's ability to juggle with in-house and subcontracted resources [3]. This means allocating scarce and costly assets to numerous tasks that belong to projects in the contractor's portfolio [4].

The general contractor's own crews can naturally carry out only the tasks that, first, correspond to their qualifications, and second, are scheduled respecting their availability. Keeping enough employees (in terms of both the number and specialization) to conduct any work the general contractor (GC) would potentially get would result in resource under-utilization [5]. For these reasons, subcontracting is used not only to provide missing in-house skills and equipment, but also to solve the problem of overallocation of in-house resources [6]. If justified by market conditions, contractors use subcontracting to survive the volatility of the economic cycles by avoiding long-term commitment of maintaining permanent employees [7].

The decision if to subcontract must be justified in terms of cost. For instance, small local companies may offer competitive prices due to low overheads, and specialized subcontractors of plant-intensive operations relieve the GC from the burden of ownership cost of rarely used equipment. Thoughtful packaging of subcontracted work speeds up delivery and cuts cost [8], and some construction companies prefer to outsource most of the work, limiting their activities to project management, e.g. coordinating work, ordering equipment and materials, monitoring progress and settling accounts for the project [9].

The GC enters into a contract with the client for the completion of all works within the project, taking full responsibility for the quality, timely delivery and safety of all works, regardless of them being physically executed by in-house or subcontracted resources. Therefore, the success of a project largely depends on selecting reliable and, at the same time, affordable subcontractors.

An analysis of the work conducted to date has shown that the issue of subcontractor selection is typically considered in relation to individual construction projects, and that the merits of subcontracting are often analysed in isolation, without taking into account the possibility of deploying the GC's own crews. In this paper a mathematical model has been proposed to assist general contractors in selecting subcontractors. The model considers costs and availability of both in-house and subcontracted resources, and the possibility to get better rates on subcontracted work if more packages are entrusted to the same subcontractor. The results of analyses based on this model are intended to assist GC in negotiating with subcontractors.

2. Methods, procedures and criteria of subcontractors selection

Under the Polish Civil Code [10], a contract (e.g. between a general contractor and a subcontractor) may be concluded through the acceptance of an offer, negotiation, auction or tender. These procedures apply to contracts not subject to the Public Procurement Law [11]. Project managers use the same formal requirements when drawing up procedures for selecting contractors for work packages [12]. Open tendering (used frequently in public procurement) is considered to ensure the greatest competition and obtain the best bids. However, due to time constraints and the burden of analysing a large number of bids, the general contractors often reduce the pool of bidders [13] and send invitation for tenders only to “trusted subcontractors” they have already cooperated with [8] or who have previously passed a pre-qualification procedure. A subcontractor hired without being screened for financial standing, reliability and capacities poses danger: their failure typically means serious disruption in a project [6].

Many general contractors cultivate long-term relationships with subcontractors, managing their order books with a focus on their partners’ resources. This allows them to take on construction projects of an increasingly large and complex nature [14]. Building trust in the GC’s supply chain has a positive effect on cost, time and quality of projects and is certainly an element of competitive advantage [15]. The cooperation extends beyond a particular projects, and partners formulate a joint policy on acquiring contracts by relying on each other’s capacities [16].

As the GC subcontracts high-value works, sometimes the entire scope of the project, the selection of suitable subcontractors is crucial to the success [17]. Therefore, in the case of starting cooperation with a new subcontractor, the decision is not based solely on the subcontractor’s tender price; the subcontractor must demonstrate ability to deliver on time and to the desired standard [18, 19]. A track record on experience with similar works, good reputation, evidence on capacity and good financial standing, and declaration on availability belong to the key input the selection decision [20, 21]. Selecting suitable subcontractors is a complex multicriterial problem [9, 22].

