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Research paper

Elastic buckling of a simply supported functionally graded beam with rigid partitions

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Abstract: The subject of this paper is a simply supported functionally graded beam in three variants: (A) without rigid partitions, (B) with two rigid partitions, and (C) with four rigid partitions. The mechanical properties, specifically Young's modulus, vary symmetrically in the depth direction of these beams. An analytical model of the beam, considering the individual nonlinear shear deformation of a plane cross-section, has been developed. Two differential equations of equilibrium for the beam, based on the principle of stationary total potential energy, were derived. This system of two equations was approximately solved for each beam variant, and the critical forces were determined with the distinguished shear effect coefficient. Detailed calculations were performed for exemplary beams with selected dimensionless sizes and mechanical properties. The results of these calculations are presented in tables and figures. The main goal of the research is to develop an analytical model of this beam and investigate the impact of rigid partitions placed within the beam on the critical force.

Keywords: analytical modelling, critical forces, elastic buckling, functionally graded beam, rigid partitions of the beam

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1. Introduction

Sandwich structures, which originated in the 20th century, have been intensively improved in the 21st century. Icardi [1] developed a sublaminate model that generalizes the zig-zag approximation for analyzing laminated and sandwich beams. He then conducted exemplary tests on laminated and sandwich beams to demonstrate the practical advantages of this model. Yang and Qiao [2] presented a higher-order analytical impact model for studying a soft-core sandwich beam under foreign body impact. They performed exemplary analytical and numerical FEM (LS-DYNA) simulations of selected beams and demonstrated that the developed model could be effectively used in the design of impact-resistant sandwich structures. Reddy [3] reformulated the classical and shear theories of beams and plates by incorporating the nonlocal differential constitutive relations of Eringen and von Karman's nonlinear strains. He derived the equilibrium equations for the nonlocal beam theories, as well as the classical and first-order equilibrium equations for the shear plate theories with von Karman nonlinearity. Phan et al. [4] formulated a new one- dimensional higher-order theory for orthotropic elastic sandwich beams, which extends the higher-order sandwich panel theory. This theory accounts for axial and transverse displacements, as well as rotation at the core center. Bardella and Mattei [5] developed an analytical model for a symmetric sandwich beam, incorporating the Jourawski (Zhuravsky) method. They performed both analytical and numerical FEM simulations of sample sandwich beams and demonstrated the consistency of the results. Goncalves et al. [6] established the foundation for analyzing linear buckling and free vibrations of sandwich beams, incorporating the modified Timoshenko beam model. Based on the exact general solution of the beam governing equations, they formulated an exact approximate stiffness matrix for finite elements. Pei et al. [7] modified the higher-order theory for functionally graded (FG) beams, which serves as a reference point for the studied problems of these beams. Birman and Kardomateas [8], based on a review of 363 published papers, highlighted contemporary trends in the development of sandwich structures, with a focus on aviation, civil and marine engineering, as well as electronics and biomedicine. Bessaim et al. [9] performed an analytical study on the buckling problem of functionally graded nanobeams embedded in an elastic foundation, considering the refined hyperbolic shear deformation theory. Mokhtar et al. [10] developed a new, simple shear deformation theory to analyze the buckling problem of single-layer graphene sheets. Their approach takes into account Eringen's nonlocal differential constitutive relations and demonstrates that the proposed theory is both accurate and straightforward for solving the buckling problem of nanoplates. Magnucka-Blandzi [11] developed an original analytical model for a seven-layer beam with three corrugated cores, considering the shear effect. She then analytically investigated the bending and buckling of both three-layer and seven-layer beams. Sayyad and Ghugal [12] presented a review of research, based on 250 published articles, on the modeling and analysis of functionally graded beams used in various engineering fields, and identified directions for further research. Bedia et al. [13] developed novel two-variable shear deformation beam theories to study the bending and buckling of nanobeams, incorporating nonlocal stress and strain gradient theory. Zhen et al. [14] highlighted the significant importance of transverse shear stresses in the analysis of buckling in laminated structures and, consequently, the limitations of models that do not satisfy the condition of interlaminar continuity of these stresses. They identified models that could be useful for future investigations. Garg and Chalak [15] proposed a new higher-order zigzag theory for the analysis of laminated sandwich beams, which satisfies the following conditions: interlaminar transverse stress continuity, and zeroing of these stress values at the top and bottom surfaces. Bellifa et al. [16] analyzed the effect of porosity on the thermal stability of a functionally graded beam under various boundary conditions. They developed an analytical model of the beam based on the Euler–Bernoulli theory. Magnucki [17] developed a shear deformation theory for homogeneous, sandwich, and functionally graded beams and analytically studied the bending of these beams. Magnucki and Magnucka-Blandzi [18] developed an analytical model for a sandwich beam with an asymmetric structure, incorporating the individual nonlinear shear deformation theory. They then analytically investigated the bending of this beam, with one end simply supported and the other clamped. Kożuch and Skrętkowicz [19] presented a general static analysis of hybrid beams with cross-sections composed of both steel and concrete elements, highlighting the significant contribution of both materials to the transfer of transverse shear. Mesbah et al. [20] developed numerical models using Finite Element Method (FEM) to study free vibration and buckling problems in functionally graded porous beams (FGP). Magnucki et al. [21] investigated analytically and numerically (FEM) the buckling problem of a sandwich plate with an individually functionally graded core, considering the nonlinear shear deformation theory. Czubacki and Lewinski [22] presented methodologies for assessing critical axial buckling loads and lateral buckling loads of straight elastic bars with open cross sections made of a homogenous isotropic material. Krystosik [23] proposed an algorithm for evaluating the load-bearing capacity and stiffness of reinforced column bases using the component method. Magnucki [24] developed two analytical models for a five-layer composite beam: one classical model incorporating the zig-zag theory and the other incorporating an individual nonlinear shear deformation theory. He then studied the bending of this beam using these two models. Addou et al. [25] enhanced the hyperbolic shear deformation theory to analyze the mechanical behavior of isotropic beams and sandwich functionally graded material (FGM) beams under various boundary conditions. Belabed et al. [26] proposed an innovative finite element model based on higher-order beam theory to analyze the bending and buckling behavior of functionally graded carbon nanotube-reinforced composite beams supported by a Winkler-Pasternak elastic foundation. Bardella [27] developed an analytical model for sandwich beams, which he used to determine novel stress distributions near the clamped end of the beam. Magnucki et al. [28] investigated analytically and numerically (FEM) the buckling problem and fundamental natural frequencies of a sandwich beam with an asymmetric structure for three variants of its end support.

