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Research paper

Hazard function, reliability function and mean time to failure of dams

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Abstract: Dams are among the largest structures constructed by human. Their disasters result in numerous casualties and large material losses. The statistical analysis of the reasons for dam disasters was carried out by the International Commission on Large Dams (ICOLD). The analysis included large dams, i.e. dams that are higher than 15 meters or taller than 5 meters and form a reservoir with a capacity of more than 3×10^6 m^3. ICOLD's study took into account dams built up to 1986 regardless of the type of construction and the material used. The hazard function $h(t)$ and next, the reliability function $R(t)$ were calculated based on that data, using the fitting of the power function in the initial mortality period and a constant function in the useful operation period. The knowledge of the reliability function allowed for calculating the mean time to failure which was 112957±12443 years. It was also demonstrated that in the operation period the annual dam failure ratio is $8.719\times10^{-6}\pm0.297\times10^{-6}$. It is a value that is proximate to the recommendations of the U.S. Army Corps of Engineers which suggests the tolerance for the annual dam disaster risk not to exceed 10^{-6} failures per year for newly-constructed dams and 10^{-4} failures per year for already existing ones.

Keywords: dam, reliability, mean time to failure, hazard function, reliability function

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1. Introduction

Dams are among the largest structures constructed by human. In case of large dams, the energy of the gathered, dammed water several times exceeds the energy of the atomic bombs dropped on Hiroshima and Nagasaki in 1945. The energy released by the explosion of this atomic bomb was equivalent to 16 kilotons of TNT [1] it's about 67 TJ, while the potential energy of water stored in Poland's largest dam Solina is about 152 TJ (Appendix A). That is 2.3 times more than the first atomic bomb. That is why disasters of dams always entail enormous human casualties as well as material losses. For example, the disaster of the Italian dam, Vaiont, of 9 October 1963, caused the death of 2043 people [2]. The disaster of the Hindu dam, Machuchhu, of 11 August 1979, resulted in the death of between 1800 and 25000 people [3, 4]. Therefore, it is very important to ensure that dams are reliable structures.

Many authors e.g. [5–9] underline the importance of the reliability of the structures and of the technical systems, which has significant impact i.a. on their functionality and lifetime, as well as on the safety of the users. Moreover, the identification of the basic reliability parameters is a significant tool supporting the proper maintenance and planning of renovations [10].

Publications referring to the reliability of dams may be divided into two groups. The first group includes an analysis of the factors that have an impact on the safety of the dam, such as loads, floods, loss of stability as a result of slide; internal erosion and suffusion [11, 12], the elaboration of new methods of reliability assessment e.g. [13, 14] and risk analysis e.g. [15] as well as the analyses of the reliability of particular elements of dams, for example of turbines as an element which is critical for ensuring the efficiency of the whole hydro- electric power station system e.g. [16]. The second group includes studies conducted based on historical data. It is based on the identification of the causes for the occurrence of disasters, usually in reference to specific types of structures e.g. [17, 18], as well as publications referring to the statistics of dam damages [19, 20].

A collection of worldwide data referring to historical dam disasters was published in Bulletin no. 99 of the International Commission on Large Dams (ICOLD), entitled: „Dam failures, Statistical Analysis" [21]. The data analyzed in the Bulletin, referring to disasters and the number of constructed dams, cover the period until the year 1986. The authors of Bulletin no. 99 excluded China from the investigated population, justifying this with the fact that the number of dams constructed in China is comparable with the number of dams constructed in other countries, and only 3 of them underwent a disaster, which is a significant discrepancy, compared to the world average. According to them, "there is no engineering explanation for such a discrepancy". Also omitted is the deliberate destruction of dams as a result of war operations, such as the Möhne and Eder dams bombed by the Royal Air Force in 1943.

All types of large dams were included in the study, regardless of the material they were built from: e.g., earthen, concrete, and the type of construction, e.g., gravity or arch. By 'large dam' was meant a dam higher than 15 m from lowest foundation to crest or a dam between 5 m and 15 m impounding more than 3×10^6 m^3. As a result of the analysis, the Bulletin's authors concluded that: until 1950, 5 268 dams were constructed, of which a disaster occurred in case of 117, i.e. 2.2%. In the years 1951–1986, 12 138 dams were constructed, of which a disaster occurred in case of 59, i.e. 0.5%. They give the following reasons for the dam failure:

overtopping – 33%, internal erosion – 40%, structural failure – 21%, foundation failure – 3%, unknown – 3%. It was also found that 70% of dam failures occur within the first ten years of operation, and most often in the first year of operation (24%). As the reported disasters caused by loss of stability of the dam body only occurred during construction, they were not included in Bulletin 99 [21]. Also, landslides of the reservoir slopes were not included. For example, the well-known landslide disaster of the slope of the Tonte Toc mountain into the Italian Vajont reservoir, which occurred in 1963, is missing from the compilation. Despite collecting abundant material and providing its broad analysis [21], does not include calculations of values and functions characterizing reliability [22–24] as:

– the hazard function (also known as hazard ratio), $h(t)$ – this is number of failures of dams per unit time per number of nonfailed dams. The hazard function is the value of the probability of dam failure as a function of time.
– the reliability function $R(t)$ is the probability of a dam not failing within a given unit time, provided that it has not failed previously,
– the mean time to failure, $MTTF$ is the average time in which a dam can fail.

One year was taken as the unit time.

A typical graph of the risk function is shown in Figure 1. Three periods can be distinguished: initial mortality period, useful operating period and weare-out period. During the initial period, the risk function drops sharply, which is due to the fact that errors made during the design or construction of the dam are revealed. Serious errors can lead to the failure of the dam especially during the first filling. As previously mentioned, 70% of dam disasters occur in the first 10 years. During the useful operating period the hazard function takes a minimum value and is a constant function $h(t) = h_0$ and is then called the hazard ratio. Dam failures are caused by random events such as overtopping or suffusion and other unknown reason [21]. The final stage is characterized by an increase in the hazard function, which is caused by the overall wear out of the overall structure of dam. It is a completely unrecognized period. There is a lack of any data on its course.

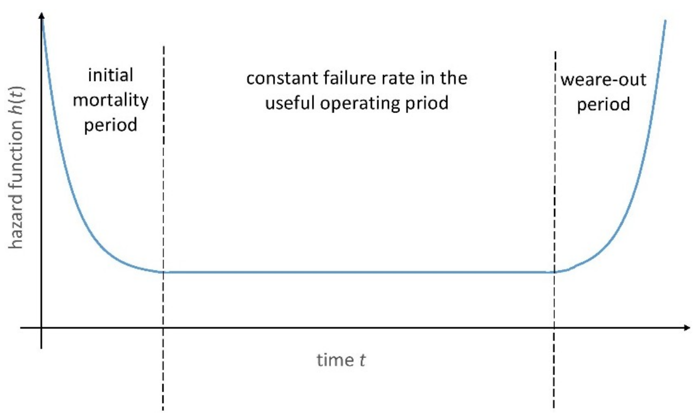

Fig. 1. A typical graph of the hazard function

The reliability function $R(t)$ is defined [22, 23] as:

$$R(t) = \exp\left(-\int_0^t h(\tau)\mathrm{d}\tau\right) \tag{1.1}$$

and $MTTF$ as:

$$MTTF = \int_0^\infty R(t)\mathrm{d}t \tag{1.2}$$

This article is devoted to the calculation of the hazard function, the reliability function and the mean time to failure of dams.

Due to the high number of mathematical formulas, only the ones referred to in the further part of the paper have been numbered. In order not to focus the reader's attention on rather complicated mathematical transformations, these have been included in the Appendices (available in the repository: https://doi.org/10.18150/XJG7A7).

2. Material and methods

2.1. General description of the methodology

The study on reliability distinguishes two basic types of structures: repairable and unrepairable ones [22]. The assumption of the present article is that dams are structures that are unrepairable, which may raise some doubts. On one hand, as was mentioned, the most frequent reason of the disaster of a concrete or a rock-filled dam is internal erosion of the dam foundation, shear stress in the dam foundation and overtopping. In case of embankment and rockfill dams, the most frequent cause of the disaster was overtopping, internal erosion of the body and internal erosion of the foundation [21]. This means that the processes leading to a dam disaster impact the dam elements that are not repairable, such as the dam body or foundation. So, a dam disaster takes place only once. Even if the dam is reconstructed after a disaster, it is no longer the same dam. Therefore, in case of dams, it is justified to apply the $MTTF$ calculation method as in case of unrepairable systems. On the other hand, dam disasters would occur much more frequently if their renovations were not performed. Due to the fact that dam renovations improve the safety of their use, dams should be treated as repairable structures. The literature devoted to the methods of dam renovations is very abundant e.g. [25]. However, collective information about the service life of dams to the first renovation and between subsequent renovations is not published. Some authors only include general information. Jansen [26] mentions that for earth dams the time to renovation is from 8 (Walter Bouldin Dam) to 85 years (Willow Creek Dam). According to Kledyński [27], for Polish dams, the mean time to the first renovation is 20 years. Jia et al. [28] believe that this time is about 50 years. A lack of detailed data makes it impossible to perform the calculation of parameters characterizing the renovated systems: such as $MTTR$ – mean time to renovation or $MTBR$ - mean time between renovation.

