



Research paper

Surface settlement prediction of short-distance double-hole tunnel based on stochastic medium theory optimization algorithm

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Abstract: It is of great significance to predict the displacement and deformation of surface settlement during the construction of short-distance double-hole metro tunnels to prevent dynamic disasters caused by surface settlement during construction. The stochastic medium theory is studied in this paper, the optimal algorithm of stochastic medium theory is proposed, which is considered the tunnel section convergence mode, the tunnel space position and the different sections of the tunnel. The accuracy of settlement monitoring data and theoretical prediction values in tunnel construction is analyzed in combination with three engineering cases. The research shows that the applicable condition of the formula is space span ratio (L/B) less than 7; the stochastic medium simplified theoretical prediction curve and the settlement monitoring curve are better fitted, the maximum settlement position and the trend It is basically consistent with the settlement monitoring curve. The theoretical prediction curve settlement groove width is basically consistent with the settlement monitoring curve settlement groove width. The optimal algorithm and the settlement monitoring data have good consistency, which can provide some reference for the project.

Keywords: metro tunnel, close double-hole tunnel, surface subsidence, stochastic medium simplification theory

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1. Introduction

With the rapid development of China's major cities, attempts to mitigate space and traffic pressure make the construction of closely spaced double-bore metro tunnels increasingly prevalent. Research indicates that during the construction of closely spaced double-bore underground tunnels, the excavation of the first tunnel induces a disturbance zone in the surrounding geological formation. Owing to the proximity between the twin bores, the subsequent tunnel lies within the disturbance zone created by the initial excavation, this proximity may lead to increased surface settlement displacements. Consequently, devising a scientifically sound and reasonable methodology to forecast surface settlement displacements associated with closely spaced twin-bore metro tunnels remains a critical issue in the field of tunnelling and underground engineering.

In predicting surface settlement displacement due to the construction of closely spaced twin-bore underground tunnels, experts and scholars internationally employ multiple prediction methods, predominantly empirical formulas, numerical analysis, and theoretical analysis. Fengfeng Guo and Yupeng Chen et al [1] using numerical simulation to study the settlement and deformation pattern of the Mingyue Mountain Tunnel underpassing the Hurong Railway Tunnel. The numerical analysis method uses numerical simulation software to calculate and analyse the tunnel construction, which can take into account the structure of the tunnel, the soil, the surrounding buildings and other influencing factors. However, in the process of calculation and analysis, the selection of soil ontological relationship and parameters is subjective, and some parameters are approximated, so it is difficult to match the numerical simulation and the actual situation completely. Peck's formula [1] stands as a principal empirical model derived from the analysis of extensive monitored data from single-bore tunnel construction. This model is most applicable to the surface settlement analysis of shallow, single-bore tunnels beneath homogeneous grounds; however, in practice, the majority of subway tunnels are closely spaced, shallow twin-bore tunnels. This interaction between closely spaced tunnels yields different settlement behaviors compared to traditional single-bore tunnels. Numerical analysis involves using simulation software to compute and assess tunnel construction, including factors such as tunnel structure, soil properties, and adjacent structures. Yet, the subjective selection of parameters and their approximations may hinder the precise correspondence between simulations and actual conditions. Theoretical analysis applies mathematical and mechanical principles to predict surface settlement. Among these methods, the stochastic medium theory, posited by Polish scholar J. Litwiniszyn based on sandbox model experiments, originally addressed mining subsidence issues, and was later expanded and applied to underground tunnel projects by scholars such as Liu Baochen and Zhang Jiasheng [3] for surface settlement prediction. Subsequent refinements by scholars including Shi Yaxin and Tao Dejing [4], Li Yalan, Song Fei [5], have progressively advanced the theory and its application in ground settlement projection within engineering projects. Since then, the theory has been studied in depth and has gradually begun to be widely used in various scenarios. Liu Baochen [6] introduced the theory of random media and its application in various scenarios. Yang Junsheng and Liu Baochen [7] extended the stochastic medium theory to the calculation of surface

movement and deformation caused by urban tunnel excavation. Shi Chenghua [8] applied this theory to the study of surface settlement and deformation caused by subway tunnel construction, studied the ground deformation under various tunnel construction conditions, and analyzed the prediction of lateral surface settlement and longitudinal settlement in the convergence mode of arbitrary sections. Zhang Fu and Chen Youliang et al [9] used the simplified formula of random medium theory to study the relationship between the convergence area and the main influence angle of the section and the buried depth or relative buried depth of the tunnel based on the monitoring data of the surface settlement of an actual tunnel. Weitao Yan and Junting Guo et al [10] applied the stochastic medium theory to establish a prediction model of land surface subsidence and verified its effectiveness. Liu Bo and Yang Weihong et al [11] proposed a parameterized inhomogeneous convergence deformation mode with the superposition of three deformation modes, introduced it into the stochastic medium theory, established a surface settlement prediction model considering the uneven convergence deformation of tunnels, and conducted in-depth research and proved its effectiveness in combination with practical engineering. Wei Gang and Zhou Yangkan [12] established a modified stochastic medium theory method for double-lane horizontal tunnels, and considered the different parameter values of the advance tunnel and the rear tunnel.

