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Research paper

Critical temperature evaluation for steel column exposed to fire – Rankine–Merchant approach

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Abstract: An alternative computational approach to the critical temperature specification for a steel column, related to the fire conditions and associated with the loss of the capacity to safely resist the loads applied to the said column is presented and discussed in detail. The algorithm recommended by the authors has been derived from the empirical Rankine–Merchant rule, with parameters calibrated so as to obtain quantitative agreement with critical temperature estimates arrived at after application of the conventional code-based approach. It has been indicated, that this approach results in an iterative calculation. Knowledge of the critical temperature determined for given structural component or a substructure under assumed fire development scenario allows for identification of its fire resistance interpreted as the forecast time of reliable service when subjected to the influence of high temperature. Numerical example presented in this paper pertains to the column devoid of any restrictions in elongation, not fire protected and heated uniformly around the whole perimeter and along the whole length. To simplify the example it has been assumed, that at any given moment of fire steel temperature is constant in the column cross-section and increases with increasing temperature of surrounding fire plume. The approach proposed by the authors allows for verification, in both qualitative and quantitative terms, of the influences exerted by geometrical imperfections of various origins on the final critical temperature specifications, in particular those related to a potential eccentricity of load application point as well as those induced by the longitudinal axis of column deviating from straightness.

Keywords: critical temperature, fire exposure, fire resistance, iterative procedure, Rankine–Merchant rule, steel column

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1. Introduction

In 1866 W.J.M. Rankine formulated an empirical rule, according to which elastic-plastic bearing capacity of a steel column ($N_{\text{R–M}}$) may be calculated based on its separated bearing capacities, related to purely elastic (N_{el}) and purely plastic (N_{pl}) destruction models, while the quantitative relation between these models is determined by the following rule:

$$\frac{1}{N_{\text{R–M}}} = \frac{1}{N_{el}} + \frac{1}{N_{pl}} \tag{1.1}$$

The purely elastic bearing capacity is in this approach specified by the critical bearing capacity according to Euler, and thus:

$$N_{el} = \frac{\pi^2 EJ}{L_e^2} \tag{1.2}$$

The purely plastic bearing capacity, however, is expressed by perfectly plastic destruction of the cross-section, thus yielding:

$$N_{pl} = f_y A \tag{1.3}$$

The buckling coefficient is expressed in the approach of this type by the following formula:

$$\chi_{\text{R–M}} = \frac{N_{\text{R–M}}}{N_{pl}} \tag{1.4}$$

As a consequence, when taking (1.1) into account, one gets:

$$\frac{1}{\chi_{\text{R–M}}} = 1 + \frac{f_y}{\pi^2 E}\frac{L_e^2}{i^2} = 1 + \overline{\lambda}^2 \tag{1.5}$$

and this in turn is equivalent to the following:

$$\chi_{\text{R–M}} = \left(1 + \overline{\lambda}^2\right)^{-1} \tag{1.6}$$

The formula (1.1) was generalized in 1954 by W. Merchant as authoritative when specifying the elastic-plastic bearing capacity of self-stable steel frames. However, this formula has been verified many times as suitable for application during analysis of fire development scenarios (among others in [1–5] when related to columns, while in [6–8] when related to frames). Under those circumstances the following occurs:

$$N_{el,fi,\theta} = \frac{\pi^2 k_{E,\theta} EJ}{L_e^2} \quad \text{as well as} \quad N_{pl,fi,\theta} = k_{y,\theta} f_y A \tag{1.7}$$

Therefore, after taking into account, that:

$$\chi_{\text{R–M},fi,\theta} = \frac{N_{\text{R–M},fi,\theta}}{N_{pl,fi,\theta}} \tag{1.8}$$

one obtains:

$$\frac{1}{\chi_{\text{R–M},fi,\theta}} = 1 + \frac{k_{y,\theta} f_y}{\pi^2 k_{E,\theta} E} \frac{L_e^2}{i^2} = 1 + \overline{\lambda_\theta}^2 \tag{1.9}$$

in turn leading to the formula [9, 10]:

$$\chi_{\text{R–M},fi,\theta} = \left(1 + \overline{\lambda_\theta}^2\right)^{-1} \tag{1.10}$$

The subscript *fi* (*fire*) denotes here reference to the fire development scenario, while the subscript θ indicates the dependence of associated factor on the steel temperature changing during the fire. The coefficient $k_{y,\theta}$ represents yield limit reduction level in steel under fire conditions, while the coefficient $k_{E,\theta}$ denotes analogous reduction level referring to the linear elasticity modulus of this material.

