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Research paper

The optimal design of deep foundation pit dewatering under the influence of waterproof curtain

Xiaofeng Li[1], Binghua Wang[2], Jianhang Xu[3], Xianmeng Zhang[4]

Abstract: Improper disposal of foundation pit dewatering can cause engineering problems such as uneven settlement, slope instability, quicksand, piping, etc., and pose a huge threat to construction safety. Therefore, it is necessary to optimize the design of foundation pit dewatering engineering. This paper introduces a simulation-optimization model for this problem, establishes a groundwater flow simulation model using MODFLOW-2005 as a platform, and uses LGR local grid densification technology to densify the dewatering area of foundation pits with different curtain depths. Under the premise of ensuring construction safety, the minimum cost is the objective function, and GWM-2005 is used to solve the optimization model of the foundation pit engineering example. The results show that with the increase of the curtain depth, the optimal design can effectively reduce the number of pumping wells and the total amount of pumping water in the foundation pit dewatering project, reasonably optimize the layout of the pumping well position, significantly improve the safety of foundation pit construction, effectively reduce the impact of foundation pit dewatering project on the surrounding geological environment, and effectively control the land subsidence problem in the precipitation process.

Keywords: foundation pit dewatering, curtain depth, simulation-optimization model, response matrix method, GWM-2005

[1]M.E., Eng., Nanning Rail Transit Construction Co., Ltd., Nanning, China, e-mail: 46413502@qq.com, ORCID: 0009-0005-6320-6430
[2]M.E., Eng., Nanning Rail Transit Construction Co., Ltd., Nanning, China, e-mail: 1099409345@qq.com, ORCID: 0009-0001-5237-6582
[3]M.E., Eng., School of Safety Engineering and Emergency Management, Shijiazhuang Tiedao University, Shijiazhuang, China, e-mail: 598864282@qq.com, ORCID: 0009-0007-4081-3929
[4]PhD., Eng., School of Safety Engineering and Emergency Management, Shijiazhuang Tiedao University, Shijiazhuang, China, e-mail: zhangxianmeng@stdu.edu.cn, ORCID: 0009-0009-3310-5961

1. Introduction

With the continuous development of China's engineering and construction, high-rise buildings, coal mine excavation, underground construction and other aspects will inevitably encounter the problem of deep foundation pit dewatering. In recent years, more and more deep foundation pits are constructed in dense urban building complexes, with a complex environment around the project, and the presence of a large number of municipal pipelines as well as other structures and buildings, which brings more uncertainties to the excavation of foundation pits and dewatering, and increases the difficulty of construction. The purpose of pit descending is to reduce the groundwater level below a certain depth to ensure that the underground engineering structure is constructed under the condition of no groundwater interference, so as to reduce the difficulty of construction and ensure the safety of pit excavation construction to the greatest extent possible. However, the descending of the foundation pit project may cause subsidence of the surrounding strata and adversely affect the existing buildings in the neighbourhood, and at the same time, pit dewatering requires a certain amount of engineering costs.

Under the condition of meeting the pit design water level drop depth, reduce the amount of engineering precipitation as much as possible in order to reduce the impact of pit precipitation on the surrounding environment, therefore, it is necessary to carry out pit precipitation optimisation research [1, 2]. Zhao Yanjun et al. carried out an optimisation design study on an open pit coal mine precipitation combing project by establishing a numerical model through MODFLOW, and the results showed that the pumping capacity of a single well, the number of pumping wells, and the layout of wells have a greater impact on the precipitation combing project. Chen Lingtong et al. used the HSS model for finite element numerical modelling of pit descents at different depths of the water barrier curtain, and the results show that the ground settlement outside the pit decreases linearly as the depth of the suspended water barrier curtain deepens [3]. Optimisation of pit dewatering is to optimise the arrangement of engineering dewatering wells (well position and water output, etc.) through the constraints of reaching the design dewatering depth of water level at each control point, and taking the cost of engineering dewatering as the objective function. Therefore, the optimisation problem of dewatering construction of foundation pit engineering is a kind of groundwater simulation optimisation problem, which can be solved by the method of groundwater simulation optimisation.