Currently the stochastic medium theory is compared with other prediction methods, the stochastic medium theory uses the difference between the cross-section area integral before deformation and the cross-section area integral after deformation of tunnel construction to predict the surface settlement, which can more accurately reflect the problem of surface settlement caused by tunnel construction. This paper proposes a prediction method for surface settlement in close-range twin-bore tunnels based on the optimisation of the inter-span ratio of stochastic medium theory by taking into account the convergence mode of tunnel sections, spatial location and different tunnel section forms, and compares the theoretical prediction value and on-site surface settlement monitoring data with the engineering examples to verify the reliability and scientific validity of the stochastic optimisation method. scientificity of the stochastic optimisation method.

2. Stochastic medium theory optimisation algorithms

2.1. Stochastic medium theory

In the theory of stochastic media, the geotechnical body is considered a “stochastic medium,” with the surface settlement caused by tunneling being viewed as a stochastic process. The tunneling process can be approximated as the excavation of an innumerable series of infinitesimal units, with surface settlement corresponding to the cumulative effect of these micro excavations. The entire surface is represented within a 3D coordinate system (X, Y, Z) , while the tunneling activities are referenced to a 3D local coordinate system (ξ, ζ, η) , where the subsidence at a specific surface location due to the excavation unit $d\xi d\zeta d\eta$ is denoted by $We(X, Y, Z)$ as illustrated in Fig. 1.

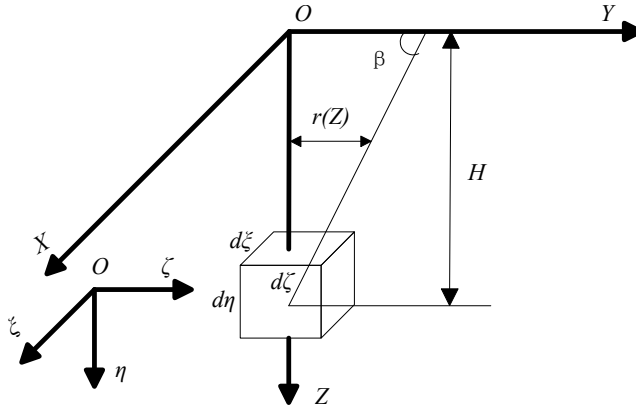


Fig. 1. Three-dimensional coordinate system of the excavation unit

When tunnelling, assuming an initial tunnel section of Ω and complete collapse of the excavation units, the ground settlement notch should be approximately equal to the sum of the volumes of all excavation units. The ground settlement when excavating a particular excavation unit is:

$$(2.1) \quad W_e(X, Y, Z) = \frac{1}{r^2(Z)} \exp\left[-\frac{\pi}{r^2(Z)}(X^2 + Y^2)\right] d\xi d\zeta d\eta$$

where $r(Z)$ is the horizontal main influence range of the excavation unit at burial depth Z , as shown in Fig. 1, which depends on the stratigraphic parameter where the excavation unit is located. Introducing the stratigraphic influence angle β , $r(Z)$ is considered to be linearly related to Z :

$$(2.2) \quad r(Z) = \frac{Z}{\tan \beta}$$

The value of $\tan \beta$ in Eq. (2.1) is related to the stratigraphic parameters at which it is located, which can be determined according to the ground investigation data and the inverse analysis method.

If only the XOZ plane is considered, i.e., assuming that the Y -axis excavation unit is of infinite length, the integral is obtained from Eq. (2.1):

$$(2.3) \quad W_e(X, Z) = \int_{-\infty}^{+\infty} \frac{1}{r^2(Z)} \exp\left[-\frac{\pi}{r^2(Z)}(X^2 + Y^2)\right] d\xi d\zeta d\eta = \frac{1}{r(Z)} \exp\left(-\frac{\pi}{r^2(Z)}X^2\right)$$

Integrating over the entire tunnel section and bringing Eq. (2.2) into Eq. (2.3) yields a surface settlement value of:

$$(2.4) \quad W(X) = \iint_{\Omega} \frac{\tan \beta}{\eta} \exp\left(-\frac{\pi \tan^2 \beta}{\eta^2} X^2\right) d\xi d\eta$$

The post-construction excavation section of a single-bore metro tunnel is ω . Using the superposition principle, the surface settlement is approximately equal to the difference between the surface settlement caused by the initial excavation section Ω and the surface settlement caused by the post-construction excavation section ω , that is:

$$(2.5) \quad W(X) = W_{\Omega}(X) - W_{\omega}(X) = \iint_{\Omega-\omega} \frac{\tan \beta}{\eta} \exp \left[-\frac{\tan^2 \beta}{\eta^2} (X^2 - \xi^2) \right] d\xi d\eta$$