The subject of the paper is an axially compressed, simply supported functionally graded beam of length L, depth h, and width b, in three variants with rigid partitions: (A) without rigid partitions, (B) with two rigid partitions, and (C) with four rigid partitions (Fig. 1).

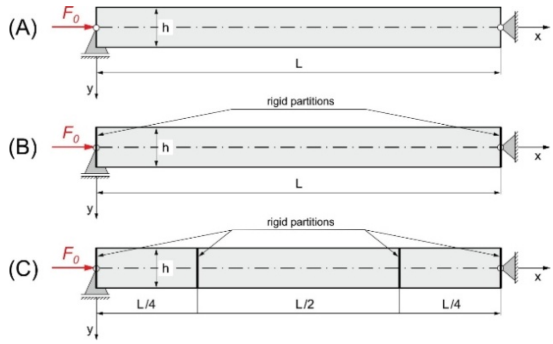

Fig. 1. Schemes of the three variants of the functionally graded beam under the compressive force F_0

The main goal of this paper is to conduct an analytical study on the effect of rigid beam partitions on the critical force value. The originality of the paper lies in developing a novel analytical model that incorporates nonlinear shear deformation for FG beams with rigid partitions and determining how these partitions influence buckling resistance.

2. Analytical model of the functionally graded beam

Young's modulus vary in the depth direction of the beam as follows:

$$E(\eta) = E f_e(\eta), \quad f_e(\eta) = \frac{1}{2+\alpha}\left[1 + \sin^2(n\pi\eta) + \alpha \sin^2(\pi\eta)\right] \tag{2.1}$$

where: $\eta = y/h$ – dimensionless coordinate $(-1/2 \le \eta \le 1/2)$, n – odd natural number, α – coefficient-real number.

The graphs of this dimensionless function for $n = 1, 3, 5$ and $\alpha = 2$ are shown in Fig. 2.