Defining criteria of subcontractor selection has been a frequent topic of research [23], a list of most frequently applied ones can be found, among others, in [6, 21]. In general, the lists of criteria are long, which reduces the relative impact of a particular criterion on the final decision. There is also no consensus among researchers and practitioners as to which criteria should be applied in practice [24]. El-Khalek *et al.* [9] having surveyed opinions of 120 randomly selected contractors, provided a list of statistically most significant criteria. A similar ranking, based on the Relative Importance Index (RII) was prepared in Taruna *et al.* [25]. Nevertheless, such rankings, though providing non-identical lists of candidate criteria, provide some guidance for the practitioners preparing subcontractor pre-qualification procedures, shortlisting subcontractors invited for tender or deciding on tender adjudication method.

As for the methods of ranking and selection, a variety of multicriteria analysis methods were presented in the literature, from the simplest Weighted Sums (as applied for defining eligibility ratings [26]), the classic Analytical Hierarchy Process [27], Analytic Network Process [28], TOPSIS [29], to customized and modified combinations of the existing methods. Among others, a combination of AHP and PROMETHEE has been proposed by [6]). Cheng *et al.* [30] developed

an evolutionary fuzzy hybrid neural network (EFHNN) to assess subcontractor performance. Ulubeyli and Kazaz [13] proposed a fuzzy multi-criteria decision making method and developed it into a computer application (CoSMo) to support the prequalification process. Arslan *et al.* [20] developed a web-based subcontractor evaluation system WEBSES using time, quality, cost and adequacy measures. A different approach was adopted by Luu and Shee [21] who used Case Base Reasoning to find patterns in actual subcontractor selection decisions of past projects.

The models and methods for pre-qualifying and selecting subcontractors presented in the literature are often complex and require a large amount of information on potential bidders. Even those described as serving actual selection (incorporating a subcontractor in the project team), and not just shortlisting, do not consider the expected impact of selecting a particular subcontractor on the time, cost and quality of the entire project [31].

Most models just assume that the subcontractors are going to be available precisely whenever the GC needs them. From GW's perspective, setting these dates at the stage of deciding on a subcontractor is a complex issue. The dates are determined on the basis of a provisional schedule drawn up for the project completion date. This means, the schedule might have been prepared without allocating teams and subcontractors to carry out specific tasks, at the GC's risk. Therefore, the commencement dates the GC specifies while searching for subcontractors are expressed as estimated time windows. Preparing their tenders, subcontractors take into account the feasibility of completing them within the time windows set by the GC, consideration given to their other commitments.

The reference schedule evolves as updated by data from subcontractor bids gradually gathered and accepted. Data from subcontractors' bids can also be used to optimise the schedule – to identify a set of resources that promises the lowest cost and the highest utilization of the GC's in-house crews while meeting deadlines set by the main contract between the client and the GC. The literature presents some optimisation models intended for analyses of this kind – a bi-criteria optimisation of schedules (or time-cost trade-off analysis) capable of analysing options of resource allocation. For instance, Hyari and El-Rayes [32] proposed multi-objective genetic algorithms to solve the trade-off problem between project duration and crew continuity for repetitive construction problems. Hegazy and Wassef [33] analysed the time-cost trade-off for various crew options with different durations and costs of work execution.

Tserng and Lin [34] integrated the problem of subcontractor selection with scheduling. They developed a web-based decision support system (Accelerated Subcontracting And Procuring, ASAP) that enables the analysis of trade-offs between risk and profit for different combinations of subcontractors. Biruk *et al.* [35] proposed a mathematical model for selecting subcontractors whose availability is defined by time windows. Subcontractors' bids include the time and cost of completing the contracted work packages. The mixed integer-binary linear programming model enables the performance of time/cost trade-off analysis. Jaśkowski [36] developed a model for solving the problem of bi-criteria optimisation of the structure of a project execution system, which can be used to select the best options for process execution methods and as a basis for deciding whether to engage subcontractors. The model was solved using a metaheuristic algorithm.

The models of the subcontractor selection problem presented in the literature seem to neglect the fact that subcontractors may offer lower prices if they are entrusted a larger number of work packages, or if they can count on a regular, or even continuous work for a particular

general contractor. Attempts to consider such a practical aspect of subcontracting were limited to the perspective of an individual projects. A proposal for modelling the issue of commissioning a larger number of work packages belonging to a number of projects by the same GC was presented by Afshar et al. [31]. They considered the possibility of recruiting subcontractors on a full-time basis, but did not analyse if the work could be alternatively entrusted to in-house crews. This model did not considered effects of potential financial incentives to speed up delivery of packages – such incentives would require analysing options of bids.