During the excavation of the shallow, closely spaced, twin-bore metro tunnel, the interaction between its tunnels must be considered. The surface settlement resulting from the excavation of the shallow, closely spaced twin-bore metro tunnel can be modeled as a linear superposition of the surface settlement phenomena resulting from the excavation of the single-bore metro tunnel. In the case of the shallow, close-spaced, double-bore underground elliptical tunnel, taking into account the spatial location and the variation in tunnel section profiles, it is postulated that the burial depths of the two tunnels are denoted by H_1 and H_2 , the initial longitudinal and transverse radii of the respective tunnel sections are A_1, B_1 and A_2, B_2 , respectively, and the distance between the centers of the two tunnels' excavations is given by L . This configuration is depicted in Fig. 2.

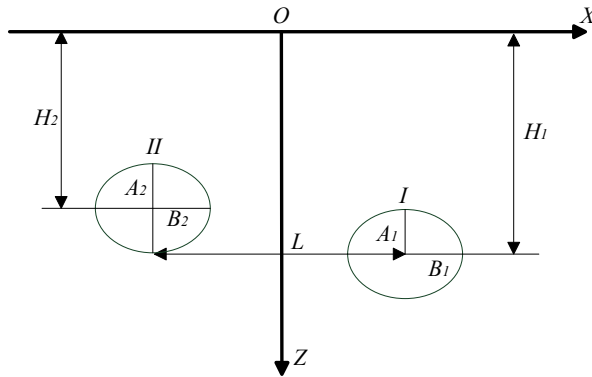


Fig. 2. Proximity double-bore parallel elliptical tunnels

Tunnel I is the advance tunnel and Tunnel II is the backward tunnel, after the tunnel construction, the section convergence values of Tunnel I and Tunnel II are Δ_1 and Δ_2 , respectively, then the value of surface settlement caused by the excavation of Tunnel I is:

$$(2.6) \quad W_I(X) = \int_{a_{11}}^{a_{12}} \int_{b_{11}}^{b_{12}} \frac{\tan \beta}{\eta} \exp \left[-\frac{\pi \tan^2 \beta (X + \frac{L}{2} - \xi)^2}{\eta^2} \right] d\xi d\eta - \int_{a'_{11}}^{a'_{12}} \int_{b'_{11}}^{b'_{12}} \frac{\tan \beta}{\eta} \exp \left[-\frac{\pi \tan^2 \beta (X + \frac{L}{2} - \xi)^2}{\eta^2} \right] d\xi d\eta$$

Style: $a_{11} = H - A_1$; $a_{12} = H + A_1$;

$$b_{11} = -B_1 \sqrt{1 - \left(\frac{H - \eta}{A_1}\right)^2} - \frac{L}{2}; \quad b_{12} = B_1 \sqrt{1 - \left(\frac{H - \eta}{A_1}\right)^2} - \frac{L}{2};$$

$$a'_{11} = H - (A_1 - \Delta_1); \quad a'_{12} = H + (A_1 - \Delta_1);$$

$$b'_{11} = -(B_1 - \Delta_1) \sqrt{1 - \left(\frac{H - \eta}{A_1 - \Delta_1}\right)^2} - \frac{L}{2}; \quad b'_{12} = (B_1 - \Delta_1) \sqrt{1 - \left(\frac{H - \eta}{A_1 - \Delta_1}\right)^2} - \frac{L}{2}.$$

The values of surface settlement caused by the construction of Tunnel II are:

$$(2.7) \quad W_{II}(X) = \int_{a_{21}}^{a_{22}} \int_{b_{21}}^{b_{22}} \frac{\tan \beta}{\eta} \exp \left[-\frac{\pi \tan^2 \beta \left(X - \frac{L}{2} - \xi\right)^2}{\eta^2} \right] d\xi d\eta - \\ \int_{a'_{21}}^{a'_{22}} \int_{b'_{21}}^{b'_{22}} \frac{\tan \beta}{\eta} \exp \left[-\frac{\pi \tan^2 \beta \left(X - \frac{L}{2} - \xi\right)^2}{\eta^2} \right] d\xi d\eta$$

Style: $a_{21} = H - A_2$; $a_{22} = H + A_2$;

$$b_{21} = -B_2 \sqrt{1 - \left(\frac{H - \eta}{A_2}\right)^2} + \frac{L}{2}; \quad b_{22} = B_2 \sqrt{1 - \left(\frac{H - \eta}{A_2}\right)^2} + \frac{L}{2};$$

$$a'_{21} = H - (A_2 - \Delta_2); \quad a'_{22} = H + (A_2 - \Delta_2);$$

$$b'_{21} = -(B_2 - \Delta_2) \sqrt{1 - \left(\frac{H - \eta}{A_2 - \Delta_2}\right)^2} + \frac{L}{2}; \quad b'_{22} = (B_2 - \Delta_2) \sqrt{1 - \left(\frac{H - \eta}{A_2 - \Delta_2}\right)^2} + \frac{L}{2}.$$