Currently, embedding methods, response matrix methods, and linked simulation optimisation methods are widely used to solve groundwater simulation optimisation problems [4–7]. In particular, the GWM-2005 programme, based on the response matrix method, is a groundwater management programme based on MODFLOW-2005, which couples the groundwater flow model with the groundwater optimisation model and provides the simplex method, sequential linear programming and the branch definition method for solving linear, nonlinear and 0–1 mixed linear groundwater management models [8–11]. Ji Yuehua et al. used GWM to solve the problem of optimal groundwater management, and the results showed that GWM has great convenience and reliability in solving the problem of optimal control of groundwater [12]. Wang Huaqi et al. used the optimisation modelling software GWM to optimise the management of groundwater in the Sarawusu Formation with the constraint of minimising the ecological environment, and the optimised exploitation scheme obtained was reasonable and feasible [13].

Taking water level and water volume as constraints and maximum pumping volume as objective function, the Wu Hongzu solved the problem of surface water and groundwater allocation in the irrigation area by using the GWM software, and the simulation results satisfy the practical applications [14]. For the pit precipitation problem, in the process of simulating the groundwater flow field, encrypting the grids around the engineering dewatering wells and water barrier curtains can improve the simulation accuracy of the regional and local key areas [15–17]. However, there are few applied researches that consider local grid encryption technology in pit precipitation optimisation design. Based on this, this paper adopts the MODFLOW-LGR2 (Local Grid Refinement2, LGR2) technique to establish a simulation model of pit precipitation, which reduces the amount of computation of the simulation model of pit precipitation under the premise of improving computational accuracy [18].

This paper firstly introduces the simulation optimisation model for the pit dewatering problem, respectively explains the LGR2 grid encryption technology of the water flow simulation model, the composition of the pit dewatering optimisation model and the optimisation solution program GWM-2005, applies the simulation optimisation model of pit dewatering to the establishment and solution of the pit dewatering engineering example, and explores the influence of the simulation optimisation results under the condition of inserting different curtain depths.

2. Simulation optimisation model for pit dewatering

Groundwater simulation optimisation models usually involve two sets of variables: decision variables and state variables. In general, the decision variables include: the amount of water pumped (or injected) by the well, the number of wells, the location of the wells, and so on. The state variable is the head, which depends on the equation of motion of groundwater flow. The groundwater simulation optimisation model is coupled by two main parts: the flow simulation model (updating the state variables) and the optimisation simulation model (selecting the optimal decision variables).

2.1. Water flow simulation models

When the direction of permeability of an anisotropic medium is consistent with the direction of the coordinate axis without considering the change in the density of water, the flow of groundwater in the underground porous medium can be expressed by the following partial differential equation:

(2.1) $$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)+W=S_s\frac{\partial h}{\partial t}$$

Among them: K_{xx}, K_{yy} and K_{zz} are the principal values of the permeability coefficient tensor in the x, y and z direction (L/T); S_S is the storage rate (L^{-1}); W is the flow rate per unit volume (T^{-1}), which represents the source-sink term; h is the head (L); and t is the time (T).

Combining the initial and boundary conditions of the real problem, numerical methods are used to solve (1). In this paper the water flow simulation model is solved using the MODFLOW-2005 program to simulate the dewatering wells with the well flow subroutine package and local grid encryption using LGR2 in the precipitation area of the foundation pit project.

2.2. Optimisation model for pit dewatering

In the process of pit descending, both from the economy and from the consideration of reducing the impact on the surrounding environment, it is required to ensure construction safety under the premise of the minimum amount of engineering descending (or engineering descending the least cost). Decision variables for the pit dewatering optimisation model include the total number of dewatering wells (including existing wells and alternative wells actually drilled), the location of the dewatering wells and the amount of water pumped.