However, buckling coefficients determined this way are much less restrictive, than the analogous coefficients determined using conventional procedure, based on recommendations contained in the code EN 1993-1-2 [11], where:

$$\chi_{\text{EN},fi,\theta} = \frac{1}{\phi_{0,\theta} + \sqrt{\phi_{0,\theta}^2 - \overline{\lambda_\theta}^2}} \tag{1.11}$$

with:

$$\phi_{0,\theta} = 0.5\left(1 + \alpha\overline{\lambda_\theta} + \overline{\lambda_\theta}^2\right) \tag{1.12}$$

at:

$$\alpha = 0.65\sqrt{\frac{235}{f_y}} \tag{1.13}$$

as well as:

$$\overline{\lambda_\theta} = \overline{\lambda}\left(\frac{k_{y,\theta}}{k_{E,\theta}}\right)^{0.5} \tag{1.14}$$

This paper aims at proper calibration of basic parameters of the procedure derived from the Rankine–Merchant formula, alternative to the conventional algorithm prescribed by the code [11], in order to obtain the buckling coefficients $\chi_{\text{R–M},fi,\theta}$ quantitatively corresponding to the coefficients $\chi_{\text{EN},fi,\theta}$. In such approach the procedure prescribed by the code is treated as a reference algorithm, sufficiently formally verified and the fitting is obtained through the modeling of substitute geometrical imperfections related to the eccentric application of compressive loads and the longitudinal axis of analyzed element deviating from straight line.

2. Buckling coefficients specified for a column deviating from perfect geometry

A substitute geometric imperfection is assumed and modelled as an initial curvature of the column axis having magnitude of e in order to more precisely estimate the bearing capacity $N_{pl,fi,\theta}$. When following such approach, the column is subjected to not only axial compression but also to bending with bending moment magnitude expressed as $N_{pl,fi,\theta}e$. This denotes an M-N interaction, in the simplest form expressed by a linear formula:

(2.1)
$$\frac{N_{pl,fi,\theta}}{k_{y,\theta} f_y A} + \frac{N_{pl,fi,\theta} e}{k_{y,\theta} f_y W_{pl}} = 1$$

which is equivalent to:

(2.2)
$$\frac{N_{pl,fi,\theta}}{k_{y,\theta} f_y A} = \left(1 + \frac{eA}{W_{pl}}\right)^{-1}$$

Such computational model leads to a correction in the equation (1.10) according to [9, 10]:

(2.3)
$$\chi_{\text{R–M},fi,\theta} = \left(1 + \frac{eA}{W_{pl}} + \overline{\lambda_\theta}^2\right)^{-1}$$

The experimental research, especially that reported in [1–5], leads to the recommendation of a slight correction in the formula (2.3), as within the scope of the following formula is postulated:

(2.4)
$$\frac{N_{pl,fi,\theta}}{k_{y,\theta} f_y A} + \frac{N_{pl,fi,\theta} e}{1.125 k_{y,\theta} f_y W_{pl}} = 1$$

Based on that the following is obtained:

(2.5)
$$\chi_{\text{R–M},fi,\theta} = \left(1 + \frac{eA}{1.125 W_{pl}} + \overline{\lambda_\theta}^2\right)^{-1}$$

When specifying the bearing capacity $N_{el,fi,\theta}$ the eccentricity of compressive load application point has been taken into account, expressed as ε, as shown in the Fig. 1.

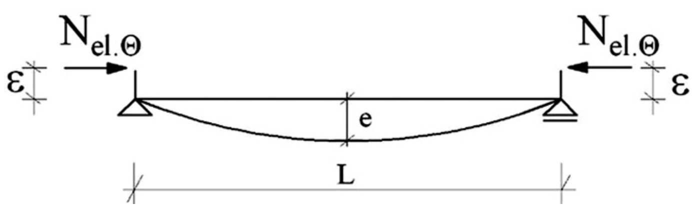

Fig. 1. A model of the substitute geometrical imperfection applied to specify the bearing capacity $N_{el,fi,\theta}$