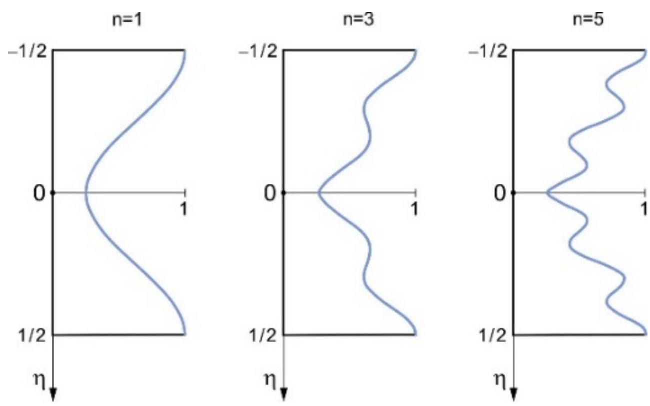

Fig. 2. Graphs of the dimensionless function $f_e(\eta)$ in (2.1)

The scheme of a planar cross section nonlinear deformation of the beam after its buckling is shown in Fig. 3.

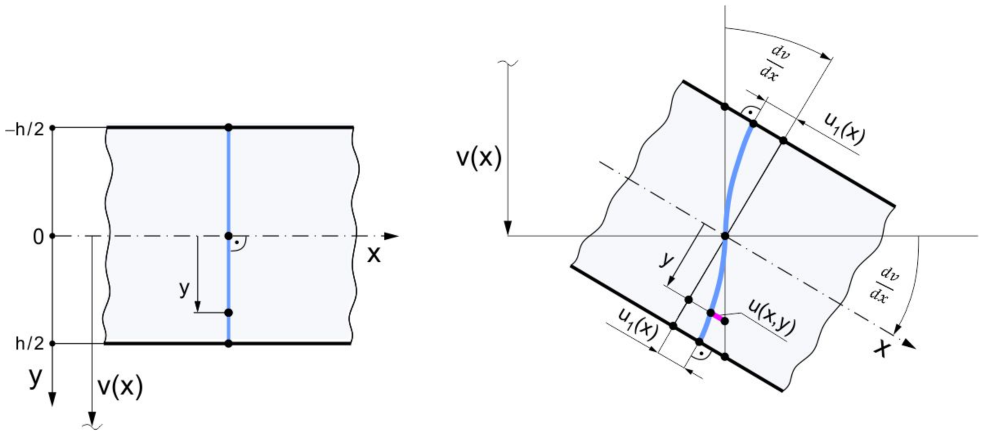

Fig. 3. The scheme of the planar cross-section deformation of the beam

According to this Fig. 3, the longitudinal displacement are as follows:

$$u\left(x,\eta\right)=-h\left[\eta\frac{\mathrm{d}v}{\mathrm{d}x}-f_d\left(\eta\right)\psi\left(x\right)\right] \tag{2.2}$$

where: $v\left(x\right)$ – the deflection function, $\psi\left(x\right)=u_1\left(x\right)/h$ – the dimensionless displacement function of the shear effect, $f_d\left(\eta\right)$ – the unknown dimensionless deformation function.

Consequently, the strains and stresses are in the following forms:

$$\varepsilon_x\left(x,\eta\right)=\frac{\mathrm{d}u}{\mathrm{d}x}=-h\left[\eta\frac{d^2v}{\mathrm{d}x^2}-f_d\left(\eta\right)\frac{\mathrm{d}\psi}{\mathrm{d}x}\right],\quad \gamma_{xy}\left(x,\eta\right)=\frac{\mathrm{d}v}{\mathrm{d}x}+\frac{\partial u}{h\partial\eta}=\frac{\mathrm{d}f_d}{\mathrm{d}\eta}\psi\left(x\right) \tag{2.3}$$

$$\sigma_x\left(x,\eta\right)=E\varepsilon_x\left(x,\eta\right)f_e\left(\eta\right),\quad \tau_{xy}\left(x,\eta\right)=\frac{E}{2\left(1+\nu\right)}\gamma_{xy}\left(x,\eta\right)f_e\left(\eta\right) \tag{2.4}$$

where: ν – the Poisson's ratio.

Taking into account papers [17, 18, 21] and [28] the unknown dimensionless deformation function $f_d\left(\eta\right)$ is determined as follows:

$$f_d\left(\eta\right)=\frac{1}{C_0}\int\frac{\int_{-1/2}^{\eta}\left[1+\sin^2\left(n\pi\eta_1\right)+\alpha\sin^2\left(\pi\eta_1\right)\right]\mathrm{d}\eta_1}{1+\sin^2\left(n\pi\eta\right)+\alpha\sin^2\left(\pi\,\eta\right)}\mathrm{d}\eta \tag{2.5}$$

where: $C_0=\int\limits_0^{1/2}\frac{\int_{-1/2}^{\eta}\left[1+\sin^2\left(n\pi\eta_1\right)+\alpha\sin^2\left(\pi\eta_1\right)\right]\mathrm{d}\eta_1}{1+\sin^2\left(n\pi\eta\right)+\alpha\sin^2\left(\pi\,\eta\right)}\mathrm{d}\eta$ – the dimensionless coefficient.