The reliability parameters of dams were calculated in the following steps:

1. Gathering data about the number of constructed dams and about the number of dam disasters.
2. Developing a histogram of the hazard function.
3. Selecting the hazard function and fitting it to the histogram.
4. Verification of the correctness of the performed fitting.
5. Calculating the reliability function $R(t)$, the $MTTF$ and the extended uncertainty [29] of $MTTF - u(MTTF)$.

All the necessary numerical calculations were performed using the MATLAB and MATHEMATICA software.

2.2. Data referring to dams and their disasters

All the data about dams and their disasters was taken from [21]. The period analyzed is up to the year 1986. The number of analyzed dams: 17 406. The number of dams that underwent a disaster: 176. As has already been mentioned, the provided data does not include China. Prior to further calculations, it was necessary to unify the data because the provided set included incomplete data, for example a lack of the date of finishing the construction, as well as data referring to disasters at the stage of constructing the dam. Due to the fact that the calculated reliability characteristics such as the hazard function or the reliability function refer to the period of operational use, the data about a disaster at the stage of construction cannot be taken into consideration. Therefore, the following cases were excluded from the analyzed set:

– incomplete data about 18 dams failures. For example: the construction date was unknown or it was not mentioned whether the disaster took place during the first filling of the reservoir or already during operational use,
– disasters in progress of the construction of dams, before putting a dam into service. There were 12 of such disasters. However, only in one case the further construction was discontinued. In the remaining cases the dams were constructed. Thus, for the further analysis, 11 dams were excluded from the set of all considered dams. And 12 cases were excluded from the set of disasters.

The following was adopted for the further analysis:

– the number of dams 17 406 – 18 – 12+1=17 377,
– the number of dam disasters 176 – 18 – 12=146.

2.3. The hazard function histogram

Basing on data, with taking into consideration the exclusion of the discussed cases, a histogram of the hazard function $H(\tau)$ [22] was developed

$$H(\tau) = \frac{n(\tau, \tau + \Delta\tau)}{N(\tau)\Delta\tau} \tag{2.1}$$

where: $H(\tau)$ – the histogram of the hazard function, n – the number of elements that were damaged in the time interval $(\tau, \tau + \Delta\tau)$, N – the sample size in the moment of time τ.

Due to the fact that only the year of construction and the year of the disaster have been provided, the time τ is not a continuous value but a set of natural numbers because the time interval over which disasters are counted is 1 year. Time $\tau = 0$ means that the failure took place during the first filling. And $\tau = 1$ means that the disaster took place during the first year of operation. It is not indicated whether it occurred in the beginning, in the middle or in the end of that year. Due to this fact, the histogram is a step function.

2.4. Fitting the hazard function

The hazard function was divided into two functions: the estimated one and the extrapolated one. The estimated function describes the initial mortality period over which the dam failure rate is variable in time – decreasing. The extrapolated function describes the operation period also known as the useful life period in which the dam failure rate is constant [22]. The division was performed in reference to the time interval. The estimation interval is the one in case of which we have data about the number of dam disasters in a given year of operation, i.e. it is the interval $(0, t_{\text{limit}}]$ where t_{limit} is the longest recorded time that has elapsed to the dam disaster. The extrapolation interval is the one in case of which the number of dam disasters is not known, i.e. it corresponds to the time interval $(t_{\text{limit}}, \infty)$.

In the estimation interval, the hazard function $h(t)$ of the following form was fitted to the histogram:

$$h(t) = at^b \tag{2.2}$$

where: $h(t)$ – the hazard function, t – time, a, b – the estimated constants.

The power function well describes the initial, fast decrease of the hazard ratio and has got only two parameters that need to be estimated. Thus, it meets the requirement of simplicity of the mathematical model [29]. The functions $h(t) = a \times e^{bt}$ and $h(t) = at^b + c$ were also considered, but the function $h(t) = at^b$ gave the best fit to the histogram. As a criterion, the sum of squares of deviations between the histogram and the fitted function was used.

A constant function was applied in the extrapolation interval

$$h(t) = h_0 \tag{2.3}$$

because after the first descent period there is a period when the hazard ratio is constant [22]. Only the parameter h_0 is estimated.

The estimated function and the extrapolated function should be continuous at the t_{limit} point. Therefore, the following condition must be met:

$$h_0 = at_{\text{limit}}^b \tag{2.4}$$

The mathematical methods of the applied estimation process and of the calculation of the standard deviation of the parameters a, b and h_0 are described in Appendix B.