Solution of an appropriate differential equation in this case leads to the formula [8]:

$$e = \varepsilon\,(\sec\beta - 1) \tag{2.6}$$

where:

$$\beta = \frac{\pi}{2}\sqrt{\frac{N_{el,fi,\theta}}{N^*_{el,fi,\theta}}} \tag{2.7}$$

at:

$$N^*_{el,fi,\theta} = N_{cr,fi,\theta} = \frac{\pi^2 k_{E,\theta} EJ}{L_e^2} \tag{2.8}$$

Based on that the value of $N_{el,fi,\theta}$, corrected with respect to the analogous value determined for the column of perfect geometry, yields:

$$N_{el,fi,\theta} = \left[\frac{2}{\pi}\cos^{-1}\left(\frac{\varepsilon}{\varepsilon + e}\right)\right]^2 N^*_{el,fi,\theta} = \xi^2 N^*_{el,fi,\theta} \tag{2.9}$$

Thus finally:

$$\chi_{\text{R–M},fi,\theta} = \left(1 + \frac{eA}{1.125W_{pl}} + \frac{\overline{\lambda_\theta}^2}{\xi^2}\right)^{-1} \tag{2.10}$$

Detailed comparative analysis of the buckling factors $\chi_{\text{R–M},fi,\theta}$ conducted using various parameters describing substitute geometric imperfection juxtaposed with appropriate coefficients $\chi_{\text{EN},fi,\theta}$ has been presented and discussed by the authors of this paper in [12].

3. Iterative procedure to determine the critical temperature of a column

The critical temperature θ_{cr} determined for a steel column is understood as such temperature of steel the column is made of, at which the reduction in bearing capacity under fire exposure proves to be sufficiently important, as to preclude the safe resistance of the considered column to the loads applied to it. When a fire development scenario forecast for given fire compartment is assumed, this temperature may be unequivocally associated with fire resistance warranted for given column, i.e. with the duration of fire taken as a period of time from fire initiation to the moment when the considered element fails to safely resist the loads applied to it (i.e. reaches the limit state of bearing capacity in fire). Usually a code formula [11] is applied to estimate this temperature. According to this formula the following occurs:

$$\theta_{cr} = 39.19\ln\left[\frac{1}{0.9674\mu_0^{3.833}} - 1\right] + 482^\circ\text{C} \tag{3.1}$$

In the above formula the dimensionless parameter μ_0 represents the bearing capacity utilization indicator, determined with respect to the steel elements subjected to fire actions and having sections assigned to the 1st, 2nd or 3rd class (according to [11]) based on the ratio:

$$\mu_0 = \frac{E_{fi,d,t}}{R_{fi,d,0}} \tag{3.2}$$

where the symbol $E_{fi,d,t}$ represents the design value of authoritative load effect, determined for the fire duration time t_{fi}, while the symbol $R_{fi,d,0}$ denotes design value of bearing capacity of the considered element formally corresponding to this effect, however determined for the fire initiation moment, that is for $t_{fi} = 0$. Thus the formula listed above, under such approach, does not take into account the reduction of bearing capacity induced by the action of high temperature on the affected steel component. Should the affected steel column preserve, during the whole duration of a fire, the capability of unrestrained deformations induced by thermal expansion of steel, one may assume that $E_{fi,d,t} = E_{fi,d,0} = \text{const}_t$. This is an equivalent to the assumption, that $\mu_0 \left(t_{fi}\right) = \mu_0 = \text{const}_t$. Nevertheless, one should remember, that effect $E_{fi,d,t}$ is always determined with respect to the rules of accidental design situation, and therefore it is quantitatively different from the analogous effect determined for the same column with respect to the persistent design situation.

The main advantage of the formula (3.1) lies in the simplicity of required calculations. However, sometimes this formula may not be applied directly. Under many design scenarios such direct application of (3.1) may lead to a grossly overestimated value of the sought temperature, in turn leading to the overly optimistic forecast of safety level guaranteed to the user. It is indicated in paper [13], that an iterative procedure has to be applied whenever under fire scenario the reduction in the element bearing capacity is disproportional to the corresponding reduction in the strength of steel grade used to make the considered structural component. Precisely this case occurs when the critical temperature of a steel column is specified, as the safety condition in such case has the following form:

$$\mu_0 \leq \frac{R_{fi,d,t}}{R_{fi,d,0}} = k_{y,\theta} \frac{\chi_{fi,\theta}}{\chi_{fi,\theta}^{(20)}} \tag{3.3}$$