The elastic strain energy is in the form

$$U_{\varepsilon,\gamma} = \frac{1}{2}Ebh\int_0^L\int_{-1/2}^{1/2}\left\{\varepsilon_x^2(x,\eta) + \frac{1}{2(1+\nu)}\gamma_{xy}^2(x,\eta)\right\}f_e(\eta)\,\mathrm{d}\eta\mathrm{d}x \tag{2.6}$$

The work of the load–axial force

$$W = \frac{1}{2}F_0\int_0^L\left(\frac{\mathrm{d}v}{\mathrm{d}x}\right)^2\mathrm{d}x \tag{2.7}$$

Taking into account the principle of stationary total potential energy $\delta\left(U_{\varepsilon,\gamma} - W\right) = 0$, with consideration of the expressions (2.3), (2.6) and (2.7), the system of two differential equations of equilibrium for this beam, after simply transformation, is obtained in the following form:

$$\overline{J}_z\frac{\mathrm{d}^2v}{\mathrm{d}\xi^2} - C_{v\psi}\frac{\mathrm{d}\psi}{\mathrm{d}\xi}L + \overline{F}_0\lambda^2 v(\xi) = 0 \tag{2.8}$$

$$C_{v\psi}\frac{\mathrm{d}^3v}{\mathrm{d}\xi^3} - C_{\psi\psi}\frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2}L + C_{\psi}\lambda^2\psi(\xi)L = 0 \tag{2.9}$$

where: $\xi = x/L$ – the dimensionless coordinate ($0 \le \xi \le 1$), $\lambda = L/h$ – the relative length, $\overline{F}_0 = F_0/(Ebh)$ – the dimensionless force, $\overline{J}_z = \int_{-1/2}^{1/2}\eta^2 f_e(\eta)\mathrm{d}\eta$,
$C_{v\psi} = \int_{-1/2}^{1/2}\eta f_d(\eta)\, f_e(\eta)\mathrm{d}\eta$, $C_{\psi\psi} = \int_{-1/2}^{1/2} f_d^2(\eta)\, f_e(\eta)\mathrm{d}\eta$,
$C_{\psi} = \dfrac{1}{2(1+\nu)}\int_{-1/2}^{1/2}\left(\dfrac{\mathrm{d}f_d}{\mathrm{d}\eta}\right)^2 f_e(\eta)\mathrm{d}\eta$ – dimensionless coefficients.

3. Analytical study of the buckling problem

The system of two differential equations of equilibrium (2.8) and (2.9) is approximately solved with two assumed functions for three beam variants (Fig. 1).

3.1. The variant (A) – without rigid partitions

The deflection function and dimensionless displacement function of the shear effect are assumed in the following forms

$$v(\xi) = v_a\sin(\pi\xi), \quad \psi(\xi) = \psi_a\cos(\pi\xi) \tag{3.1}$$

where: v_a, ψ_a – coefficients of this functions.

The forms of these functions are shown in Fig. 4.

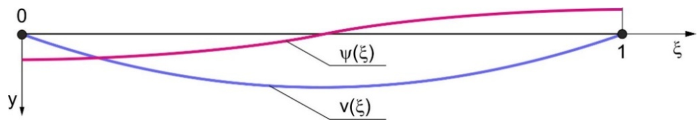

Fig. 4. The forms of the functions in (3.1)

Substituting these functions (3.1) into two differential equations of equilibrium (2.7) and (2.9), after simple transformation, one obtains the coefficient of the shear effect and the dimensionless critical force in the following forms:

$$C_{\text{se}}^{(\text{A})} = \frac{\pi^2 C_{v\psi}^2}{\pi^2 C_{\psi\psi} + \lambda^2 C_{\psi}} \frac{1}{\overline{J}_z}, \qquad \overline{F}_{0,\text{CR}}^{(\text{A})} = \left(1 - C_{\text{se}}^{(\text{A})}\right) \frac{\pi^2 \overline{J}_z}{\lambda^2} \tag{3.2}$$

Exemplary calculations are carried out for this beam with the selected data: $\nu = 0.3$, $\alpha = 2$, $\lambda = 20$. The results of the analytical calculations for three values of the natural number $n = 1, 3, 5$ are specified in Table 1.