In practice, GCs explore any cost-saving option and negotiate the terms of subcontracts. Their procurement includes the following steps: packaging works and specifying requirements per package, preparing a shortlist of potential subcontractors based on case-relevant criteria, issuing invitations to tender, evaluating bids, negotiations, and finally concluding subcontracts on the basis of the best and final offers [12]. Negotiations provide the opportunity for adjusting contract conditions, including the price [37] in search for trade-offs acceptable by both parties. For instance, the GC may ask the subcontractor to reduce prices in exchange for entrusting them with a larger number of packages: the subcontractor would benefit from the economies of scale and supplementing their order portfolio with larger order. Since the negotiating parties strive for maximizing their own profits, the process might be lengthy. Negotiations involve analysing how proposed terms would affect the project outcomes – from the point of the GC, the project duration, budget, and quality. To formulate proposals and assess subcontractor counterproposals, the GC needs more than experience: analysis can be accelerated using reliable decision-support tools.

3. Assumptions and problem description

The proposed approach is intended to assist the general contractor at the stage of analysing bids for work packages received from previously approved subcontractors. Allocation of in-house and subcontracted crews to work packages of all projects of the GC's portfolio is crucial for operational management and preparing schedules that meet project milestone dates agreed with clients. It is not possible to prepare detailed project schedules or determine the costs without allocating resources. Assuming that subcontractor selection process is time-consuming and cannot be completed before the GC's tenders for the clients are due, the GC relies on own estimates of the time and cost of their project to determine the tender price and the timeframe for the execution of the packages.

Let us assume that, within a predefined planning horizon $[0, H]$, the GC is to deliver a set of work packages $V = \{1, 2, \dots, n\}$. These belong to projects commissioned by different clients, and represent processes that belong to different projects. Let $E \subset V \times V$ be a relation defining the precedence between packages across the portfolio. They arise from the logic of works within each project (intra-project relationships) and the time windows for packages if set as milestones (inter-project relationships); they do not involve any resource availability constraints. Therefore, the scope of works within the portfolio and their sequence can be represented by a directed, acyclic and consistent unigraph $G = \langle V, E \rangle$ with no loops, with a single dummy start and a single dummy finish node.

Each package can be constrained by a time window for execution that results from programmes of particular projects as agreed with clients. Such time window for package i is described by two dates: the earliest possible start of the package es_i , and the latest allowable finish lf_i to meet contractual milestones. These dates are crucial input for the subcontractors – on their basis, the subs verify their resource availability and decide whether to bid for a package.

Each process (work package) $i \in V$ will be delivered by one resource of set C_i . The elements of this set are subcontractors who, having passed the pre-qualification procedure, submitted a bid for this package, plus the GC's own crews capable of completing this package. The set of all subcontractors interested in any of the packages is SC , and the GC's own crews form another set, GCC .

Let us assume that the GC encourages the subcontractors to present alternative bids (variants) that cover different sets of packages, and not just a one. The bids submitted by subcontractors $s \in C_i \cap SC$ declare the duration $t_{i,s,w}$ and the price $k_{i,s,w}$ for work package $i \in V$ according to variant $w \in W_{s,i}$ ($W_{s,i}$ is a set of variants – alternative bids submitted by subcontractor s that include package i). To compare with subcontractor bids, the GC calculates the duration $t_{i,c}$ and cost $k_{i,c}$ of delivering this package using capable in-house resources $c \in C_i \cap GCC$.

The decision on allocating a particular subcontractor to a particular package is modelled using binary variables $x_{i,s,w} \in \{0, 1\}$. Its value is 1 if package i is entrusted to subcontractor s on the basis variant w of their bid; otherwise, it is set to 0. Similarly, binary variable $y_{i,c} \in \{0, 1\}$ models the decision to allocate an in-house crew to a particular process: it equals 1 if package i is to be delivered by crew c , and 0 if otherwise.