The final surface settlement value $W(X)$ due to the construction of the shallow buried close proximity twin-bore metro elliptical tunnel is:

$$(2.8) \quad W(X) = W_I(X) + W_{II}(X)$$

2.2. Analysis of convergence patterns of tunnel sections

The surface settlement displacements resulting from the construction of the closely spaced twin-bore elliptical underground tunnel, as deduced above, yield non-integrable functions; hence, the Gauss–Legendre numerical integration technique is employed to address the issue. However, this method's complexity, stemming from the function's integration bounds, leads to significant computational demands, rendering it impractical for routine engineering applications. Consequently, this study addresses the convergence behavior of the elliptical tunnel section in shallow, closely spaced twin-bore underground tunnels, considering spatial positioning, varying cross-sectional profiles, and additional variables, and introduces a stochastic medium theory optimization approach.

In current practices for calculating tunnel surface deformation and displacement, the convergence models for tunnel sections applied in surface settlement predictions can be categorized into three types: the uniform convergence model [13, 14], the non-bulging arch bottom convergence model [15–17], and the bulging arch bottom convergence model [18, 20], as illustrated in Fig. 3. These three convergence models presuppose small deformations in the

section, meaning the section’s center remains unchanged pre and post deformation, without considering any potential lining deformations. Upon comparison and analysis of the three tunnel section convergence models, it is apparent that the bulging arch bottom model (Fig. 3c) aligns more accurately with observed tunnel deformations.

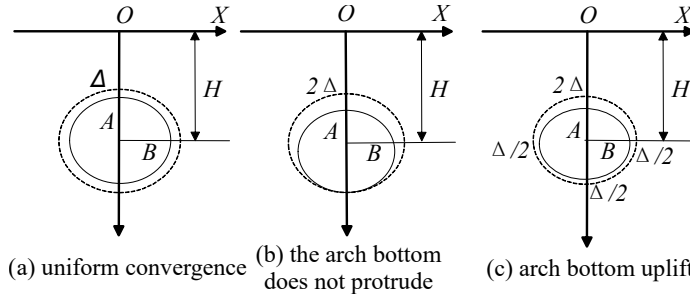


Fig. 3. Convergence model of tunnel section: a) uniform convergence, b) the arch bottom does not protrude, c) arch bottom uplift

Research by K.H. Park [18], Tong Lei [20], and others indicates that during actual tunnel construction, the release of stress at the arch bottom’s soil leads to a degree of bulging. Additionally, the initial tunnel lining, under the peripheral rock pressure above and the soil pressure beneath the arch, will experience elliptical deformation. Therefore, the convergence of the tunnel section’s deformation encompasses homogeneous radial deformation, lining’s elliptical deformation, and the section’s vertical settlement, as illustrated in Fig. 4.

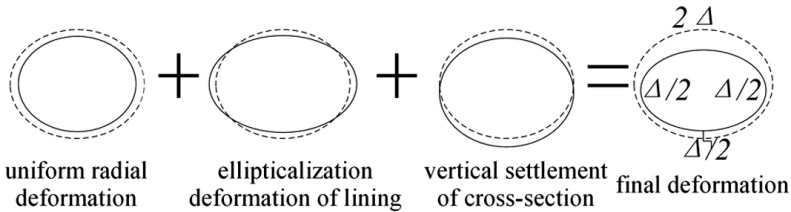


Fig. 4. Convergence model of arch bottom bulge

2.3. Stochastic medium theory optimisation algorithm model

The principle of superposition is used in calculating the surface settlement displacement caused by metro tunnel construction. In Eq. (2.5) and Eq. (2.6) for the prediction of surface settlement caused by underground tunnel excavation, it is considered that the tunnel section consists of an infinite number of infinitesimal microelements $d\xi d\eta$.

The initial excavation section of the metro tunnel is Ω , and the deformation of the section after excavation is ω . Assuming that the tunnel section is an excavation unit in the surrounding rock, and the tunnel section area is equivalent to the excavation unit $d\xi d\eta$ as shown in Fig. 5, the

surface settlement caused by the complete collapse of the tunnel I section before deformation is:

$$(2.9) \quad W_{I\Omega}(X) = \frac{\pi A_1 B_1 \tan \beta_1}{H_1} \exp \left[-\frac{\pi \tan^2 \beta_1}{H_1^2} \left(X + \frac{L}{2} \right)^2 \right]$$