2.5. Verifying the correctness of the fitting

The verification of the correctness of the fitting was performed using the χ^2 test by comparing the calculated value χ^2 with the critical value $\chi^2_{\text{crit}\,\alpha}$ for the adopted level of significance α. If $\chi^2 < \chi^2_{\text{crit}\alpha}$, we find no grounds for rejecting the hypothesis of the equality of the functions $H(t)$ and $h(t)$. Otherwise, the hypothesis is rejected. The calculation methods are given in Appendix C.

2.6. The reliability function

Depending on the time interval, the reliability function is expressed by the formulas: for the estimation interval $t \in [0, t_{\text{limit}}]$

$$R(t) = \exp\left(-a\frac{t^{b+1}}{b+1}\right) \quad \text{for} \quad 0 \le t \le t_{\text{limit}} \tag{2.5}$$

for the extrapolation interval $t \in (t_{\text{limit}}, \infty)$

$$R(t) = \exp\left(-a\frac{t_{\text{limit}}^{b+1}}{b+1} - h_0 t + h_0 t_{\text{limit}}\right) \quad \text{for} \quad t_{\text{limit}} < t \tag{2.6}$$

The calculations are provided in Appendix D.

2.7. The mean time to failure

The $MTTF$ is calculated by integrating the reliability function $R(t)$ (Eq. (2.5) and Eq. (2.6)) in the range from zero to infinity [22].

$$MTTF = \int_0^\infty R(t)\mathrm{d}t = \int_0^{t_{\text{limit}}} \exp\left(-a\frac{t^{b+1}}{b+1}\right)\mathrm{d}t + \frac{1}{h_0}\exp\left(-a\frac{t_{\text{limit}}^{b+1}}{b+1}\right) \tag{2.7}$$

The exact formulas and calculations are given in Appendix E.

2.8. The uncertainty of the *MTTF*

The uncertainty of the $MTTF - u(MTTF)$ results from the standard uncertainty of the estimated parameters $u(a)$ and $u(b)$ of the hazard function $h(t)$. Standard uncertainty $u(a)$ and $u(b)$ shall be understood as the standard deviation of these parameters [30] i.e. σ_a and σ_b.

$$u(MTTF) = \sqrt{\left(\frac{\partial MTTF}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial MTTF}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial MTTF}{\partial h_0}\right)^2 \sigma_{h_0}^2} \tag{2.8}$$

Extended uncertainty $U(MTTF)$ is an interval [30] containing $MTTF$:

$$U(MTTF) = (MTTF - k \times u(MTTF), MTTF + k \times u(MTTF)) \tag{2.9}$$

where k is the extension coefficient depending on the adopted confidence level β. Parameter β indicates what is the probability that the $MTTF$ value lies inside the $U(MTTF)$ interval. The parameters β and k are closely related. For example, if $\beta = 0.9997$ then $k = 3$.

Calculations are provided in Appendix F.

3. Results

3.1. Histogram of hazard function

The hazard function histogram was calculated based on Eq. (2.1). The histogram in the time interval from 0 to 65 years is presented on the Figure 2.

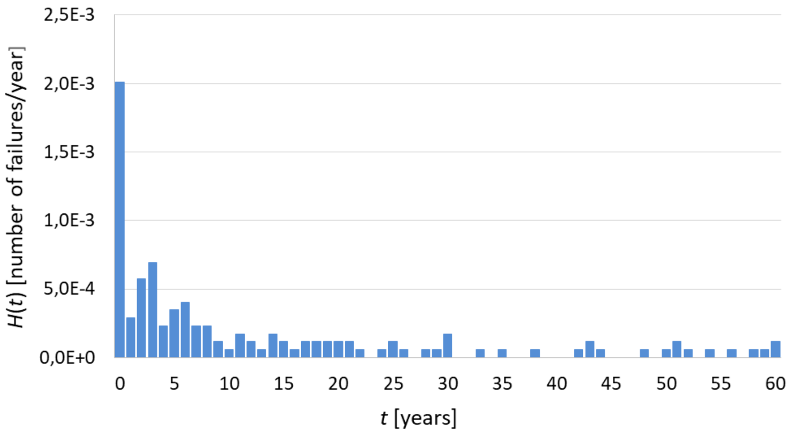

Fig. 2. The hazard histogram. The graph has been demonstrated in the time interval from 0 to 65 years in order to present the initial variability of the hazard