The unknown start and finish dates of package i are denoted by s_i and f_i , respectively. – The duration (t_i) and cost (k_i) of each package depend on the resource assigned to it.

If resource $k \in C_i \cap C_j$ is selected to deliver a pair of processes (i, j) , it cannot deliver them at the same time. To model this constrain, a binary variable, $P_{i,j,k}$, is introduced; it equals 1 if package i precedes package j if both executed by the same resource k .

While analysing the selection of packages to be awarded to a given subcontractor, the GC is likely to focus on subcontractor prices. We assume that the GC might want to negotiate a price discount of d_s (expressed as a part of the bid price) if the number of packages the subcontractor s wins exceed a predefined number of NP_s , and would like to analyse how a hypothetical discount would affect configuration of resources and thus the schedule and budget. Experimenting with various values of d_s would help set targets for price negotiations. Therefore, another binary variable D_s is introduced. It equals 1 if there is a basis for considering a discount (i.e. number of packages for a subcontractors s is greater than NP_s), and 0 otherwise.

4. The model

The mathematical model includes an objective function and constraints expressed as linear relationships between decision variables. It also takes into account parameters – constant values describing, among other things, work completion times, costs, etc. – determined on the basis of, for example, subcontractors' bids. The following notations of the model parameters are used in the mathematical formulation:

H	the planning horizon,
V	the set of work packages (processes) within the project portfolio,
n	the number of work packages,
E	the set of pairs of packages related by direct precedence relationships,
G	the graph representing the scope of works scheduled for the entire planning horizon,
es_i	the earliest possible start of package i ,
lf_i	the latest acceptable finish of package i ,
C_i	the set of resources competent to deliver package $i \in V$,
SC	the set of all subcontractors that bid for any package,
GCC	the set of all in-house crews,
$W_{s,i}$	the set of variants of delivering package i by subcontractor s (i.e. the set of bids presented by subcontractor s that include package i),
$t_{i,s,w}$	the duration of package i if entrusted to subcontractor s according to variant of their bid $w \in W_{s,i}$,
$k_{i,s,w}$	the cost (price to be paid by GC) of package i if entrusted to subcontractor s according to variant of their bid $w \in W_{s,i}$,
$t_{i,c}$	the duration of package i if entrusted to in-house crew c ,
$k_{i,c}$	the cost of package i if entrusted to in-house crew c ,
NP_s	the number of work packages that involves a price discount offered by the subcontractor s ,
d_s	the rate of desired discount to be the target of price negotiation between the GC and a subcontractor s if the number of his work packages is greater than NP_s ,
A	the minimum value of packages to be delivered by in-house crews within the planning horizon,
M	a sufficiently large number.

The decision variables assuming non-negative values:

s_i	the start date of package i ,
$s_{i,r}$	the start date of package i delivered by resource r ,
f_i	the finish date of package i ,
t_i	the duration of package i ,
k_i	the cost of package i ,
$x_{i,s,w}$	a binary variable that models the decision to entrust package i to subcontractor s according to variant w of their bid,
$dx_{i,s,w}$	an auxiliary binary variable that models the decision to entrust package i to subcontractor s according to variant w of their bid with the discount,
$y_{i,c}$	a binary variable that models the decision to entrust package i to in-house crew c ,
$P_{i,j,r}$	a binary variable that models the sequence of package i and j if both delivered by the same resource r ,
D_s	a binary variable that models the fact of granting the discount related with entrusting more than NP_s packages to subcontractor s ,
dd_s	an auxiliary binary variable representing the discount rate offered by subcontractor s .