In spite of the fact, that the buckling coefficient $\chi_{fi,\theta}^{(20)}$ refers here to the accidental fire situation, it is determined under the assumption, that $\theta = 20°\text{C}$. Thus it remains constant during the whole time of fire exposure. When the thermal expansion remains unrestricted the parameter μ_0 remains constant as well. This means, that in the subsequent steps of the iterative procedure the value of the reduction coefficient $k_{y,\theta}$, listed for given steel temperature, has to be adjusted according to the below rule:

$$k_{y,\theta} \geq \mu_0 \frac{\chi_{fi,\theta}^{(20)}}{\chi_{fi,\theta}} \tag{3.4}$$

4. Computational example

A 6.0 m high axially compressed column made of S235 steel on articulated support at the bottom and sliding articulated support at the top subjected to the action of high temperature under fire conditions has been selected for detailed analysis. The freedom of axial movement at the top support allows for unrestrained thermal expansion of the column in the vertical direction, and thus additional internal forces are not introduced in the column, therefore one may assume that the load effect $E_{fi,d,t}$ does not change during the whole fire action. The core of the column was made of a HEB 180 I beam, having the cross section of $A = 65.3\ \text{cm}^2$. This is a class 1 component under both accidental and persistent design situations. Under presented assumptions the relative slenderness of the column at 20°C, following (1.14), equals $\overline{\lambda}_\theta^{(20)} = 1.398$. It is assumed, that an axial vertical force comprising of $G_k = 100$ kN permanent load and variable service load of $Q_k = 250$ kN is applied to the column. Under persistent design situation this yields a representative design effect of $E_d = 1.35 \cdot 100 + 1.5 \cdot 250 = 510$ kN, while under accidental design situation of a fully developed fire a quantitatively different design effect is obtained, equal to $E_{fi,d,t} = 100 + 0.6 \cdot 250 = 250$ kN, under the assumption that $\psi_{fi} = \psi_{2,1} = 0.6$ [14]. Thus, in this case, the suitable conversion coefficient is equal to $\eta_{fi} = 250/510 = 0.490$. One may prove, that the considered column is capable to safely resist the loads applied to it under persistent design situation, as then the following holds:

$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{510}{520.21} = 0.98 < 1 \tag{4.1}$$

However, under fire conditions, after application of the code formula (1.11), one gets $\chi_{\text{EN},fi,\theta}^{(20)} = 0.306$, leading to the following specification:

$$N_{\text{EN},b,fi,\theta,Rd}^{(20)} = 0.306 \cdot 65.3 \cdot 10^{-4} \cdot 235 \cdot 10^3 / 1.0 = 469.57\ \text{kN} \tag{4.2}$$

Should one now assume:

$$\mu = \frac{N_{Ed}}{N_{\text{EN},b,fi,\theta,Rd}^{(20)}} = \frac{510}{469.57} = 1.086 > 1 \tag{4.3}$$

then, after application of the persistent design situation rules to arrive at the representative load effect, the bearing capacity of the column would prove to be insufficient to safely resist the compressive force applied to it even at room temperature. However, under accidental conditions the coefficient $\mu_0 = \eta_{fi}\mu = 0.490 \cdot 1.086 = 0.532 < 1$ is authoritative when a safety analysis is conducted. When this value is entered in the formula (3.1) one gets $\theta_{\text{cr,EN}} = 574.6$°C. Nevertheless this value is not a proper solution of the problem posed here, but only a first approximation of this solution. Based on (3.4) the value of the reduction coefficient $k_{y,\theta}^{(574.6)}$ should be corrected, and after taking into account $\chi_{\text{EN},fi,\theta}^{(574.6)} = 0.237$ one gets:

$$k_{y,\theta}^{(574.6)} \geq 0.532 \frac{0.306}{0.237} = 0.687 \Rightarrow \theta_{\text{cr,EN}} = 530.0^\circ\text{C} \tag{4.4}$$

At this temperature $k_{y,\theta}^{(530.0)} = 0.513$ and $\chi_{\mathrm{EN},fi,\theta}^{(530.0)} = 0.249$ occur, in turn leading to the specification:

$$N_{\mathrm{EN},b,fi,\theta,Rd}^{(530.0)} = 0.249 \cdot 65.3 \cdot 10^{-4} \cdot 0.687 \cdot 235 \cdot 10^{3}/1.0 \\ = 262.5\ \mathrm{kN} > N_{E,fi,d} = 250\ \mathrm{kN} \tag{4.5}$$