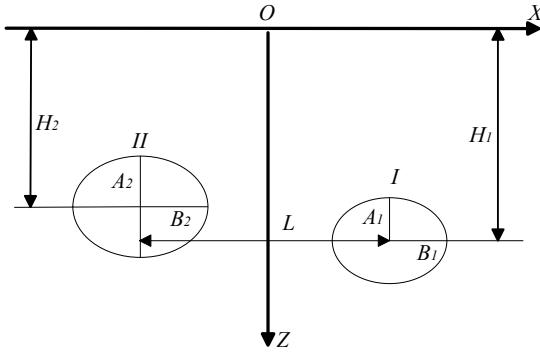


Fig. 5. Schematic diagram of a close-range twin-bore tunnel

The convergence model of the underground tunnel section after tunnel construction adopts the arch bottom bulge convergence model. Assuming that the tunnel arch sinks by $2\Delta_1$, the left and right sides are deformed by $\Delta_1/2$, and the arch bottom bulge is $\Delta_1/2$, the surface settlement caused by the complete collapse after deformation of the tunnel section I will be:

$$(2.10) \quad W_{I\omega}(X) = \frac{\pi (A_1 - 5\Delta_1/4) (B_1 - \Delta_1/2) \tan \beta_1}{H_1} \times \exp \left[-\frac{\pi \tan^2 \beta_1}{H_1^2} \left(X + \frac{L}{2} \right)^2 \right]$$

The total ground settlement displacement due to the construction of Tunnel I will be:

$$(2.11) \quad \begin{aligned} W_I(X) &= W_{I\omega}(X) - W_{I\Omega}(X) \\ &= -\frac{\pi [(2A_1 + 5B_1) \Delta_1 + \Delta_1^2] \tan \beta_1}{4H_1} \exp \left[-\frac{\pi \tan^2 \beta_1}{H_1^2} \left(X + \frac{L}{2} \right)^2 \right] \end{aligned}$$

Since Δ_1 is very small relative to the rest of the tunnel parameters, Δ_1^2 is negligible. Similarly, the total surface settlement due to the construction of Tunnel II is:

$$(2.12) \quad \begin{aligned} W_{II}(X) &= W_{II\omega}(X) - W_{II\Omega}(X) \\ &= -\frac{\pi (2A_2 + 5B_2) \Delta_2 \tan \beta_2}{4H_2} \exp \left[-\frac{\pi \tan^2 \beta_2}{H_2^2} \left(X - \frac{L}{2} \right)^2 \right] \end{aligned}$$

As the tunnel construction will create a certain range of disturbance zone in the surrounding geotechnical body. When the construction of Tunnel I of the proximity twin-bore metro tunnel is completed, due to the relatively close spacing between the two tunnels, Tunnel II is located

within the influence of Tunnel I. The total surface settlement caused by the construction of the proximity twin-bore metro tunnel is:

$$(2.13) \quad W(X) = W_I(X) + W_{II}(X)$$

In the stochastic medium theory, the stratigraphic influence angle β is closely related to the width of the sinkhole. The upper geotechnical body of the shallow close-distance twin-bore underground tunnel in the actual project generally consists of multiple soil layers, and the stratum influence angle of each soil layer is calculated by the empirical formula [20], and then the weighted average method is used to find out the average stratum influence angle, which is more closely related to the actual stratum influence angle.

3. Analysis of engineering examples

The methodology of this paper is used to compare and analyse the surface settlement monitoring data of three shallow buried close proximity twin-bore metro tunnel projects, including inclined alignment tunnels, twin-bore variable section tunnels and twin-bore parallel tunnels.

3.1. Project case 1

A metro interval [21] has been constructed using double-bore parallel shield tunneling, featuring a shallow buried tunnel section with an inclined alignment, as depicted in Fig. 6. The soil layers above the tunnel consist of artificial fill, sandy silty soil, and gravelly clay, respectively. Tunnel I has a center depth of 11.0 m, while Tunnel II's center depth measures 16.2 m. The transverse center-to-center distance between the tunnels is $L = 11.44$ m. Each tunnel section has a longitudinal radius of $A = 3.12$ m and a transverse radius of $B = 3.12$ m. Based on surface parameters, the angle of strata influence is determined to be $\tan \beta = 1.12$. The convergence values of the two shield tunnel sections, derived from on-site monitoring data, are $\Delta_1 = 7.28$ mm and $\Delta_2 = 6.86$ mm.

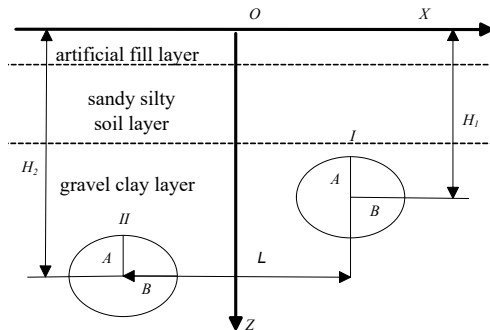


Fig. 6. Proximity double bore inclined alignment tunnel

The prediction curves of the stochastic medium theory optimisation algorithm and the field surface monitoring data are shown in Fig. 7.