Table 1. Values of the coefficient of the shear effect and the dimensionless critical force

n	**1**	**3**	**5**
$C_{\text{se}}^{(\text{A})}$	0.0129005	0.0102834	0.0101530
$\overline{F}_{0,\text{CR}}^{(\text{A})}$	0.00173123	0.00159836	0.00158757

3.2. The variant (B) – with two rigid partitions

The deflection function and dimensionless displacement function of the shear effect are assumed in the following forms

$$v(\xi) = v_a \sin(\pi\xi), \qquad \psi(\xi) = \psi_a \sin(2\pi\xi) \tag{3.3}$$

where: v_a, ψ_a – coefficients of this functions.

The forms of these functions are shown in Fig. 5.

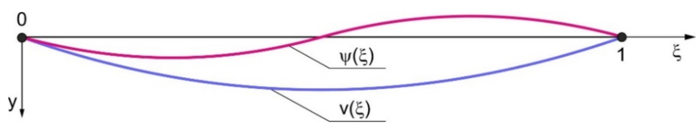

Fig. 5. The forms of the functions in (3.3)

Substituting these functions (3.3) into two differential equations of equilibrium (2.8), (2.9), and using the Galerkin method

$$\int_0^1 \Re\,[\text{Eq. (2.8)}] \sin(\pi\xi)\,d\xi = 0, \qquad \int_0^1 \Re\,[\text{Eq. (2.9)}] \sin(2\pi\xi)\,d\xi = 0 \tag{3.4}$$

where: $\Re$ [Eq. (2.8)], $\Re$ [Eq. (2.9)] – left parts of these equations.

Consequently, after simple transformation, one obtains the coefficient of the shear effect $C_{se}^{(B)}$ and the dimensionless critical force $\overline{F}_{0,CR}^{(B)}$ in the following forms:

$$C_{se}^{(B)} = \left(\frac{8}{3}\right)^2 \frac{C_{v\psi}^2}{4\pi^2 C_{\psi\psi} + \lambda^2 C_\psi} \frac{1}{\overline{J}_z}, \qquad \overline{F}_{0,CR}^{(B)} = \left(1 - C_{se}^{(B)}\right) \frac{\pi^2 \overline{J}_z}{\lambda^2} \tag{3.5}$$

Exemplary calculations are carried out for this beam with the same data as for the variant (A). The results of the analytical calculations for three values of the natural number $n = 1, 3, 5$ are specified in Table 2.

Table 2. Values of the coefficient of the shear effect and the dimensionless critical force

n	1	3	5
$C_{se}^{(B)}$	0.00893803	0.00718056	0.00709373
$\overline{F}_{0,CR}^{(B)}$	0.00173818	0.00160337	0.00159248

3.3. The variant (C) – with four rigid partitions

The deflection function and dimensionless displacement function of the shear effect are assumed in the following forms

$$v(\xi) = v_a \sin(\pi\xi), \qquad \psi(\xi) = \psi_a \sin(4\pi\xi) \tag{3.6}$$

where: v_a, ψ_a – coefficients of this functions.

The forms of these functions are shown in Fig. 6.

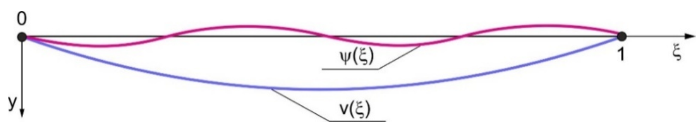

Fig. 6. The forms of the functions in (3.6)

Substituting these functions (3.6) into two differential equations of equilibrium (2.8), (2.9), and using the Galerkin method, similarly to the beam variant (B)

$$\int_0^1 \Re\,[\text{Eq. (2.8)}] \sin(\pi\,\xi)\,d\xi = 0, \qquad \int_0^1 \Re\,[\text{Eq. (2.9)}] \sin(4\,\pi\,\xi)\,d\xi = 0 \tag{3.7}$$

and after simple transformation, one obtains the coefficient of the shear effect $C_{\mathrm{se}}^{(\mathrm{C})}$ and the dimensionless critical force $\overline{F}_{0,\mathrm{CR}}^{(\mathrm{C})}$ in the following forms:

$$C_{\mathrm{se}}^{(\mathrm{C})} = \left(\frac{16}{15}\right)^2 \frac{C_{v\psi}^2}{16\pi^2 C_{\psi\psi} + \lambda^2 C_{\psi}} \frac{1}{\overline{J}_z}, \quad \overline{F}_{0,\mathrm{CR}}^{(C)} = \left(1 - C_{\mathrm{se}}^{(\mathrm{B})}\right) \frac{\pi^2 \overline{J}_z}{\lambda^2} \tag{3.8}$$

Exemplary calculations are carried out for this beam with the same data as for the variant (A). The results of the analytical calculations for three values of the natural number n =1, 3, 5 are specified in Table 3.