Thus the critical temperature determined at this iteration is underestimated. After subsequent application of the iterative procedure one gets:

$$k_{y,\theta}^{(530.0)} \geq 0.532\frac{0.306}{0.249} = 0.654 \Rightarrow \theta_{\mathrm{cr,EN}} = 540.6^{\circ}\mathrm{C} \tag{4.6}$$

yielding $k_{y,\theta}^{(540.6)} = 0.482$ and $\chi_{\mathrm{EN},fi,\theta}^{(540.6)} = 0.247$, therefore:

$$N_{\mathrm{EN},b,fi,\theta,Rd}^{(540.6)} = 0.247 \cdot 65.3 \cdot 10^{-4} \cdot 0.654 \cdot 235 \cdot 10^{3}/1.0 \\ = 247,9\ \mathrm{kN} < N_{E,fi,d} = 250\ \mathrm{kN} \tag{4.7}$$

So here an overly optimistic estimate of the critical temperature has been obtained. During the next iteration the following yields:

$$k_{y,\theta}^{(540.6)} \geq 0.532\frac{0.306}{0.247} = 0.659 \Rightarrow \theta_{\mathrm{cr,EN}} = 539.0^{\circ}\mathrm{C} \tag{4.8}$$

Based on that $k_{y,\theta}^{(539.0)} = 0.487$ and $\chi_{\mathrm{EN},fi,\theta}^{(539.0)} = 0.247$, leading in turn to:

$$N_{\mathrm{EN},b,fi,\theta,Rd}^{(539.0)} = 0.247 \cdot 65.3 \cdot 10^{-4} \cdot 0.659 \cdot 235 \cdot 10^{3}/1.0 \\ = 249,8\ \mathrm{kN} < N_{E,fi,d} = 250\ \mathrm{kN} \tag{4.9}$$

In the authors' opinion the estimate of $\theta_{\mathrm{cr,EN}} = 539.0^{\circ}\mathrm{C}$ may be considered sufficiently accurate, as the difference between the column bearing capacity specified for this temperature and the corresponding representative effect of the load applied to the column is sufficiently small.

5. Critical temperature values determined by the Rankine–Merchant rule

The iterative procedure presented in this example allows to determine the critical temperature, both the one denoted as $\theta_{\mathrm{cr,EN}}$ and referring to the conventional code algorithm, as well as the one denoted as $\theta_{\mathrm{cr,R-M}}$ and determined based on the alternative approach using Rankine–Merchant rule. The change in $\theta_{\mathrm{cr,EN}}$ following the change in the initial relative slenderness of the column $\overline{\lambda}_{\theta}^{(20)}$ (i.e. that specified at the initial temperature of 20°C) is depicted in Fig. 2. The degree of formally unjustified overestimation of this value resulting from treating the result obtained in the first step of the iterative procedure as reliable instead of performing

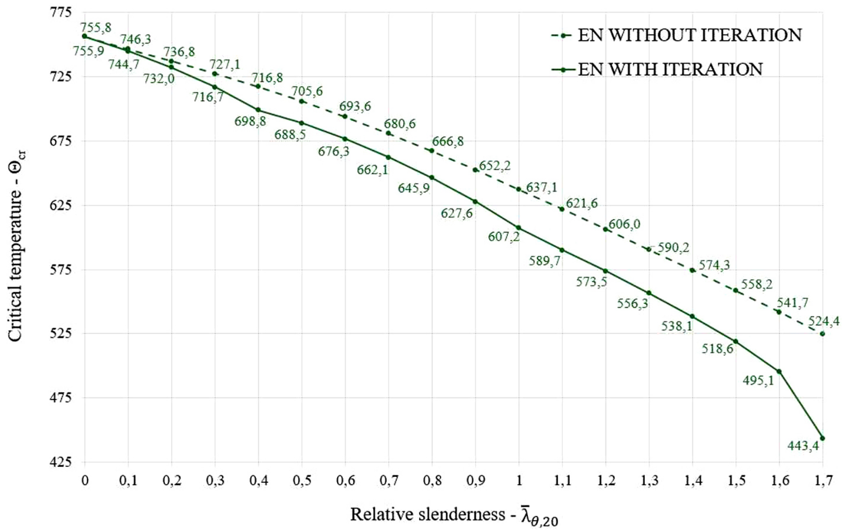

Fig. 2. The critical temperature values $\theta_{\text{cr,EN}}$ of the column considered in the example depending on its relative slenderness $\overline{\lambda_{\theta}^{(20)}}$ determined at 20°C. Continuous line denotes here the results obtained by iterative approach, while the dotted line denotes the formally corresponding results overestimated when this approach is not applied

the appropriate number of iterations required to obtain the convergence is shown there as well. One may easily notice that difference to the detriment of the safety in the degree of safety warranted to the user increases with increasing column slenderness.

