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Research paper

Determining the duration of a construction project using Markov chain

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Abstract: In this article, practical application of the Markov chain to estimate the timeframe of a construction project is proposed. This methodology involves forecasting the duration of work based on the probability of transitions between different states in the implementation of the construction project. In this context, each state can represent a specific phase of the construction work, such as the different condition during construction work with or without risks. Transition probabilities between these states can be determined based on statistical data from previous projects, expert opinions, and other sources. In this article, the transition probabilities between states were established based on expert opinions. The proposed approach allows predicting the likely completion time of the project and assessing the impact of multiple elements on the project's timeframe on its duration. The article also includes numerical examples illustrating the operation of the proposed approach. The obtained results demonstrate the practical application possibilities in terms of efficient planning of a construction project considering risk factors.

Keywords: Markov chain, technological and organizational solution, construction work duration, risk, network planning, construction implementation planning

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1. Introduction

A construction project is a set of interdependent activities aimed at satisfying the construction needs of an investor or property owner according to the definition by Professor T. Kasprowicz [1]. For each investor, during the construction phase, the main needs are ensuring the construction of the object within the scope, time, and budget established during the planning stage. However, in practice, it often turns out that the duration of a construction project is longer than what was determined during the planning stage. Many researchers, for instance in the study [2], point out that the reason for the increased realized construction duration in comparison with the planned timeframe is the emergence of various risk factors that negatively affect the duration. Additionally, for example, in the work [3, 4], it is noted that there are risk factors that are difficult to foresee during the construction planning phase.

The assessment of the construction project execution time at the planning stage is based on technological and organizational solutions (*TOS*) [5, 6], developed by the planner. TOS refers to the technological and organizational dependencies between different construction works. Then, the normative duration for each work is determined based on standards. In Poland, for the normative assessment of duration, Construction Norms and Regulations (*KNR*) [7] are used, which contain average values for duration and do not account for the occurrence of various adverse factors during the work process. The calculation of the total duration is performed using the critical path method (*CPM*) [8, 9]. When analyzing each individual work, it can be concluded that during the execution process, the work can be in several states: a normal state without the influence of risk factors or a state influenced by one or more risk factors. The total duration of each individual construction work is the sum of the durations of each state of individual construction work. Each construction job is in a different state during execution. For example, if the work is in normal working condition without exposure to negative factors, then its duration will be equal to the standard duration. But if at some point in time the work moves from a normal operating state to a state under the influence of negative external factors, then the duration of such work may lengthen.

There are many different studies that allow for the consideration of risk factors, for example, using construction projects planning with fuzzy decision nodes and fuzzy sets [10–14]. However, they do not account for the duration spent in different states for each individual work and the possibility of returning to normal work mode. There are several other approaches to determining the duration of each individual construction work (task) is adding additional time to the normative time allocated for completing the construction work in case of the occurrence of a risk event. In this approach, several options for determining additional time are distinguished:

– in the research by [15], it is proposed to consider additional time as lag time (*LT*) without taking into account the probability of the emergence of a risk event,
– in the research by [16], it is proposed to consider additional time as the product between LT and the probability of the emergence of a risk event,
– In the research by [17], it is proposed to consider additional time as a fuzzy product between lag time, the probability of the likelihood of a risk event occurring and the severity of the threat it poses to the work using a risk matrix.

In the study [17], it is shown that the difference between the four approaches can vary within 10%. The main drawback of these approaches is that when several risk events affect the construction work simultaneously, the additional time is determined as the maximum possible for one of the several risk events.

The study focuses on introducing an application for precise calculation of the construction timeline. This framework accounts for additional time needed to complete the work with risks, especially when multiple risk events impact the construction process simultaneously, leveraging the Markov chain.

Key contributions of this work include:

1. Development of a novel application for estimating construction project durations. This application considers the incorporation of extra time to manage risks and addresses the simultaneous impact of multiple risk events. It utilizes the Markov, allowing for the examination of various states and transitions while enabling the assessment of risk.
2. Validation of the proposed methodology through practical analysis conducted on a specific construction project case study.

The article is structured as follows: Part 2 presents a comprehensive review of existing research. Part 3 describes the proposed application for construction project execution time using Markov chains, and the basic theory of Markov Chains and Critical Path Method. Practical numerical examples are presented in Part 4. Part 5 outlines the findings of the practical case study analysis, followed by a detailed discussion. Finally, Part 6 offers a brief summary, concluding the article.

2. Literature review

The construction work technological process is associated with the state, because it unfolds over time, typically following a sequential timeline from start to finish, and changes the state with or without the effect of various risk events is something that is uncertain and matters, such as bad weather, crane failure, and so on. An example of the construction process in different states is presented in Figure 1, taking into account condition monitoring once a day.