Examination of Figs. 6 and 7 reveals that, due to the inclined alignment of the tunnel's spatial location, the settlement curve is asymmetrically distributed along the centerline of the dual tunnels, but exhibits a bias towards the deeper side of the tunnel. Notably, the width of the subsidence troughs is consistent, and minor differences are observed in the location of the inflection points. The theoretically predicted settlement curve exhibits a favorable correlation with the empirical on-site monitoring data, with both the position of maximum settlement and the overall trend showing substantial agreement.

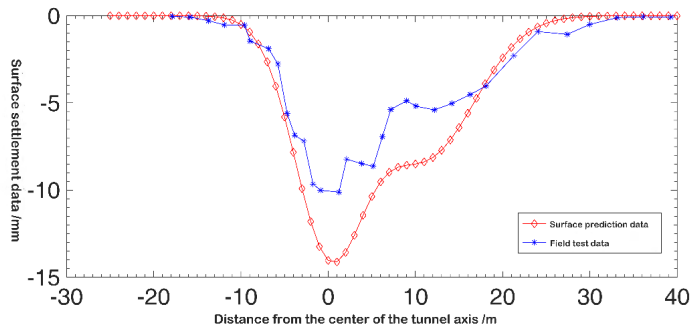


Fig. 7. Surface monitoring curve and theoretical prediction value curve

The correlation coefficient serves as a critical index to assess the magnitude of the relationship between theoretical predictions and monitoring values. Calculation of the correlation coefficient between monitoring values and theoretical predictions provides insight into the strength of their association. In the above cases, the correlation coefficient between the monitored value and the theoretical predicted value is 0.84, indicating that there is a good correlation between the monitored value and the theoretical predicted value.

3.2. Project case 2

An underground interval [22] was constructed by shallow double-borehole concealed excavation method, and the soil layers in the upper part of the tunnel were miscellaneous fill, brown chalky clay, brownish-yellow chalky clay, and medium sand, respectively. The longitudinal radius $A_1 = 5.31$ m and transverse radius $B_1 = 7.08$ m in the section of right tunnel I. The longitudinal radius $A_2 = 3.3$ m and transverse radius $B_2 = 3.1$ m in the section of left tunnel II. The transverse spacing of the tunnel centre is $L = 17.38$ m, and the depth of burial in the centre of the right tunnel is $H_1 = 10.9$ m, and the depth of burial in the centre of the left tunnel is $H_2 = 12.5$ m. This is shown in Fig. 8. According to the surface parameters, the stratigraphic influence angle is found to be $\tan \beta_1 = 0.621$, $\tan \beta_2 = 0.75$, and the convergence values of the two tunnel sections are $\Delta_1 = 20.3$ mm and $\Delta_2 = 25.5$ mm by the on-site monitoring data.

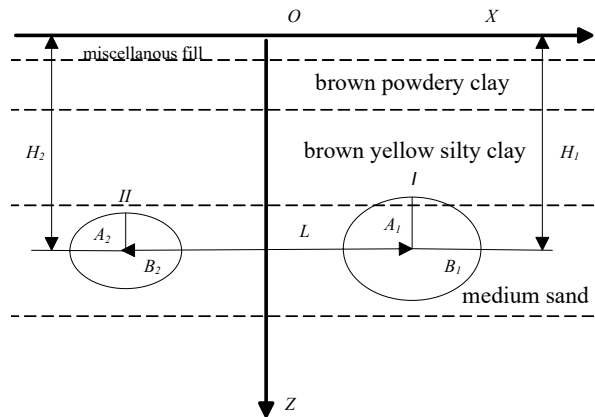


Fig. 8. Schematic diagram of a close-range twin-bore tunnel

The prediction curves of the stochastic medium theory optimisation algorithm and the field surface monitoring data are shown in Fig. 9.