A subcontractor delivers a package of works according to a particular variant of their bid, therefore:

$$(4.1) \quad \sum_{w \in W_{s,i}} x_{i,s,w} \leq 1, \quad \forall i \in V, \forall s \in SC \cap C_i$$

Each package can be allocated only one resource, either a subcontractor or an in-house crew:

$$(4.2) \quad \sum_{s \in SC \cap C_i} \sum_{w \in W_{s,i}} x_{i,s,w} + \sum_{c \in GCC \cap C_i} y_{i,c} = 1, \quad \forall i \in V$$

The duration of a particular package depends on who delivers it and is calculated as follows:

$$(4.3) \quad t_i = \sum_{s \in SC \cap C_i} \sum_{w \in W_{s,i}} t_{i,s,w} \cdot x_{i,s,w} + \sum_{c \in GCC \cap C_i} t_{i,c} \cdot y_{i,c}, \quad \forall i \in V$$

Packages must be delivered within their allowable time windows to assure that no project due dates are exceeded:

$$(4.4) \quad s_i \geq es_i, \quad f_i \leq lf_i, \quad \forall i \in V$$

Packages are assumed to be delivered in a continuous manner, thus:

$$(4.5) \quad f_i - s_i \leq t_i, \quad \forall i \in V$$

The package start dates depend on the logic of works defined by the graph modelling the relationships within the GC's portfolio of projects:

$$(4.6) \quad s_i + t_i \leq s_j, \quad \forall (i, j) \in E$$

If resource r is allocated to a pair of packages (i, j) , these packages cannot be executed simultaneously, therefore:

$$(4.7) \quad \sum_{w \in W_{r,i}} x_{i,r,w} + \sum_{w \in W_{r,j}} x_{j,r,w} \leq 1 + P_{i,j,r} + P_{j,i,r}, \quad \forall r \in C_i \cap C_j \cap SC$$

$$(4.8) \quad y_{i,r} + y_{j,r} \leq 1 + P_{i,j,r} + P_{j,i,r}, \quad \forall r \in C_i \cap C_j \cap GCC$$

$$(4.9) \quad P_{i,j,r} + P_{j,i,r} \leq 1, \quad \forall r \in C_i \cap C_j, \forall i, j \in V$$

The sequence of packages (i, j) delivered by resource k and defined using binary variable $P_{i,j,r}$ affects their start and finish dates. In the case of the resource being a subcontractor, the package dates are calculated as follows:

$$(4.10) \quad s_{i,r} + \sum_{w \in W_{r,i}} t_{i,r,w} \cdot x_{i,r,w} \leq s_{j,r} + M(1 - P_{i,j,r}), \quad \forall r \in C_i \cap C_j \cap SC, \forall i, j \in V$$

$$(4.11) \quad s_{i,r} \leq s_i, \quad \forall r \in C_i \cap SC, \forall i \in V$$

$$(4.12) \quad s_{i,r} \leq M \cdot \sum_{w \in W_{r,i}} x_{i,r,w}, \quad \forall r \in C_i \cap SC, \forall i \in V$$

$$(4.13) \quad s_{i,r} \geq s_i - M \cdot \left(1 - \sum_{w \in W_{r,i}} x_{i,r,w} \right), \quad \forall r \in C_i \cap SC, \forall i \in V$$

Similar formulas are used for determining start and finish dates of a process entrusted to an in-house crew:

$$(4.14) \quad s_{i,r} + t_{i,r} \cdot y_{i,r} \leq s_{j,r} + M(1 - P_{i,j,r}), \quad \forall r \in C_i \cap C_j \cap GCC, \forall i, j \in V$$

$$(4.15) \quad s_{i,r} \leq s_i, \quad \forall r \in C_i \cap GCC, \forall i \in V$$

$$(4.16) \quad s_{i,r} \leq M \cdot y_{i,r}, \quad \forall r \in C_i \cap GCC, \forall i \in V$$

$$(4.17) \quad s_{i,r} \geq s_i - M \cdot (1 - y_{i,r}), \quad \forall r \in C_i \cap GCC, \forall i \in V$$

If resource r is not involved in execution of package i , $s_{i,r}$ equals 0.