3.2. Hazard ratio function

The ICOLD Bulletin [21] mentions that the longest time to a dam disaster was 236 years. It was assumed that it was in the middle of that year, i.e.

$$t_{\text{limit}} = 236.5 \tag{3.1}$$

In the estimation interval, i.e. for $t \in (0, 236.5]$, function 2.2 was fitted to the histogram. The following parameter values were obtained:

$$a = 1.062 \times 10^{-3}, \quad b = -0.8786 \tag{3.2}$$

and 95% confidence intervals which were: for parameter $a \in (1.02 \times 10^{-3}; 1.11 \times 10^{-3})$ and for parameter $b \in (-0.91823, -0.838967)$. Based on formula (B1) the standard deviations were calculated:

$$\sigma_a = 2.24 \times 10^{-5}, \quad \sigma_b = 1.98 \times 10^{-2} \tag{3.3}$$

In the extrapolation interval, i.e. $t \in (236.5, \infty)$, the hazard function is expressed by Eq. (2.3). After substituting the value of the parameters t_{limit} Eq. (3.1) and a, b Eq. (3.2), the dam failure rate was obtained:

$$h_0 = 1.062 \times 10^{-3} \times 236.5^{-0.8786} = 8.719 \times 10^{-6} \tag{3.4}$$

Standard deviation σ_{h_0} is calculated using formula (B2) and is equal:

(3.5) $$\sigma_{h_0} = 2.97 \times 10^{-7}$$

Therefore, taking into account the standard uncertainty $\sigma_h 0$, the dam failure rate can be written down as:

(3.6) $$h_0 = (8.719 \pm 0.297) \times 10^{-6} \text{ failure per year}$$

To sum up, the hazard function $h(t)$ is composed of two functions:

(3.7) $$h(t) = 1.062 \times 10^{-3}\, t^{-0.8786} \quad \text{for} \quad t \in (0; 236.5]$$

(3.8) $$h(t) = 8.719 \times 10^{-6} \quad \text{for} \quad t \in (236.5; \infty)$$

The hazard function graph is demonstrated in Figure 3.

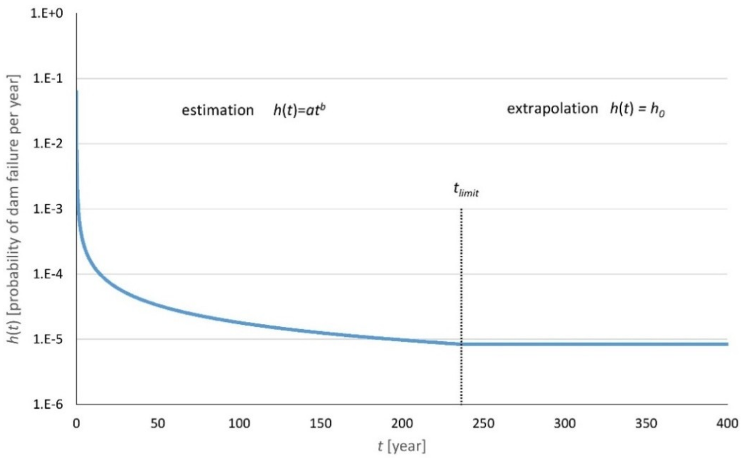

Fig. 3. A graph of the hazard function, $t_{\text{limit}} = 236.5$ – the border time between the interpolation and the extrapolation of the hazard function $h(t)$. The calculated parameters are: $a = 1.062 \times 10^{-3} \pm 2.24 \times 10^{-5}$; $b = -0.8786 \pm 1.98 \times 10^{-2}$; $h_0 = 8.719 \times 10^{-6} \pm 2.97 \times 10^{-7}$

3.3. Correctness of fitness

The correctness of fitness was verified using the chi-squared χ^2 statistical test which is the sum of the squares of the differences between $H(t)$ and $h(t)$. The tested hypothesis is: „Adopting the significance level $\alpha = 0.05$ there is no statistically significant difference between the empirical histogram $H(t)$ and the estimated function $h(t)$". The high value of χ^2 proves against the tested hypothesis. The value of the χ^2 statistics was calculated based on formulas (C1), (C2) and (C3). The necessary standard deviations, σ_h, σ_H, were calculated using formulas (C4), (C5) and (C6). They were calculated for every point separately. The ndf – number degree of freedom value was calculated based on formula (C7).

The following was obtained:

$$ndf = 237 - 2 - 1 = 234 \tag{3.9}$$

$$\chi^2 = 90.3 \tag{3.10}$$

The test critical value, calculated using the R package function is:

$$\chi^2_{\text{crit}}(\alpha = 0.05; ndf = 234) = 270.68 \tag{3.11}$$

Because

$$\chi^2 < \chi^2_{\text{crit}} \tag{3.12}$$

it has been found that there are no grounds for rejecting the hypothesis of the compliance of $H(t)$ and $h(t)$.