In the actual pit dewatering optimisation design, the locations of alternative dewatering wells are generally given in advance based on the site conditions, so the above decision variables are redefined to generally include two types of decision variables: (1) flow decision variables, which indicate the amount of water pumped at the location of the dewatering wells; (2) 0–1 decision variable that determines whether the alternative dewatering wells are drilled ("on/off").

The mathematical model for the optimal design of pit dewatering can be formulated as follows: in the case of satisfying the constraints and control conditions, a set of decision variable values is sought to minimise the total amount of water influx in the pit or the cost of the dewatering project. In this paper, the minimum total cost of precipitation is used as the objective function, from which the optimisation mathematical model is established as follows:

$$\text{Minimize} \sum_{n=1}^{N} \beta_n Q_n T_{Q_n} + \sum_{l=1}^{L} K_l I_l \tag{2.2}$$

Subject to

Control point head restraint:

$$H_{\min} \leq H_i \leq H_{\max} \tag{2.3}$$

Single-well water output constraints:

$$Q_{\min} \leq Q_n \leq Q_{\max} \tag{2.4}$$

Among them: Q_n and I_l denote the amount of water pumped from the nth dewatering well ($L3/T$), and whether the lth alternative well was drilled (0–1 variable), respectively; β_n is the cost of pumping per unit volume of the nth dewatering well (USD Million/$L3$); K_l is the cost of drilling the lth well (USD Million/$L3$); T_{Q_n} is the total operating time of the nth dewatering well (T); N and L are the number of descending wells (existing and alternative), and the number of alternative descending wells, respectively. $H_{\max}$ and $H_{\min}$ are the upper and lower limits of the water level at the point of constraint, and $Q_{\max}$ and $Q_{\min}$ are the upper and lower limits of the amount of water pumped from the pumping wells.

3. Engineering case studies

3.1. Site overview

The ground elevation of an underground station in Nanning is 104.58–119.59 m. The station is a four-storey underground station combining open and concealed excavation with separated island platform, and the depth of foundation pit design is 54 m. The site consists of clayey soils and fills above a depth of 11.0 m, with mudstone and muddy siltstone predominating in the range 11.0 m to 60.0 m. The main stratigraphic distribution of the site is shown in Fig. 1 and Table 1.

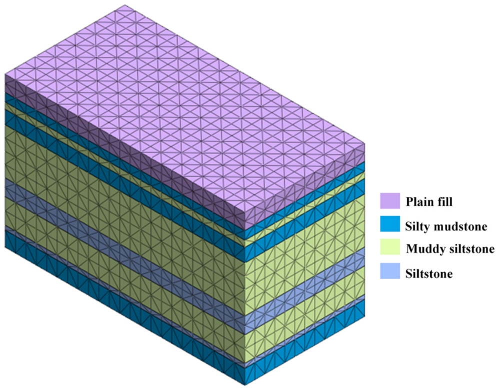

Fig. 1. Stratigraphic distribution of the site

Table 1. Stratigraphic distribution details

Floor number	Geotechnical properties	Floor elevation	Thicknesses (m)
⑦$_2$	Plain fill	101.42	6.6
⑦$_{1-3}$	Silty mudstone	97.92	3.5
⑦$_{2-3}$	Muddy siltstone	95.32	2.6
⑦$_{1-3}$	Silty mudstone	89.72	5.6
⑦$_{2-3}$	Muddy siltstone	70.52	19.2
⑦$_{3-3}$	siltstone	63.72	6.8
⑦$_{2-3}$	Muddy siltstone	53.42	10.3
⑦$_{3-3}$	siltstone	51.62	1.8
⑦$_{1-3}$	Silty mudstone	45.42	6.2

Site aquifers include phreatic aquifers and confined aquifers. The target aquifer for this pit descending is the pressurised aquifer, which is endowed with ⑦$_{2-3}$ muddy siltstone, ⑦$_{3-3}$ siltstone, and the lower ⑦$_{1-3}$ silty mudstone is the lower water barrier, and the average depth of the pressurised aquifer is about 18.8 m.