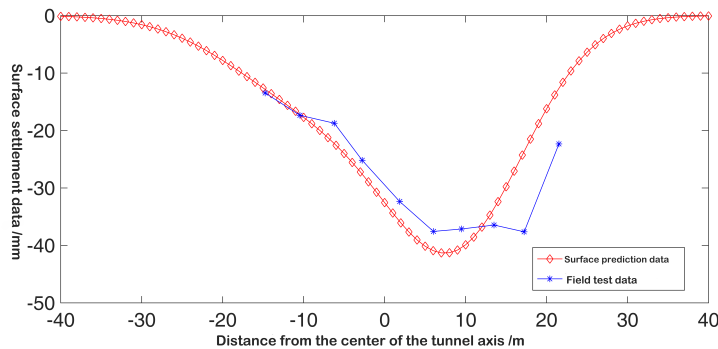


Fig. 9. Surface monitoring curve and theoretical prediction value curve

An underground section, referred to as [23], was constructed using the shallow double-borehole concealed excavation technique, with the soil layers above the tunnel comprising miscellaneous fill, brown chalky clay, brownish-yellow chalky clay, and medium sand, sequentially. In the section of right tunnel I, the longitudinal radius, A_1 , is 5.31 m, and the transverse radius, B_1 , is 7.08 m. In the section of left tunnel II, the longitudinal radius, A_2 , measures 3.3 m, and the transverse radius, B_2 , measures 3.1 m. The two tunnel centers are separated transversely by a distance L of 17.38 m. The burial depth at the center of the right tunnel, H_1 , is 10.9 m, while for the center of the left tunnel, H_2 , it is 12.5 m. This configuration is depicted in Fig. 8. Based on surface parameters, the stratigraphic influence angles are calculated as $\tan \beta_1$ of 0.621 and $\tan \beta_2$ of 0.75, and the convergence values for the two tunnel sections, as determined by on-site monitoring data, are Δ_1 at 20.3 mm and Δ_2 at 25.5 mm.

3.3. Project case 3

A metro interval, designated [24], was constructed using the double-bore parallel tunneling method, with the overlying soil layers consisting of artificial fill, pulverized clay, clay, fine sand, medium-coarse sand, and rounded gravel, as depicted in Fig. 10. The center depth of the tunnel, H , is 11.0 meters, and the transverse distance from the center of the tunnel, L , is 14 meters. The longitudinal and transverse radii of the tunnel section are both 3 meters, designated A and B respectively. Based on the surface parameters, the angle of influence of the stratum, $\tan \beta$, was found to be 1.274, and the convergence values for the double-bore tunnel section, as determined by on-site monitoring, are Δ_1 at 7.382 millimeters and Δ_2 at 6.644 millimeters.

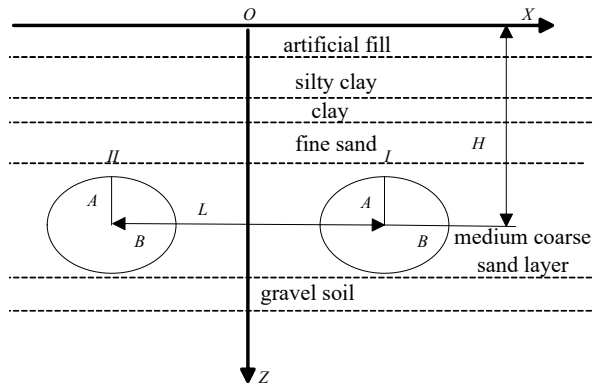


Fig. 10. Proximity twin-bore parallel tunnel

The prediction curves of the stochastic medium theory optimisation algorithm and the field surface monitoring data are shown in Fig. 11.

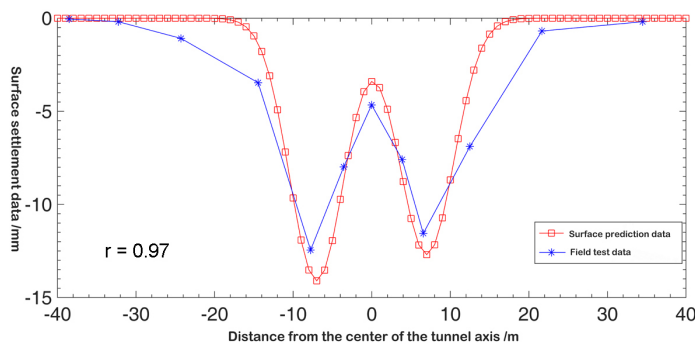


Fig. 11. Surface monitoring curve and theoretical prediction value curve

As illustrated in Fig. 10 and Fig. 11, the tunnel configuration comprises a double-bore, parallel alignment, with the axes of the two tunnels in close proximity. Consequently, the settlement curves are symmetrically aligned along the central axes of the tunnels, forming

a distinctive ‘W’ shape. The sinkhole widths are uniform, and the positions of the inflection points show minimal variance. The simplified theoretical prediction of a stochastic medium and the empirical field surface settlement data exhibit a close fit. Furthermore, the location of the maximum settlement and the differential trends correspond closely with the field monitoring data. The calculated correlation coefficient R of 0.97 signifies a strong correlation between the monitored values and the theoretical predictions.

3.4. The scope of application of stochastic medium theory optimisation algorithms

The Stochastic Medium Theory Optimization Algorithm model is primarily applied to shallow-buried, closely proximate twin-bore tunnels; it is therefore essential to investigate whether these twin-bore tunnels fulfill the criteria for close proximity. To assess the range of close proximity in the three scenarios, this study examines the surface deformation induced by the Stochastic Medium Theory Optimization Algorithm in constructing twin-bore tunnels with varied spacings, as demonstrated in Figs. 12, 13, and 14.