Decision to entrust more than NP_s packages to subcontractor s implies a discount:

$$(4.18) \quad \sum_{i \in V} \sum_{w \in W_{s,i}} x_{i,s,w} \geq NP_s - M(1 - D_s), \quad \forall s \in C_i \cap SC, \forall i \in V$$

$$(4.19) \quad \sum_{i \in V} \sum_{w \in W_{s,i}} x_{i,s,w} \leq NP_s + M \cdot D_s, \quad \forall s \in C_i \cap SC, \forall i \in V$$

The binary variable representing the discount dd_s (its values being either 0 or d_s) can be introduced using the following formulas:

$$(4.20) \quad dd_s \leq d_s, \quad \forall s \in SC$$

$$(4.21) \quad dd_s \leq M \cdot D_s, \quad \forall s \in SC$$

$$(4.22) \quad dd_s \geq d_s - M \cdot (1 - D_s), \quad \forall s \in SC$$

Since dd_s – in this form – is a decision variable, using it as a multiplier to reduce subcontractors' tender prices in the formula for calculating the cost of works would result in the formula being non-linear. For this reason, additional binary variables are introduced $dx_{i,s,w} \in \{0, 1\}$; they equal 1 if package i goes to subcontractor s according to bid variant w modified by the discount, and 0 otherwise. The values of these variables are determined on the basis of the following formulas:

$$(4.23) \quad dx_{i,s,w} \leq 1 - d_s \cdot D_s, \quad \forall s \in C_i \cap SC, \forall i \in V, \forall w \in W_{s,i}$$

$$(4.24) \quad dx_{i,s,w} \leq M \cdot x_{i,s,w}, \quad \forall s \in C_i \cap SC, \forall i \in V, \forall w \in W_{s,i}$$

$$(4.25) \quad dx_{i,s,w} \geq 1 - d_s \cdot D_s - M \cdot (1 - x_{i,s,w}), \quad \forall s \in C_i \cap SC, \forall i \in V, \forall w \in W_{s,i}$$

The cost of a package can be thus expressed as:

$$(4.26) \quad k_i = \sum_{s \in SC \cap C_i} \sum_{w \in W_{s,i}} k_{i,s,w} \cdot dx_{i,s,w} + \sum_{c \in GCC \cap C_i} k_{i,c} \cdot y_{i,c}, \quad \forall i \in V$$

The general contractor strives to maximise profits not only by reducing the costs of subcontracted work, but also by keeping in-house resources occupied. This objective can be incorporated into the model in various ways (for instance, by introducing conditions to minimize idle time of in-house crews and minimize the value of subcontracted work). In this model, the assumption is that the capacity utilization levels of in-house crews over the planning horizon should not drop below levels observed previous periods, A , with the utilization level being expressed by the value of output attributable to in-house crews:

$$(4.27) \quad \sum_{i \in V} \sum_{c \in GCC \cap C_i} k_{i,c} \cdot y_{i,c} \geq A$$

The objective function minimizes the total cost of delivering all packages:

$$(4.28) \quad \min K : K = \sum_{i \in V} k_i$$

The mathematical model of the subcontractor allocation and package scheduling problem encompasses the objective function Eq. (4.28), constraints Eqs. (4.1)–(4.19) and Eqs. (4.23)–(4.27), and conditions on the decision variables being non-negative. By introducing additional constraints and auxiliary variables, the model is brought to a linear form. As such, it can be solved using standard solvers. It can be incorporated into a computerized decision support system, enabling easy modification of parameters (including the discount rate, the number of packages ordered, at which the discount will be negotiated, etc.) and an assessment of the impact of decisions on the cost of completing the portfolio of projects. Considering the complexity of the problem and the large number of parameters, such system would facilitate the general contractor's negotiation process and updating schedules.

Operation of the model should be illustrated using a numerical example. However, the problem is relatively complex, and a clear presentation of inputs and outputs would significantly exceed the editorial constraints on the maximum volume of the paper. Consequently, the authors have elected to provide only a comprehensive description of the model itself.

5. Conclusions

Traditional scheduling systems based on the Critical Path Method frequently fail to support allocation of resources. However, the way the limited resources are managed determines project completion time and cost. This issue is particularly significant for the general contractor, who simultaneously undertakes a number of projects disposing of a particular pool of in-house resources with limited capacities. Subcontracting is a common practice worldwide. However, selecting subcontractors is a complex and risky task as decisions in this respect affect the timeliness, quality, and cost of the work – and thus the success or failure of the general contractor.