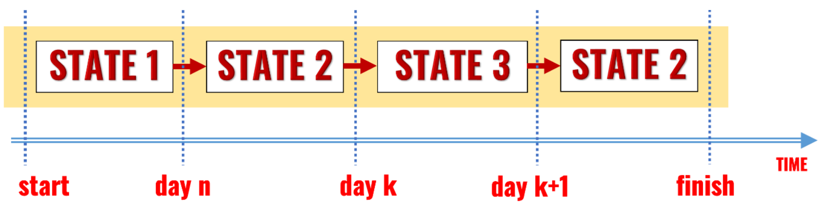

Fig. 1. The construction work process from start to finish in different states

Thus, during the implementation of construction work over time from start to finish, it can be in several states: a functional or normal state without the occurrence of risk events, states with individual risk events, and states with a combination of risk events. During the implementation of construction work, state transitions occur with a defined probability. The overall representation of this transition is shown in Figure 2. Such a process allows for describing Markov process. In literature, a Markov process with discrete state and time spaces is referred to as [18, 19].

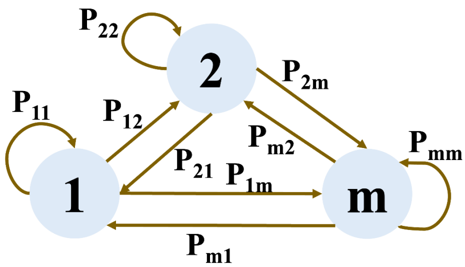

Fig. 2. Example of graph of states and transitions of an object

A Markov chain is a mathematical framework used to describe sequences of events in which the probability of transitioning to the next state depends exclusively on the current state. In other words, it is a stochastic process that moves from one state to another in a sequence of discrete time intervals, where the transition probabilities are constant and depend only on the present state. Markov chains are widely used in various fields such as mathematics, statistics, physics, biology, economics, and computer science. They have applications in modeling systems with random behavior, such as financial markets, weather patterns, genetic sequences, and network traffic, among others.

In the field of construction, there are few applied studies using Markov processes. In the study [20], the probability of accidents over a certain period of time is determined, and then, using the property of a Markov process, an analytical expression for the risk probability density is derived. In the article [21], a Markov process with discrete states in continuous space was used to forecast geological conditions. In the article [22], a Markov process is considered as a probabilistic method for assessing the reliability of construction structures.

Overall, the literature review underscores the significance of Markov processes and Markov chains in construction project management. These models offer valuable insights into project dynamics, facilitating improved decision-making, enhanced risk management, and efficient resource allocation across the entire project lifecycle.

3. Application for estimating the construction project execution time using markov chain

3.1. Basic information of the Markov chain

A Markov process is a stochastic process where the probability of transitioning from one state to another depends solely on the current state [18]. It exhibits the Markov property, meaning future behavior depends only on the present state and is unaffected by past events. Markov processes are often characterized by a set of states, transition probabilities between these states, and an initial state distribution. A special case of a Markov process is a Markov chain. A Markov chain is a specific type of Markov process that consists of a finite or countable set of states and transition probabilities between these states. It is a discrete-time stochastic process where transitions occur at discrete time steps. Markov chains are commonly represented using transition probability matrices, representing the probabilities of transitioning between states within a single time step [19]:

$$P = \left[p_{ij}(n)\right] = \begin{bmatrix} p_{11} & p_{21} & p_{1j} \\ p_{21} & p_{22} & p_{2j} \\ p_{i1} & p_{i2} & p_{ij} \end{bmatrix} \tag{3.1}$$

where: p_{ij} – transition probabilities.

The transition probabilities matrix is called stochastic, if the following conditions are met [19]:

$$p_{ij} > 0, \quad i.j \in X \tag{3.2}$$

$$\sum_j p_{ij} = 1, \quad i \in X \tag{3.3}$$

The stationary distribution of a Markov chain with transition probability matrix $[p_{ij}]$ is defined as the probability distribution $\{v_i\}$that remains unchanged in the Markov chain as time progresses. It is generally expressed as a row vector with probability values summing to 1, and for a given transition matrix P, it satisfies the following condition [18, 19], for which at each n

$$\sum_{i=0}^{\infty} \upsilon_i \cdot p_{ik}(n) = \upsilon_k, k = 0, 1, ..., \tag{3.4}$$

Transitioning from one state to another is accompanied by certain incomes $[r_{ij}]$, then such a set can be accepted as a revenue matrix R of the form:

$$R = \left[r_{ij}(n)\right] = \begin{bmatrix} r_{11} & r_{21} & r_{1j} \\ r_{21} & r_{22} & r_{2j} \\ r_{i1} & r_{i2} & r_{ij} \end{bmatrix} \tag{3.5}$$

where: p_{ij} – income.