3.4. Reliability function

The reliability function is expressed by Eq. (2.5) and Eq. (2.6). After substituting the values of the obtained parameters a, b Eq. (3.2) and h_0 Eq. (3.4) we receive:

$$R(t) = \exp\left(-0.008748 \times t^{0.1214}\right) \quad \text{for} \quad 0 \le t \le 236.5 \tag{3.13}$$

$$R(t) = \exp\left(-0.00685 - 8.72 \times 10^{-6} t\right) \quad \text{for} \quad 236.5 < t \tag{3.14}$$

The probability of the dam disaster is $1 - R(t)$.

3.5. Mean time to failure

$MTTF$ calculated from Eq. (2.7) is:

$$MTTF = 112957 \text{ years} \tag{3.15}$$

Details of the calculation are given in Appendix G.

3.6. Uncertainty of the $MTTF$

The $u(MTTF)$ uncertainty and the extended uncertainty $U(MTTF)$ was calculated from Eq. (2.8) and Eq. (2.9).

$$u(MTTF) = 12443 \text{ years} \tag{3.16}$$

$$U(MTTF) = (88072, 137842) \text{ years} \tag{3.17}$$

Calculations are provided in Appendix H

4. Discussion

It is very difficult to discuss the obtained result: $MTTF = 112957 \pm 12443$ years. Are dams actually more durable than the Egyptian pyramids, which already demonstrate the impact of the time of 4500 years? The oldest concrete dams are about 130 years old. Even though the normal service period of concrete structures is assumed to be 50 years [28], Wieland [31] assesses concrete performance to 150-200 years. However, he believes that it is possible to achieve a concrete dam service life of up to 1000 years. Jia et al. [28] mention that some dams in China are ca. 1000 years old and they claim that the lifespan of an earth dam or a rock- filled dam is slightly greater than that of a concrete dam. Despite that, an $MTTF$ value of about 100 000 years is unlikely.

The resulting $MTTF$ value is due to the fact, that only failure modes that have already caused the failure of a dam in the past have been included in the calculation, i.e. overtopping, internal erosion, structural failure, foundation failure, as already mentioned. There is no record of any disaster caused by dam ageing. Similar, apparently unrealistic, results are obtained by examining other accidents. For example, Hoskin et al [32] studied the risk of death for workers involved in the remediation of hazardous waste sites. Selected hazard ratio (death rate) and $MTTF$ calculations are given in Table 1. $MTTF$ was calculated according to a commonly [22, 23] used formula:

$$MTTF = \frac{1}{h_0} \tag{4.1}$$

Table 1. The risk of fatal accident during hazardous waste site remediation [32], with $MTTF$ calculation

Occupational title	**Hazard ratio $h0$ [1/year]**	**$MTTF$ [years]**
Truck driver	3.876×10^{-4}	2580
Dozer operator	2.840×10^{-4}	3521
Inspector	1.059×10^{-4}	9443
Civil engineer	0.328×10^{-4}	30488
Secretary	0.029×10^{-4}	344828
Safety officer	0.000×10^{-4}	infinity

The above results do not mean that the life expectancy of a truck driver is 2,580 years, a secretary more than 340,000 years and a health and safety worker never dies. These results mean that if accidents at work were the only factor causing the death of a worker, their life expectancy would be as long as indicated in Table 1.

The hazard function for structures during their use is best described by the bathtub curve [22, 33], which includes three phases. In the beginning of the first one, the risk of a failure is very high but it quickly goes down. This results from the existence of hidden defects of the materials, of the construction or execution errors. The second phase is the period of operation for which the failure level is basically constant. In the last phase of use, the failure risk rapidly increases, which is the effect of the materials getting old. As may be concluded from the dam hazard function graph demonstrated in Figure 2, one can clearly distinguish the first phase which ends after about five years. In this graph one can also notice the second phase, with a constant hazard. However, the third phase, in which the hazard increases, is absent. The lack of knowledge of the whole hazard function for dams has an impact on the obtained $MTTF$ result, although it is not known how strong this impact is.

The analysis performed in this article included all dams, regardless of their type, i.e. the material they were constructed from, as well as the type of construction and the height. This is because, as has been demonstrated in [21], the ratio:

$$\frac{\text{all destroyed dams of a given type}}{\text{all destroyed dams}} \approx \frac{\text{all constructed dams of a given type}}{\text{all constructed dams}} \tag{4.2}$$

is approximately constant. The mentioned Bulletin no 99 [21] also shows that the number of dam disasters is independent of the dam height. Therefore, conducting $MTTF$ calculations for a uniform set, regardless of the type and the height of dams is justified.