Subcontracting decisions affect also the utilization and costs of in-house resources. Therefore, the GC must consider a number of factors: apart from the obvious eligibility (determined by experience, capabilities, financial standing etc.) of potential subcontractors and their price expectations, the general contractor must also find a satisfactory combination

of multiple subcontractors, and mind the demand for work of in-house crews. This is a complex task calling for support with reliable analytical tools, facilitating concurrent analysis of schedules and cost estimates.

The decision support methods presented in the recent literature typically separate the subcontractor selection from scheduling, and do not take into account the simultaneous execution of several contracts. The mathematical model of the problem of allocating resources to work packages of the entire portfolio of orders, presented in this article, assumes that the potential subcontractors offer different prices and durations of packages depending on the combination of their services the GC can choose from. The model provides data necessary for the construction of work schedules and financial plans. By manipulating the model's parameters, the planner can assess the impact of decisions made during the negotiation stage on the cost of executing the order portfolio.

Unlike previous studies, the developed model allows for the analysis of a construction company's entire order portfolio and assists the GC in deciding whether to outsource specific work packages, thereby ensuring maximum utilization of in-house crews and timely completion of orders at minimal cost. The model in the form presented in the paper is just a mathematical formulation based on a set of assumptions that do originate from the construction practice, but certainly do not cover all aspects of the problem, which are worth considering in the decision-making process, but which require more complex mathematical formulations. Therefore, further work is needed to refine it – in the first place, to consider financial losses associated with idle time of in-house and subcontracted resources. Future work includes developing an algorithm and building a user-friendly computer application to facilitate subcontractor selection for project portfolios.

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Wspomaganie doboru podwykonawców przy realizacji zleceń przedsiębiorstwa budowlanego

Słowa kluczowe: obór podwykonawców, harmonogramowanie procesów budowlanych, alokacja zasobów, modelowanie matematyczne, wspomaganie decyzji generalnego wykonawcy

Streszczenie:

Przedsiębiorstwa budowlane zaangażowane w realizację jednocześnie kilku przedsięwzięć budowlanych na rzecz różnych inwestorów nie są w stanie samodzielnie realizować całego zakresu robót. Wynika to zarówno z ograniczeń w dostępności własnych brygad, jak i braku wykwalifikowanej specjalistycznej kadry roboczej. Proces selekcji podwykonawców jest czasochłonny i złożony, dlatego jest często przeprowadzany w trakcie realizacji zleceń przez generalnego wykonawcę (GW). Dobór podwykonawców wpływa na czas, koszt oraz jakość realizacji robót i wymaga wspomagania narzędziami pozwalającymi na uwzględnienie ograniczeń w zakresie terminów dyrektywnych wykonania pakietów robót oraz terminów ustalonych w kontraktach. Klasyczne procedury przetargowe, nawet ograniczone do wybranego kręgu rzetelnych podwykonawców, nie pozwalają na modyfikację złożonych ofert, w celu dostosowania ich do potrzeb GW. W artykule zaproponowano model matematyczny wspomagający proces podejmowania decyzji o zlecaniu pakietów robót i przeprowadzania negocjacji z podwykonawcami, po przeprowadzeniu procedury prekwalifikacji podwykonawców. Uwzględniono w nim możliwość analizy wariantowych ofert oraz negocjowania upustu przy powierzaniu jednemu podwykonawcy większej liczby pakietów robót. Zaproponowane podejście pozwala na minimalizację kosztu realizacji zleceń przez przedsiębiorstwo w przyjętym okresie planowania, ale jednocześnie zapewnia wysoki stopień wykorzystania własnego

potencjału wykonawczego, tak aby zapewnić front robót dla zatrudnionych brygad roboczych poprzez ustalenie minimalnej wartości robót realizowanych własnymi siłami. Opracowany model pozwala na alokację zasobów GW oraz określenie harmonogramu robót dla ustalonego horyzontu planowania.

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