The total expected income $d_i(n)$ for n transitions is equal [18, 19]:

$$d_i(n) = q_i + \sum_{j=1}^{N} p_{ij} \cdot d_j(n-1) \tag{3.6}$$

where: q_i – the average one-step income.

The average one-step income q_i represents the amount that an object (conditionally) will bring when exiting state i [18, 19]:

$$q_i = \sum_{j=1}^{N} p_{ij} \cdot r_{ij} \tag{3.7}$$

3.2. Basic information of the critical path method

The Critical Path Method (*CPM*) serves as an essential instrument for project planning and schedule management [17, 23]. At its core lies the longest dependent path, representing the most extended sequence of dependent tasks that determines the project's completion. It forms a sequential workflow sequence, where the initiation of the next activities awaits the completion of its predecessor [17]. By employing the *CPM* algorithm, the minimal expected duration of a project can be established, and for each task within the project, both the earliest feasible initiation time and the latest permissible completion time can be determined. In this study, *CPM* is utilized alongside the Precedence Diagramming Method (*PDM*), a strategic approach for constructing a project workflow network diagram. This method, known as Activity-on-Node (*AON*), employs checkpoints to represent activities, interconnected by arrows to illustrate their sequential dependencies. Each checkpoint is given a distinct identifier, usually a letter or number, representing a specific activity within the project schedule. Fundamentally, an activity-on-node diagram delineates the prerequisites for initiating subsequent activities, often referred to as the "finish-to-start" precedence. This signifies that an activity must conclude before the following one can commence. The *PDM* encompasses four fundamental types of dependencies or logical connections between activities: finish-to-start (*FS*), start-to-start (*SS*), finish-to-finish (*FF*), and start-to-finish (SF). *FS* denotes an activity that cannot commence before its preceding activity concludes. *SS* represents a defined relationship between the start times of activities, while FF indicates a specified connection between their completion dates. *SF* signifies a defined relationship between the start of one activity and the end date of its successor. Each activity is characterized by four states: early start (*ES*), late start (*LS*), early finish (*EF*), and late finish (*LF*). An activity's early start is the earliest time it can commence, whereas its late start is the latest time it can begin without impacting the critical path. Similarly, the early finish denotes the earliest completion time, while the late finish represents the maximum allowable time for activity completion without delaying the project. An example of a precedence diagram is provided in Figure 3.

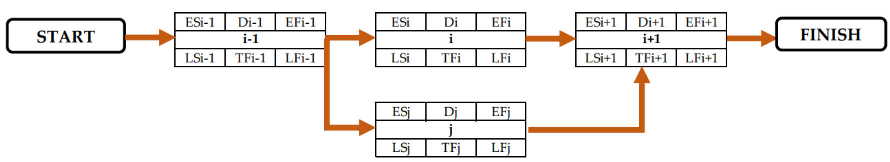

Fig. 3. Example of precedence diagram [17]

The *ES* for the initial activity following the project start is always set to zero. The *ES*, *E*F, *LS*, and *LF* the subsequent activities, the calculations are performed using the following Equations (3.8)–(3.10) [17]:

$$ES_i = EF_{i-1} \tag{3.8}$$

where: ES_i – early start for activity i (in days), EF_{i-1} – early finish for activity $i-1$ (in days)

(3.9) $$EF_i = EF_i + D_i$$

where: EF_i – early finish for activity i (in days), ES_i – early start for activity i (in days), D_i – duration for activity i (in days)

(3.10) $$ES_{i+1} = \max\{EF_i; EF_j\}$$

where: ES_{i+1} – early start for activity $i+1$ (in days), EF_i – early finish for activity i (in days), EF_j – early finish for activity j (in days).

Once all *EF* values are determined, the next step is to calculate *LF*. The *PDM* backward pass method establishes the latest possible dates for each activity to be completed without extending the project's minimum duration, using the given Equation (3.11)–(3.13) [17].

(3.11) $$LS_{i+1} = LF_{i+1} - D_{i+1}$$

where: LS_{i+1} – late start for activity $i+1$ (in days), LF_{i+1} – late finish for activity $i+1$ (in days), D_{i+1} – duration for activity $i+1$ (in days)

(3.12) $$LF_i = LF_j = LS_{i+1}$$

where: LF_i – late finish for activity i (in days), LF_j – late finish for activity j (in days), LS_{i+1} – late start for activity $i+1$ (in days)

(3.13) $$LF_{i-1} = \min\{LS_i; LS_j\}$$

where: LF_{i-1} – late finish for activity $i-1$ (in days), LS_j – late finish for activity j (in days), LS_i – late start for activity i (in days).