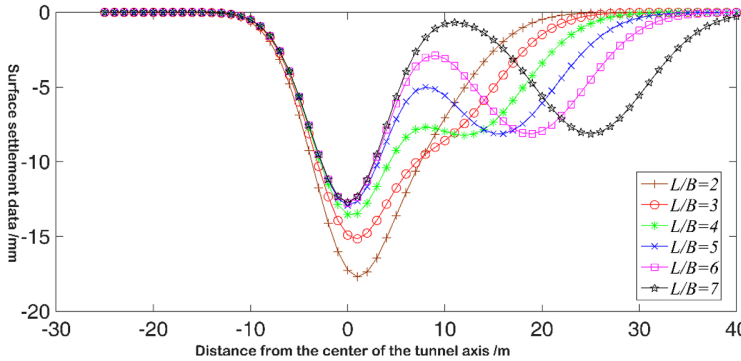


Fig. 12. Settlement curves at different tunnel spacings for Case 1

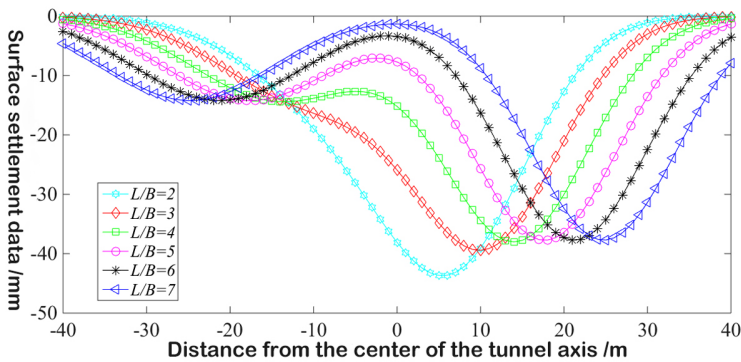


Fig. 13. Settlement curves at different tunnel spacings for Case 2

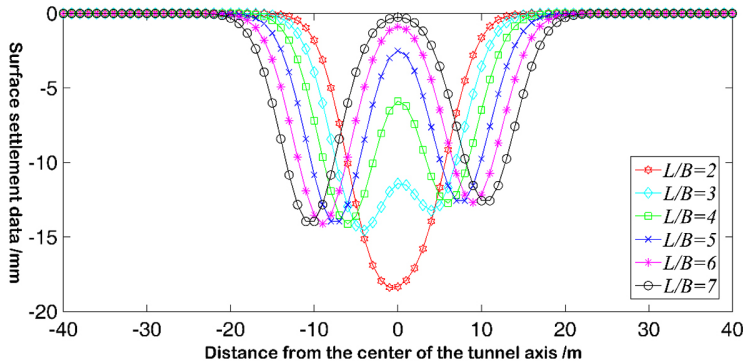


Fig. 14. Settlement curves at different tunnel spacings for Case 3

In the three cases discussed above, when the ratio of tunnel spacing (L) to lateral radius (B) (the inter-tunnel ratio) is 2, the interaction between the twin tunnels is at its peak, resulting in superimposed tunnel-induced surface settlements, and the settlement curves appear as a pronounced “V” shape. As the inter-tunnel ratio increases, the mutual influence between the two tunnels diminishes, the settlement curve begins to fluctuate and takes on a “W” shape, and when the inter-tunnel ratio increases, the tunnel-induced settlement begins to resemble the pattern observed from two separate single-bore tunnels.

Therefore, this paper takes the inter-span ratio less than 7 as the applicable condition of the optimisation algorithm of stochastic medium theory.

4. Conclusions

1. In order to scientifically and reasonably predict the surface settlement displacement caused by the construction of the short-range double-hole tunnel, the stochastic medium theory is studied more deeply, and the stochastic medium theoretical optimization algorithm suitable for the prediction of surface settlement caused by the construction of shallow-buried close-range double-hole tunnel is proposed, considering the tunnel section convergence mode, the spatial location of the tunnel and the different cross-section types of the tunnel.
2. By using the stochastic medium theory optimization algorithm proposed in this paper, the surface settlement monitoring data of three shallow-buried short-distance double-hole subway tunnels, namely, inclined alignment tunnel, double-hole variable section tunnel and double-hole parallel tunnel, are compared and analyzed. The correlation coefficient between the predicted value of the stochastic media theory optimization algorithm and the settlement monitoring data is above 0.8. The prediction curve of the stochastic media optimization algorithm fits well with the settlement monitoring curve. The maximum settlement position and change trend are basically consistent with the monitoring curve, and the width of the sedimentation tank of the theoretical curve

is basically consistent with that of the field monitoring curve. The surface settlement caused by the construction of shallow buried short-distance double-hole subway tunnel can be predicted scientifically and effectively.

3. Based on the stochastic medium theory optimization algorithm, the ground deformation law caused by double-hole tunnel construction in three engineering cases in this paper is discussed under different tunnel spacing, found that the applicable conditions of stochastic medium theory optimization algorithm are span ratio less than 7.

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Received: 2024-06-03, Revised: 2024-11-07