The first important result of the performed calculations is obtaining the parameters of the reliability function $R(t)$. This will allow for studying in more detail how the probability of a dam disaster, which is $1 - R(t)$, changes over time. Knowing the possible losses, it will be possible to calculate the risk generated by dams. The presented calculations may constitute valuable supplementation from the side of numerous authors e.g. [34–36]; for the rapidly developing domain of dam risk analysis.

The second important result of the performed calculations obtained for the operational period of dams is the annual dam failure ratio, which is $8.719{\times}10^{-6}{\pm}0.297{\times}10^{-6}$ Eq. (3.4). Analyzing the studies performed in the years 1963-1977, Donnelly [37] indicates that the dam failure rate fits in the interval from $2{\times}10^{-4}$ to $7{\times}10^{-4}$, and its mean is $4{\times}10^{-4}$. The corresponding $MTTF$ calculated according to formula (3.7) ranges from 5000 to 1430 years, and its mean is 2500 years. A similar value is indicated by Xiao [38] for China – it is said to be $8.761{\times}10^{-4}$, $MTTF$ is 1140 years. Foster et al [39] assessing dam failure rate due to internal erosion by embankment, foundation, embankment into foundation separately obtained an annual disaster probability value of 10^{-6} for each, giving a total value of $3{\times}10^{-6}$. This corresponds to the $MTTF$ value of 333333 years. As may be easily noticed, these are values that are a double order of magnitude lower than that obtained in this article. The reason for this difference may be the fact that during the calculation of dam disasters the mentioned authors did not perform the division into the initial mortality period and the operation period, while, after all, the dam failure rate should be provided for the operation period. Bowles [40] indicates that, according to the recommendations of the U.S. Army Corps of Engineers, the tolerance for the annual risk of a dam disaster should not exceed 10^{-6} for newly- constructed dams and 10^{-4} for already existing ones. This would mean that existing dams on average meet this requirement. Therefore, in the light of Kent's Words of Estimative Probability [41] a dam disaster is an incident that is 'almost certainly impossible'.

5. Conclusions

The paper demonstrates that dam durability is much higher up to 100 years - the value commonly adopted as the economic life span. The paper also includes the calculation of the dam reliability function which may be applied regardless of the dam type: arch, gravity, or the material it is made of, such as earth, concrete, a rock-filled dam. The knowledge of the reliability function allows for calculating the probability of a dam disaster, i.e. a value that is necessary for quantitative risk analysis. This, in turn, can find practical application in calculating the cost of insurance for damage caused by a dam failure.

Data availability statement

Data and Appendices are available in the repository: https://doi.org/10.18150/XJG7A7.

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Funkcja intensywności uszkodzeń, funkcja niezawodności oraz średni czas życia zapór

Słowa kluczowe: funkcja intensywności uszkodzeń, funkcja niezawodności, niezawodność, średni czas życia, zapory

Streszczenie:

Zapory należą do największych budowli wzniesionych przez człowieka. W przypadku dużych zapór energia zgromadzonej, spiętrzonej wody wielokrotnie przewyższa energię bomb atomowych zrzuconych na Hiroszimę i Nagasaki w 1945 roku. Energia uwolniona podczas eksplozji tej bomby atomowej była równoważna 16 kilotonom trotylu [1], to jest około 67 TJ, podczas gdy energia potencjalna wody zmagazynowanej w największej polskiej zaporze Solina wynosi około 152 TJ (Załącznik A). Wielu autorów, np. [5–9] podkreśla znaczenie niezawodności konstrukcji i systemów technicznych, co ma istotny wpływ m.in. na ich funkcjonalność i żywotność, a także na bezpieczeństwo użytkowników. Ponadto identyfikacja podstawowych parametrów niezawodności jest istotnym narzędziem wspomagającym właściwą konserwację i planowanie remontów [10]. Publikacje odnoszące się do niezawodności zapór można podzielić na dwie grupy. Pierwsza grupa obejmuje analizę czynników mających wpływ na bezpieczeństwo zapory, takich jak obciążenia, powodzie, utrata stateczności w wyniku osuwiska; erozja wewnętrzna i podsycanie [11, 12], opracowanie nowych metod oceny niezawodności, np. [13, 14] i analizę ryzyka, np. [15], a także analizy niezawodności poszczególnych elementów zapór, np. turbin jako elementu krytycznego dla zapewnienia sprawności całego systemu elektrowni wodnej, np. [16]. Drugą grupę stanowią badania prowadzone na podstawie danych historycznych. Opierają się one na identyfikacji przyczyn występowania katastrof, zwykle w odniesieniu do konkretnych typów budowli, np. [17, 18], a także publikacji odnoszących się do statystyk uszkodzeń zapór [19, 20]. Zbiór danych światowych odnoszących się do historycznych katastrof zapór został opublikowany w Biuletynie nr 99 Międzynarodowej Komisji ds. Dużych Zapór (ICOLD) [21]. Pomimo zebrania obfitego materiału i przedstawienia jego szerokiej analizy [21], nie uwzględniono obliczeń wartości i funkcji charakteryzujących niezawodność [22–24] jak: funkcja intensywności uszkodzeń $h(t)$, funkcja niezawodności $R(t)$ a także $MTTF$ – średni czas do awarii. Metodzie obliczenia tych funkcji poświęcona jest niniejsza praca. W trakcie przeprowadzonych badań wykonano: zbieranie danych o liczbie wybudowanych zapór i liczbie katastrof zapór; opracowanie histogramu funkcji intensywności uszkodzeń. Dokonano wyboru funkcji intensywności uszkodzeń i dopasowanie jej do histogramu oraz weryfikację poprawności wykonanego dopasowania. Obliczono funkcję niezawodności $R(t)$, oraz $MTTF$ – średni czas do katastrofy. W wyniku przeprowadzonych obliczeń otrzymano funkcję intensywności uszkodzeń h(t) opisaną wzorami (3.7) i (3.8), której wykres pokazany jest na rys. 3, oraz funkcję niezawodności R(t), podaną za pomocą formuł (3.13) i (3.14). Znajomość funkcji niezawodności umożliwiło obliczenie średniego czasu życia zapory

$MTTF$ (3.15), który wynosi 112957 lat. Pierwszym ważnym, otrzymanymwynikiem przeprowadzonych obliczeń jest uzyskanie parametrów funkcji niezawodności $R(t)$. Pozwala to na szczegółowe obliczenie, jakie jest prawdopodobieństwo katastrofy zapory, które wynosi 1 - $R(t)$, w czasie. Znając możliwe straty spowodowane katastrofą, można obliczyć ryzyko stwarzane przez zapory. Przedstawione obliczenia mogą stanowić cenne uzupełnienie prac wielu autorów np. [34–36]; dla dynamicznie rozwijającej się dziedziny analizy ryzyka zapór. Drugim istotnym wynikiem przeprowadzonych obliczeń uzyskanym dla okresu eksploatacji zapór jest roczny wskaźnik awaryjności zapór, który wynosi: $8{,}719\times10^{-6}\pm0{,}297\times10^{-6}$ (13). Foster et al [39] oceniając wskaźnik awaryjności zapory z powodu erozji wewnętrznej przez nasyp, fundament, nasyp do fundamentu oddzielnie uzyskali roczną wartość prawdopodobieństwa katastrofy 10^{-6} dla każdego z nich, co daje całkowitą wartość 3×10^{-6}. Odpowiada to wartości $MTTF$ wynoszącej 333333 lat. Ta wartość tylko pozornie wydaje się nierealna. Podobnie, pozornie nierealne wyniki otrzymuje się przy badaniu innych wypadków. Na przykład badając częstotliwość wypadków śmiertelnych pracowników składowisk materiałów niebezpiecznych otrzymuje się wynik [32], że ze wszystkich pracowników najczęściej wypadkom ulegają kierowcy ciężarówek, a ich średni czas życia wynosi 2580 lat. Oczywiście ani średni czas życia człowieka nie wynosi ponad dwa i pół tysiąca lat ani zapory ponad trzysta tysięcy lat. Takie wartości wynikają z powszechnie stosowanej metodyki obliczeń, w której zakłada się, że pracownik składowiska może umrzeć jedynie w skutek śmiertelnego wypadku, a w przypadku zapory jedynym powodem jej katastrofy może być jej przelanie, erozja wewnętrzna, utrata stateczności czy błąd posadowienia. W obydwu przypadkach nie uwzględnia się czynnika starzenia. Niestety w przypadku zapór, jeszcze nie posiadamy wystarczającej wiedzy w tym zakresie. Można jednak zauważyć, że 4,8 razy jest większe prawdopodobieństwo śmiertelnego wypadku kierowcy ciężarówki na składowisku odpadów niebezpiecznych od prawdopodobieństwa katastrofy zapory.

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