Total float for each activity is calculated using the given Equation (3.14) [17].

(3.14) $$TF_{i+1} = LF_{i+1} - EF_{i+1}$$

where: TF_{i+1} – total float for activity $i+1$ (in days), EF_{i+1} – early finish for activity $i+1$ (in days), LF_{i+1} – late finish for activity $i+1$ (in days).

In the current research, it was previously stated that the actual work duration consists of the normal duration and the risk lag time (*RLT*). Figure 4 presents an example of a precedence diagram incorporating *RLT* for activity *j* [17].

The *EF* and *LS* for activities with risk lag such as *j* on Figure 4 are determined using Equations (3.15) and (3.16) instead of Equations (3.9) and (3.11) [17].

(3.15) $$EF_j = ES_j + D_j + RLT_j$$

where: EF_j – early finish for activity j (in days), ES_j – early start for activity j (in days), D_j – duration for activity j (in days), RLT_j – risk lag time for activity j (in days)

(3.16) $$LS_j = LF_j - D_j - RLT_{j1}$$

where: LS_j – late start for activity j (in days), LF_j – early finish for activity j (in days), D_j – duration for activity j (in days), RLT_j – risk lag time for activity j (in days).

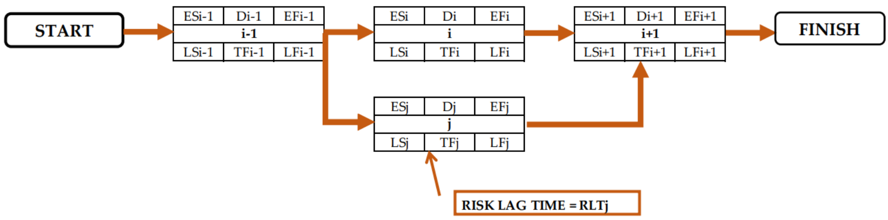

Fig. 4. Example of input *RLT* into precedence diagram

3.3. Author's application with Markov chain

The theoretical implementation of the application model was designed to provide an accurate estimation of the overall construction project execution time. Figure 5 demonstrates the model used to calculate the overal project timeline.

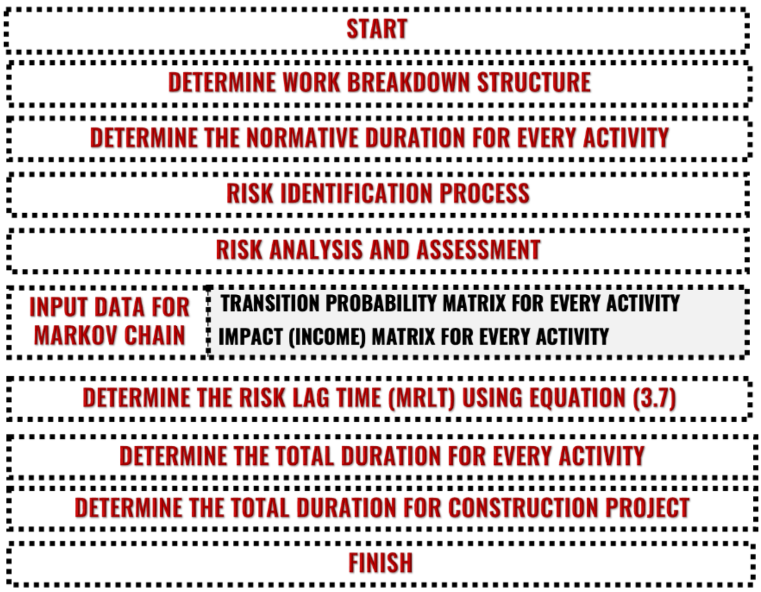

Fig. 5. Application flowchart

To initiate a newly initiated construction project, managed by the construction project manager or planner first develops a project breakdown hierarchy, systematically segmenting the project into smaller sub-projects organized hierarchically to accomplish the project objectives [20]. This structure encompasses work packages, work units, and activities. Next, the connections between activities are defined, and the standard (normative) duration of each task is determined. The Early Start (*ES*), Early Finish (*EF*), Late Start (*LS*), and Late Finish