Table 3. Values of the coefficient of the shear effect and the dimensionless critical force

n	1	3	5
$C_{\mathrm{se}}^{(\mathrm{C})}$	0.00123970	0.00102265	0.00101234
$\overline{F}_{0,\mathrm{CR}}^{(\mathrm{C})}$	0.00175168	0.00161331	0.00160223

The results of the exemplary calculations: values of the coefficient of the shear effect C_{se} and the dimensionless critical force $\overline{F}_0$ for three beams (Tab. 1, Tab. 2, Tab. 3) are graphically presented in Fig. 7 and Fig. 8.

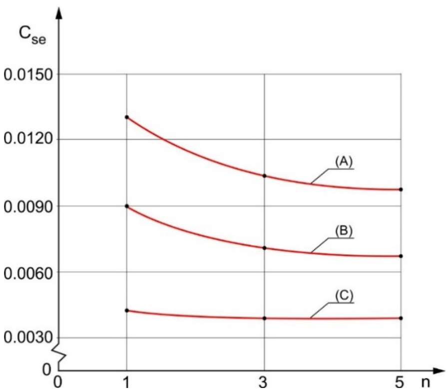

Fig. 7. The values of the coefficient of the shear effect

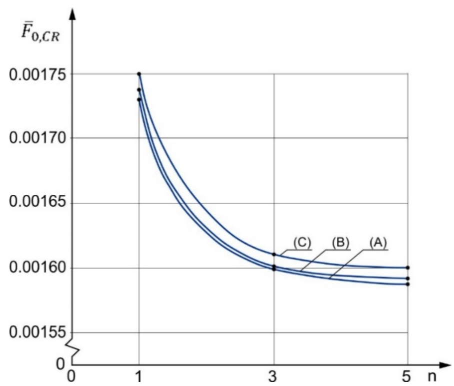

Fig. 8. The values of the dimensionless critical force

4. Conclusions

Analyzing the results of the calculations for three sample beams: (A) without rigid partitions, (B) with two rigid partitions, and (C) with four rigid partitions, it is clear that the number of rigid partitions significantly affects the shear coefficient C_{se} and consistently influences the critical force $\overline{F}_0$ of the beam. The value of the shear coefficient C_{se} decreases significantly as the number of rigid partitions increases (Fig. 7), while the value of the critical force $\overline{F}_0$ increases (Fig. 8).

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Wyboczenie sprężyste przegubowo podpartej belki funkcjonalnie stopniowana z przegrodami sztywnymi

Słowa kluczowe: modelowanie analityczne, siły krytyczne, wyboczenie sprężyste, belka funkcjonalnie stopniowana, przegrody sztywne

Streszczenie:

Przedmiotem artykułu jest belka swobodnie podparta funkcjonalnie stopniowana w trzech wariantach: (A) – bez przegród sztywnych, (B) – z dwiema przegrodami sztywnymi, (C) – z czterema przegrodami sztywnymi. Właściwości mechaniczne – moduł Younga zmienia się symetrycznie w kierunku głębokości tych belek. Opracowano model analityczny tej belki z uwzględnieniem indywidualnego nieliniowego odkształcenia ścinającego płaskiego przekroju poprzecznego. Na podstawie zasady stacjonarności całkowitej energii potencjalnej otrzymano dwa równania różniczkowe równowagi tej belki. Ten układ dwóch równań rozwiązano w sposób przybliżony dla każdego wariantu belki, a siły krytyczne zapisano z wyróżnieniem współczynnika efektu ścinania. Szczegółowe obliczenia przeprowadzono dla przykładowych belek o wybranych bezwymiarowych wymiarach i właściwościach mechanicznych. Wyniki tych obliczeń przedstawiono w tabelach i na rysunkach. Wykazano wpływ sztywnych przegród umieszczonych w belce na siłę krytyczną i na współczynnik efektu ścinania. Stwierdzono, że zwiększanie liczby sztywnych przegród zwiększa wartość siły krytycznej, natomiast zmniejsza wartość współczynnika efektu ścinania.

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