Juxtaposition of the relationships presented in Fig. 3 yields a comparison of $\theta_{\text{cr,EN}}$ values with corresponding $\theta_{\text{cr,R-M}}$ values, obtained after application of (1.10) based on the assumption of perfect column geometry. The iterative procedure had been applied to determine both sets of these values, however the quantitative difference between the corresponding values is nevertheless significant. Should one treat the $\theta_{\text{cr,EN}}$ values as formally authoritative, since in authors' opinion as code recommendations these are supported by appropriate experimental verification, then the corresponding $\theta_{\text{cr,R-M}}$ values should be seen as overly optimistic. Therefore, to assure the sufficient level of safety of these estimates, analogous to that associated with $\theta_{\text{cr,EN}}$ estimates the authors of this paper propose an appropriate calibration of the parameters present in the computational model proposed above, based on application of substitute geometrical imperfection to the column subjected to analysis. This finally leads to the replacement of the formula (1.10) by a corresponding formula (2.10). The latter formula highlights the role of this imperfection in differentiation of the $\theta_{\text{cr,R-M}}$ value. Thus the remaining thing here is the determination of the parameters e and ξ to obtain the values of the temperature $\theta_{\text{cr,R-M}}$ quantitatively convergent with the corresponding values of $\theta_{\text{cr,EN}}$. Calculations conducted by the authors seem to indicate that sufficient convergence of results is obtained when $\frac{eA}{W_{pl}} = 0,1$ and $\xi = \sqrt{0.9} = 0.949$ are assumed (as it is shown in Fig. 4).

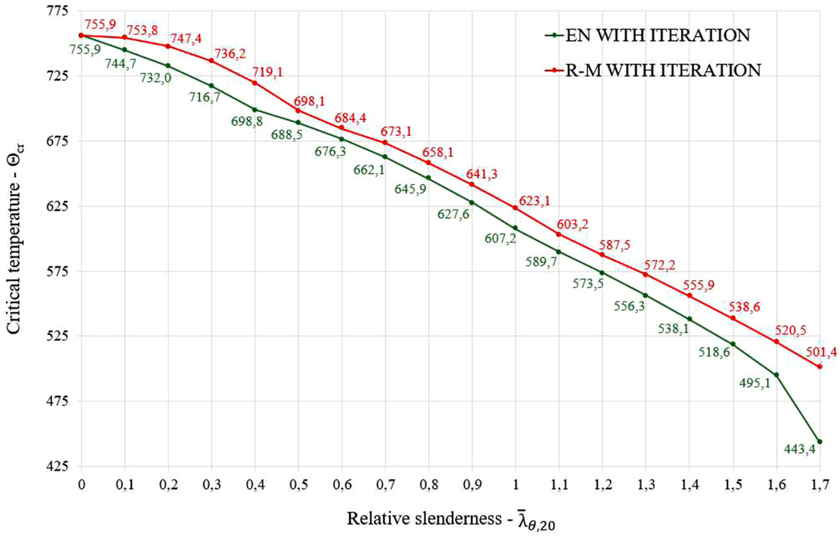

Fig. 3. Critical temperature values determined for the column considered in the example depending on its relative slenderness $\overline{\lambda_{\theta}^{(20)}}$ determined at 20°C, i.e. $\theta_{\mathrm{cr,EN}}$ (in green) and $\theta_{\mathrm{cr,R-M}}$ (in red). The values $\theta_{\mathrm{cr,R-M}}$ have been determined according to (1.10)