(*LF*) times are then calculated for each activity, as well as the critical path and overall project duration, using the Precedence Diagramming Method (*PDM*) and Equations (3.8)–(3.14). The following step involves identifying the risks associated with each construction activity. After this, the project leads and planners in construction investigate the causes and evaluate these identified risks for each activity, dividing the risk assessment. The project manager estimates the probability level for every states and transition between states for each activity, based on their expertise or professional judgment input. The impact assessment should reflect the extent of severity or potential damage that could influence the project. If a risk event impacts the activity completion time, its effect can be measured in days. The probability and impact levels are represented in matrix form using Equations (3.1) and (3.5). Determining the risk lag time using Markov Chain theory (*MRLT*) is conducted through Equations (3.2)–(3.4) and (3.6). For subsequent calculations of *EF* and *LS* using Equations (3.15) and (3.16), the *MRLT* equals *RLT*. The total duration of construction projects is then computed using Equations (3.8)–(3.14).

4. Numerical example

4.1. Input parameters

The proposed application was deployed to conduct a risk assessment for the construction of a six-story commercial building in Warsaw [17]. The overal standard duration of construction is 243 days, not accounting for the occurrence and impact of risks on the work. The TOS for the building's construction entails dividing the project into two sections of equal work volume for the execution of foundation and monolithic works. These tasks are scheduled to be carried out concurrently by two independent contractors. The remaining construction activities will be completed by a single team. The task list and their standard durations are determined in accordance with the Construction Norms and Regulations (*KNR*). This information is systematically presented in aggregated categories in Table 1 and Figure 6, depicted as a Critical Path Method (*CPM*) diagram. The Early Start (*ES*), Early Finish (*EF*), Late Start (*LS*), Late Finish (*LF*), and Total Float (*TF*) for this construction project are calculated using Equations (3.8)–(3.14).

Table 1. Construction tasks and their time allocations

Task	**Description of Task**	**Normal Duration, days**	**Previous Task**
A	Site Survey and Preparatory Works	30	–
B	Execution of Foundation Works in the Section 1	45	A
C	Execution of Foundation Works in the Section 2	45	A
D	Execution of Monolithic Frame Works in the Section 1	46	B

Table continued on the next page

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Task	Description of Task	Normal Duration, days	Previous Task
E	Execution of Monolithic Frame Works in the Section 2	46	C
F	Structural Masonry Works	30	D, E
G	Interior and Exterior Finishing Tasks	72	F
I	Environmental and Aesthetic Site Works	20	G

Table continued on the next page

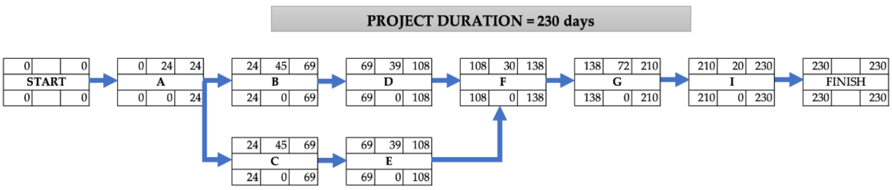

Fig. 6. Aligning Work Processes with the *CPM* Scheduling Diagram [17]

4.2. Risk identification

Based on the chosen organizational and technological approach, a risk identification process is carried out to assess potential impacts on the construction duration. This process involves recognizing risk events that may arise during the construction phase. For this case study, project planners have identified six activities susceptible to risks: Site Survey and Preparatory Works; Execution of Foundation Works in the Section 1; Execution of Foundation Works in the Section 2; Execution of Monolithic Frame Works in the Section 1; Execution of Monolithic Frame Works in the Section 2; Structural Masonry Works. The risk events and risk factors (risk factors can be defined as conditions that may lead to a risk occurring) identified and illustrated in Table 2.

Table 2. The project activities along with their associated risk events and influencing factors

Description of Task	Risk event	Risk Factor
(A) Site Survey and Preparatory Works	Bad weather	The amount of rainfall
(B) Execution of Foundation Works in the Section 1	Unreliable soil information	Error from survey team Unexpected underground objects
(C) Execution of Foundation Works in the Section 2	Unreliable soil information	Error from survey team Unexpected underground objects

Table continued on the next page

Table continued from the previous page

Description of Task	Risk event	Risk Factor
(D) Execution of Monolithic Frame Works in the Section 1	Tower crane failure	Lack of maintenance Carry overload
	Bad weather	The amount of rainfall
(E) Execution of Monolithic Frame Works in the Section 2	Misunderstanding of technical documentation by workers	Low language proficiency Lack of technical education
	Bad weather	The amount of rainfall
(F) Structural Masonry Works	Worker absenteeism	Worker illness Rule and regulation

4.3. Risk analysis and assessment

Project management professionals and schedulers evaluated and estimated the likelihood of risk occurrence based on their expertise, data from subcontractors, and various expert assessment methods for each identified risk event. The probability of a risk event for A, B, C, D, E, F activities with risk factors using Equation (3.1) is shown in matrix form as Equations (4.1)–(4.6):

$$[p_{ij}(n)]^A = \begin{bmatrix} 0.52 & 0.48 \\ 0.50 & 0.50 \end{bmatrix} \tag{4.1}$$

$$[p_{ij}(n)]^B = \begin{bmatrix} 0.75 & 0.25 \\ 0.00 & 1.00 \end{bmatrix} \tag{4.2}$$

$$[p_{ij}(n)]^C = \begin{bmatrix} 0.70 & 0.30 \\ 0.50 & 0.50 \end{bmatrix} \tag{4.3}$$

$$[p_{ij}(n)]^D = \begin{bmatrix} 0.55 & 0.10 & 0.15 & 0.20 \\ 0.15 & 0.60 & 0.15 & 0.10 \\ 0.15 & 0.15 & 0.15 & 0.55 \\ 0.25 & 0.10 & 0.15 & 0.50 \end{bmatrix} \tag{4.4}$$

$$[p_{ij}(n)]^E = \begin{bmatrix} 0.45 & 0.15 & 0.15 & 0.25 \\ 0.10 & 0.65 & 0.15 & 0.10 \\ 0.15 & 0.05 & 0.05 & 0.75 \\ 0.25 & 0.15 & 0.15 & 0.45 \end{bmatrix} \tag{4.5}$$

$$[p_{ij}(n)]^F = \begin{bmatrix} 0.75 & 0.25 \\ 0.60 & 0.40 \end{bmatrix} \tag{4.6}$$

Furthermore, the effect of risk events has been quantified. The impact level is measured in days and indicates the potential increase in workdays if a risk event happens. For this example, the impact level and the calculated impact level as an income matrix are determined using Equation (3.5) and summarized in Equations (4.7)–(4.12).

$$[r_{ij}(n)]^A = \begin{bmatrix} 0 & 10 \\ 0 & 10 \end{bmatrix} \tag{4.7}$$

$$[r_{ij}(n)]^B = \begin{bmatrix} 0 & 18 \\ 0 & 18 \end{bmatrix} \tag{4.8}$$

$$[r_{ij}(n)]^C = \begin{bmatrix} 0 & 8 \\ 0 & 8 \end{bmatrix} \tag{4.9}$$

$$[r_{ij}(n)]^D = \begin{bmatrix} 0 & 6 & 12 & 12 \\ 0 & 6 & 12 & 12 \\ 0 & 6 & 12 & 12 \\ 0 & 6 & 12 & 12 \end{bmatrix} \tag{4.10}$$

$$[r_{ij}(n)]^E = \begin{bmatrix} 0 & 15 & 15 & 8 \\ 0 & 15 & 15 & 8 \\ 0 & 15 & 15 & 8 \\ 0 & 15 & 15 & 8 \end{bmatrix} \tag{4.11}$$

$$[r_{ij}(n)]^F = \begin{bmatrix} 0 & 18 \\ 0 & 18 \end{bmatrix} \tag{4.12}$$

The outcomes of the calculations will be detailed in the Results and Discussion section.

5. Results and discussion

Initially, we will determine the stationary probability distribution for each state of each task using Equation (3.4) and the total expected lag time (*MRLT*) using Equation (3.6). The results will be presented in Table 3.

Table 3. Construction tasks and their time allocations with *RLT*

Task	**Description of Task**	**The stationary distribution of a Markov chain for each state**				**Total expected lag time (MRLT), days**
		1	2	3	4	
A	Site Survey and Preparatory Works	0.51	0.49	–	–	4.9
B	Execution of Foundation Works in the Section 1	0	1	–	–	18
C	Execution of Foundation Works in the Section 2	0.63	0.37	–	–	3.4
D	Execution of Monolithic Frame Works in the Section 1	0.31	0.21	0.15	0.33	7.0
E	Execution of Monolithic Frame Works in the Section 2	0.24	0.27	0.14	0.35	8.5
F	Structural Masonry Works	0.71	0.29	–	–	5.3

The duration of each individual task will be determined using Equation (3.9) and (3.11).