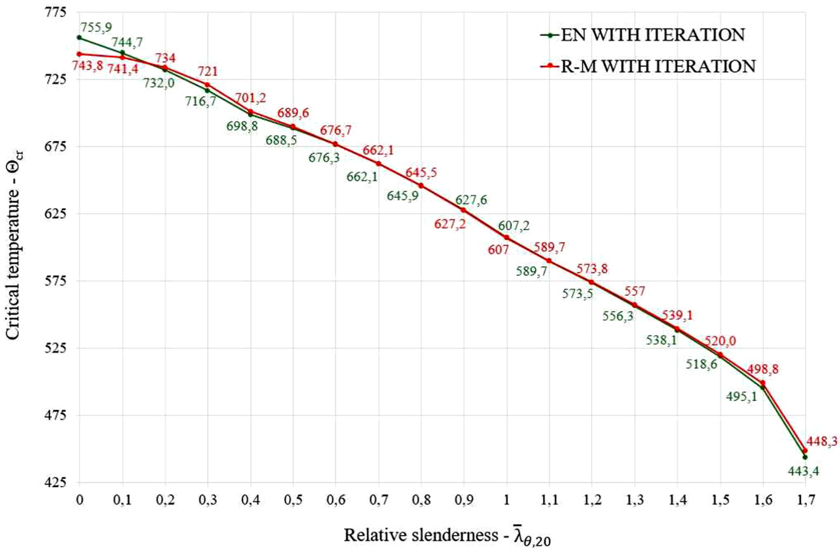

Fig. 4. Values of $\theta_{\mathrm{cr,EN}}$ (in green) and $\theta_{\mathrm{cr,R-M}}$ (in red) with $\theta_{\mathrm{cr,R-M}}$ adjusted by substitute geometrical imperfection following (2.10). Parameters $\frac{eA}{W_{pl}} = 0,1$ and $\xi = \sqrt{0.9} = 0.949$ have been assumed

6. Concluding remarks

It should be kept in mind, that when in the column considered in the example the steel temperature reaches the level of θ_{cr} it will not be immediately destroyed, but only the probability of such random phenomenon occurring becomes sufficiently high, that it can no longer be tolerated by the user. However, this probability depends on how the relevant effect of the loads applied to the column exposed to fire conditions will be determined a'priori. In the code [14] it is a rule that in the case of variable service loads the frequent or quasi permanent components of these loads should be treated as authoritative. Out of these two in Polish conditions the quasi permanent values were usually recommended for application (Fig. 4). However, recently, in point NB.7 of the informative National Annex (NB) associated with the Polish Code PN-EN 1991-1-2:2006 [15], it has been written as follows: "It is recommended that when determining the appropriate effects of actions in an accidental design situation such as a fire, related to the reliable combination of mechanical actions, consideration should be given to the frequent component $\psi_{1,1} Q_1$, interpreted as a representative value of the random variable load Q_1". The alternative assumption that this time $\psi_{fi} = \psi_{1,1} = 0.7$ occurs, as it is usually accepted for typical variable load classified to standard category C (in place of the previous assumption according to which the equality $\psi_{fi} = \psi_{2,1} = 0.6$ has been set), significantly changed the resulting critical temperature estimate identified for the column considered in the example. It is now presented in detail in Fig. 5. As one can see, any change in authoritative load effect specification affects the critical temperature value determined in calculations, in spite of the fact that it is identified for the same column.

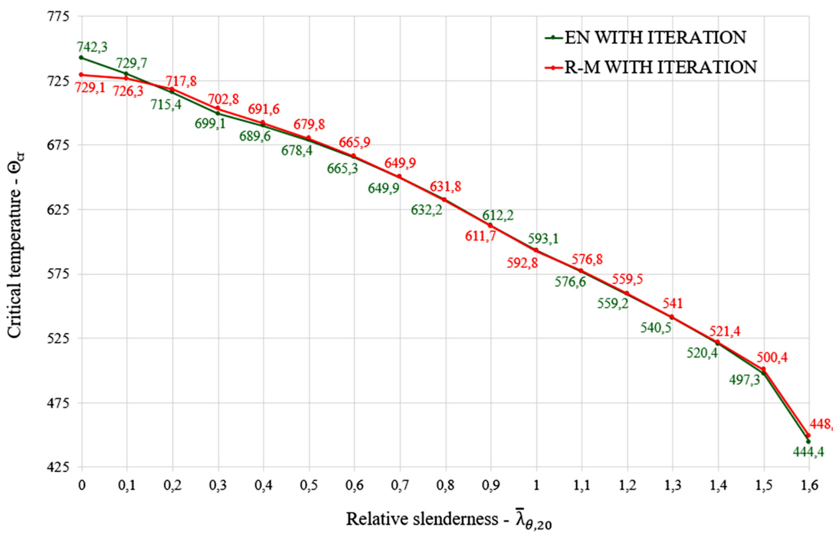

Fig. 5. Values of $\theta_{\text{cr,EN}}$ (in green) and $\theta_{\text{cr,R-M}}$ (in red) identified analogously to those presented previously in Fig. 4, but this time under alternative assumption that $\psi_{fi} = \psi_{1,1} = 0.7$ occurs