Table 4. Construction tasks and their time allocations with RLT

Activity	Normal duration, days (1)	The activity duration with authors' framework, days (2)	The activity duration with PxLT, days (3)	The activity duration with LT, days (4)	The differences between (1) and (2), %	The differences between (1) and (3), %	The differences between (2) and (3), %
A	30	35	35	40	16.7	16.7	0.0
B	45	63	50	63	40.0	11.1	26.0
C	45	49	48	53	8.9	6.7	2.1
D	46	53	49	58	15.2	6.5	8.2
E	46	55	49	61	19.6	6.5	12.2
F	30	36	35	48	20.0	16.7	2.9
G	72	72	72	72	0.0	0.0	0.0
I	20	20	20	20	0.0	0.0	0.0
Total duration	243	279	261	301	14.8	7.4	6.9
P – probability for states without transition, %. Data from (4.1)–(4.5) LT – lag time, days. Data from (4.6)–(4.11)							

The data in Table 3 will be analyzed. Initially, the results for individual construction tasks D and E, which are affected by 2 risks, will be considered. The difference between the normative duration values for tasks D and E and the actual duration of tasks D and E, considering risks using the proposed framework calculation, is 15.2% and 19.6%, respectively. Moreover, the difference between the normative duration values for tasks D and E and the actual duration of tasks D and E, considering risks using the calculation based on the product of probability and risk impact, is 6.5% for both tasks. The discrepancy between the values obtained using two different methods for estimating the duration of individual tasks ranges from 0% to 2.9%, except for task B, where this difference is 26.0%. This difference arises because, according to expert estimation, there might be a probability of occurrence of a risk event in task B, resulting in two states – normal execution without a risk event and execution with risk events. The expert indicates that returning to normal work mode in case of a risk event is not feasible. Furthermore, the algorithm for estimating the duration of individual tasks considering risks using the product of probability does not account for the possibility or impossibility of returning to normal work mode (without risk), resulting in a significantly lower estimation of additional time. Assuming that the additional time equals the product of probability and impact, then the matrix (4.2) should have the following form:

$$\left[p_{ij}(n)\right]^B = \begin{bmatrix} 0.75 & 0.25 \\ 0.75 & 0.25 \end{bmatrix} \tag{5.1}$$

So, according to this approach, the probability of returning to normal work mode without risk is 75%, but the expert assessed that it is not possible to return to normal work mode at all when risks occur.

6. Conclusions

Estimating the normative duration for executing a construction project during the design phase to closely match the actual implementation time is a pertinent and significant task globally. As demonstrated in the example, Markov chains can be effectively utilized to model the various states in which construction activities might exist during execution. This characteristic of Markov processes enables the simultaneous consideration of multiple risks what makes it different from other methods. Furthermore, when considering a single risk, the duration values for work impacted by risk using these methodologies are approximately the same, provided that the expert has assessed the likelihood of returning to a normal state after a risk event occurs. This indicates that the proposed framework based on the Markov process can be a valuable tool for estimating the duration of construction activities by accounting for the influence of both single and multiple risks simultaneously, offering a more objective assessment of work duration considering the actual transition probabilities between different states.

In prospective research, the authors aim to refine the proposed methodology for estimating the duration of individual construction tasks by integrating fuzzy assessment techniques to evaluate the probability and impact levels of risk events.

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Określenie czasu trwania przędsięwzięcia budowlanego z wykorzystaniem łańcucha Markowa

Słowa kluczowe: łańcuch Markowa, rozwiązania technologiczno- organizacyjne, czas trwania robót budowlanych, ryzyko, planowanie sieciowe przedsięwzięcia, planowanie realizacji budowy

Streszczenie:

Przedsięwzięcie budowlane to złożony proces obejmujący szereg wzajemnie powiązanych działań, które mają na celu zaspokojenie określonych potrzeb budowlanych inwestora lub właściciela obiektu budowlanego. Zazwyczaj w kontekście realizacji przedsięwzięcia budowlanego zaspokojenie potrzeb stanowi dotrzymanie planowanych (obliczonych) wartości czasu i kosztu. Z analizy literatury wynika, że najczęstszymi powodami opóźniającymi czas trwania robót (procesów) budowlanych są występowania czynników ryzyka, których można na etapie planowania uwzględniać i oszacować. Istotne znaczenie w tym kontekście mają właściwe ustalone rozwiązania technologiczno-organizacyjne (RTO) przedsięwzięcia budowlanego, z których oblicza się czas trwania. RTO składa się z przyjętych rozwiązań dla poszczególnych robót budowlanych, dla których powstaje harmonogram oraz odpowiedni model sieci zależności, gdzie czas trwania przedsięwzięcia oblicza się z wykorzystaniem metody drogi krytycznej