The critical temperature determined for a steel column may be considered as an objective measure of its safety in fire. This temperature is unequivocally assigned to the column after taking into account the boundary conditions as well as the level and type of the loads applied. Knowledge of this temperature allows for evaluation of the fire resistance of given column, that is determination of the time during which, beginning with fire initiation, the column will be capable to safely resist the loads applied to it until reaching the limit state of bearing capacity in fire. Of course, this resistance is significantly affected by the fire development scenario forecast.

The critical temperature $\theta_{\mathrm{cr,R-M}}$ determination method proposed by the authors and based on the empirical Rankine–Merchant rule with parameters calibrated so as to obtain the quantitative agreement with corresponding values of $\theta_{\mathrm{cr,EN}}$ determined in an conventional manner, may be treated as an interesting and valuable alternative to the traditional code-based approach. Qualitative and quantitative highlighting of the influences exerted by geometric imperfections of many types on the final results may be considered as a basic advantage of this approach.

The estimates of critical temperature specific for given column will be reliable, regardless of the computational algorithm applied in practice, only if the iterative procedure described in this paper is applied to determine these. Such a process can be terminated when the critical temperature values obtained in subsequent steps differ from each other by tenths of a degree Celsius.

A generalization of the Rankine–Merchant approach to determine the bearing capacity of compressed steel columns under fire conditions, taking into account the geometric imperfections resulting additionally in biaxial bending (with respect to the so-called "strong" as well as the "weak" axis), has been proposed in [16].

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Ocena temperatury krytycznej ogarniętego pożarem słupa stalowego – podejście Merchanta–Rankina

Słowa kluczowe: temperatura krytyczna, oddziaływanie pożaru, odporność ogniowa, procedura iteracyjna, podejście Merchanta-Rankina, słup stalowy

Streszczenie:

Zaprezentowano i szczegółowo przedyskutowano alternatywne podejście obliczeniowe do specyfikacji temperatury krytycznej słupa stalowego, odniesionej do warunków pożaru i skojarzonej z wyczerpaniem możliwości bezpiecznego przenoszenia przez ten słup przyłożonych do niego obciążeń. Rekomendowany przez autorów algorytm postępowania został wyprowadzony z empirycznej reguły Merchanta-Rankina, z parametrami kalibrowanymi w taki sposób, aby po jej zastosowaniu uzyskać ilościową zgodność finalnego wyniku z analogicznymi oszacowaniami temperatury krytycznej, otrzymywanymi po zastosowaniu konwencjonalnego podejścia normowego. Wskazano na konieczność wykorzystania w opisywanych obliczeniach procedury iteracyjnej. Znajomość temperatury krytycznej wyznaczonej dla danego elementu lub podukładu konstrukcyjnego, przy założonym scenariuszu rozwoju pożaru, pozwala na identyfikację jego odporności ogniowej interpretowanej jako prognozowany dla tego scenariusza czas niezawodnej pracy w warunkach oddziaływania wysokiej temperatury. Zamieszczony w artykule przykład obliczeniowy dotyczy słupa ze swobodą realizacji indukowanych termicznie wydłużeń, pozbawionego izolacji przeciwogniowej i ogrzewanego w sposób równomierny z czterech stron, zarówno na całym obwodzie, jak i na całej wysokości. Dla uproszczenia rozważań przyjęto założenie, że w danej chwili pożaru temperatura stali w całym przekroju poprzecznym słupa jest wyrównana i narasta wraz ze wzrostem temperatury otaczających ten słup gazów spalinowych. Podejście proponowane przez autorów daje

możliwość weryfikacji wpływów, tak w sensie ilościowym, jak i jakościowym, jakie na wynikową wartość temperatury krytycznej mają różnego rodzaju imperfekcje geometryczne, w szczególności te kojarzone z ewentualnym mimośrodem przyłożenia obciążeń a także te wynikające z braku prostoliniowości.

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