na podstawie normatywnych czasów trwania poszczególnych robót, czyli przeciętnych norm pracy dla rozpatrywanych robót. Takie podejście pozbawione jest uwzględnienia zakłócających czynników ryzyka. Dlatego, celem artykułu jest przedstawienie algorytmu szacowania czasu trwania przedsięwzięcia budowlanego z uwzględnieniem różnych czynników ryzyka i ich wspólnego oddziaływania z wykorzystaniem łańcucha Markowa. Przy analizowaniu poszczególnych robót budowlanych można zauważyć, że w czasie trwania robót mogą zaistnieć różne stany - stan bez oddziaływania czynników ryzyka, stan z oddziaływaniem jednego czynnika ryzyka lub stan z oddziaływaniem kilku czynników ryzyka jednocześnie. Z tego wynika, że czas trwania danej roboty budowlanej będzie składać się z summy czasów trwania każdego stanu. Obliczenie czasu przebywania w każdym stanie i przejście z jednego stanu do drugiego z możliwością lub bez możliwości powrotu można dokonać z wykorzystaniem łańcucha Markowa. Łańcuch Markowa jest szczególnym przypadkiem procesu Markowa, który polega na tym, że ciąg zdarzeń, w którym prawdopodobieństwo każdego zdarzenia zależy jedynie od wyniku poprzedniego zdarzenia. Proponowany przez autorów algorytm obliczenia czasu trwania polega na tym, że planista określa różne czynniki ryzyka dla poszczególnych robót. Następnie ustala poziom prawdopodobieństwa przejścia pomiędzy stanami na podstawie wiedzy eksperckiej. Przykładowo rozważmy następującą sytuację: robota budowlana w czasie realizacji może być w trzech stanach: stan normalny - bez oddziaływania czynników, stan z oddziaływaniem czynnika nr 1 i stan z oddziaływaniem czynnika nr 2. Planista na podstawie wiedzy eksperckiej ustalił możliwości przejścia z jednego stanu do drugiego i poziom prawdopodobieństwa, z którym to przejście może zdarzyć się. Także planista na podstawie doświadczenia ustalił, że nie jest możliwie równoległe oddziaływanie dwóch czynników ryzyka. Wtedy omówiony przypadek można przedstawić w postaci grafu stanów i przejść, gdzie wierzchołki przedstawiają poszczególne stany, a łuki Pij przejścia. Autorski algorytm polega na prognozowaniu czasu trwania robót na podstawie prawdopodobieństwa przejść między różnymi stanami w realizacji przedsięwzięcia budowlanego. W tym kontekście każdy stan może reprezentować określoną fazę trwania roboty budowlanej. Prawdopodobieństwa przejść między tymi stanami mogą być wyznaczone na podstawie danych statystycznych z poprzednich przedsięwzięć, na podstawie opinii eksperckich lub z innych źródeł. W niniejszym artykule prawdopodobieństwo przejścia między stanami ustalono na podstawie opinii ekspertów. Proponowane podejście pozwala przewidywać prawdopodobny czas zakończenia przedsięwzięcia oraz oceniać wpływ różnych czynników na jego czas trwania. Artykuł także zawiera przykład liczbowy, który przedstawia działanie proponowanego podejścia. W obliczeniach przy użyciu proponowanego algorytmu wartości czasu trwania robót, dla których określono wpływ dwóch czynników ryzyka, były wyższe o 10-12% niż normatywny czas trwania. Natomiast przy porównaniu wartości normatywnego czasu trwania z wartościami czasu trwania z uwzględnieniem ryzyka przy wykorzystaniu operacji iloczynu między poziomem ryzyka a prawdopodobieństwem jego wystąpienia, różnica zmieściła się w granicach 5-7%. Z kolei, przy uwzględnieniu tylko jednego czynnika ryzyka, wartości czasu trwania dla robót z uwzględnieniem czynników ryzyka przy użyciu tego samego podejścia są mniej więcej takie same. Otrzymane wyniki pokazują praktyczne możliwości zastosowania w zakresie efektywnego planowania przedsięwzięcia budowlanego z uwzględnieniem czynników ryzyka. Pozwala to stwierdzić, że zaproponowane podejście oparte na procesie Markowa może być narzędziem do szacowania czasu trwania robót budowlanych z uwzględnieniem wpływu zarówno pojedynczych, jak i wielu czynników ryzyka, oraz dostarcza bardziej obiektywną ocenę czasu trwania roboty co do faktycznej możliwości przejścia z jednego stanu do drugiego. W dalszych badaniach autorzy planują udoskonalać przedstawioną metodologię oceny czasu trwania poszczególnych robót budowlanych z wykorzystaniem elementów teorii zbiorów rozmytych.

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