3.2. Water flow simulation models

According to the stratigraphic distribution conditions of the site, the hydrogeological conceptual model of the calculation area is established, and the calculation area is numerically modelled in three dimensions in accordance with the requirements for pit descending. In the process of numerical calculation of groundwater, first of all, it is necessary to reasonably set up the regional scope of the calculation model, in the process of establishing the water flow model, respectively, the use of different coarse mesh model to determine the simulation range of the groundwater model simulation range of 1200 × 1000 m can be eliminated from the calculation process of the influence of the boundary effect. In the vertical direction, with reference to the geotechnical investigation data, the depth of 51.62 m below the ground surface is the boundary (the value is taken with reference to the elevation of ⑦$_{3-3}$ siltstone base plate), and the phreatic aquifer, the weakly permeable aquifer and the pressurised aquifer are in order from the top to the bottom.

As the topography of the site varies greatly in undulation from south to north, and there is a certain hydraulic gradient in the groundwater, the north and south of the model are set as Class 1 boundaries, and the east and west as Class 2 boundaries. The present model has fixed head boundaries in the north (h_b = 104 m) and south (h_n = 95.5 m), and water separation boundaries in the east and west, with an initial water level of h_o = 104 m.

According to the calculation plane range, stratigraphic distribution and groundwater initial and boundary conditions, the calculation area is discretised using a finite-difference grid to establish a three-dimensional groundwater flow numerical model. In the grid dissection, localised grid encryption was carried out in the pit dewatering areas where curtains were inserted. As shown in Fig. 2, the parent model (2a) is divided into 4 layers, 120 rows and 100 columns, totalling 48,000 grid units; The 1:5 encrypted sub-model was divided into 4 layers, 135 rows and 50 columns with a total of 27,000 grid units, and the grid size of the encrypted area was 2 × 2 m.

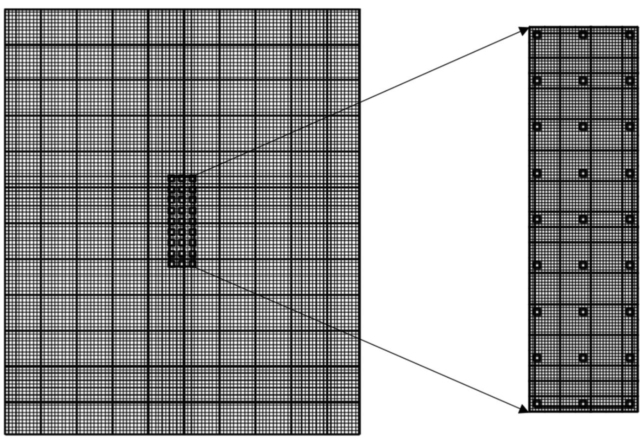
(a) (b)

Fig. 2. Local grid encryption in the simulated area (plan view): (a) parent model; (b) sub model

Based on the on-site pumping test, the inversion of hydrogeological parameters was carried out by trial estimation-correction method under the coarse grid model, and the hydrogeological parameters were obtained as shown in Table 2.

Table 2. Hydrogeological parameters

Type of groundwater	**Calculated parameter**	
	permeability coefficient *K*(m/d)	**water storage coefficient *S* / water supply degree μ**
phreatic	0.7	0.042
weakly permeable layer	0.02	3.5×10^{-5}
pressure-bearing water	0.95	1.5×10^{-4}

3.3. Optimisation models

Considering the limited water discharge capacity of the upper submersible aquifer, the main consideration for pit rainfall is the design of pressure-relief for pressurised aquifers, and according to the relationship between the depth of pit excavation and the safe pressurised water, the pit pressure-bearing head needs to be reduced to an elevation of 54 m. Based on the pumping test, the radius of influence of the pumping wells was determined to be 45 m, and the spacing of the wells was 30 m. According to 1 pumping well for every 535 m^2, 27 alternative dewatering wells were selected for 14,400 m^2 (Fig. 3). The pit dewatering optimisation model is expressed as:

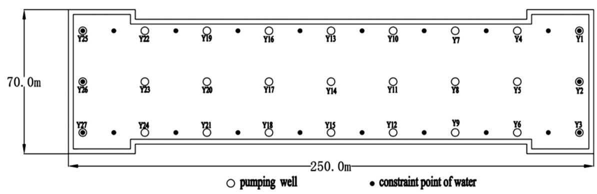

Fig. 3. Layout of dewatering wells

$$\text{Minimize} \sum_{n=1}^{N} \beta_n Q_n T_{Qn} + \sum_{l=1}^{L} K_l I_l + H_i E_i \tag{3.1}$$

$$\text{Subject to} \leq Q_n \leq 150, \;\; n = 1, 2, \ldots, 27 \tag{3.2}$$

$$H_j \leq 54, \;\; j = 1, 2, \ldots, 24 \tag{3.3}$$

Among them: Q_n is the volume of water pumped from the n-th dewatering well (m^3/d), I_l is whether the l-th alternative dewatering well is drilled (0–1 variable); β_n is the cost of pumping per unit volume of the nth dewatering well (USD Million /m^3), this model β_n is 3×10^{-4}; k_l is the cost of drilling the l-th dewatering well (USD Million), and k_l is 0.3 in this model. T_{Qn} is the total operating time (d) of the nth dewatering well, and this model T_{Qn} is 78d; H_i is the depth of curtain insertion of the i-th curtain material (m); $E_i$is the cost per unit depth of curtain type i (USD Million); H_j is the positional head of the control point (m), the model is set to 54 m.

3.4. Analysis of results

The pit construction requirements in 78d after the pit construction location of the pressurised aquifer down to the elevation of 54 m below, the use of GWM-2005 on the pit precipitation area model cell grid to take 1:5 way to encrypt, the simulated curtains are all hanging curtains., curtains are diaphragm walls, the curtain permeability coefficient is a 1×10^{-7} cm/s, respectively, the depth of the curtain is set to 33 m, 38 m, 43 m, 48 m, based on the objective function response matrix method to construct the optimization model to find the results are shown in Table 3 below.

Table 3. Optimisation results of descending water under different curtain depth conditions

Well number \ Curtain depth	Without curtain	Depth 33 m	Depth 38 m	Depth 43 m	Depth 48 m
1	150.00	150.00	142.05	147.51	131.07
2	150.00	150.00	145.41	150.00	128.08
3	150.00	150.00	150.00	150.00	150.00
4	150.00	150.00	148.02	150.00	147.48
5	150.00	150.00	150.00	150.00	0.00
6	150.00	150.00	150.00	150.00	150.00
7	150.00	0.00	0.00	0.00	0.00
8	150.00	150.00	150.00	150.00	150.00
9	150.00	142.19	150.00	0.00	0.00
10	0.00	0.00	150.00	0.00	146.09
11	150.00	146.13	0.00	150.00	0.00
12	0.00	0.00	0.00	0.00	146.09
13	0.00	0.00	0.00	0.00	0.00
14	30.47	94.48	0.00	150.00	150.00
15	0.00	0.00	0.00	0.00	0.00
16	0.00	0.00	0.00	0.00	0.00
17	144.58	150.00	150.00	150.00	150.00
18	0.00	150.00	150.00	0.00	0.00

Continued on next page

Table 3 – *Continued from previous page*

Curtain depth / Well number	Without curtain	Depth 33 m	Depth 38 m	Depth 43 m	Depth 48 m
19	150.00	0.00	0.00	150.00	150.00
20	150.00	150.00	150.00	150.00	150.00
21	149.85	150.00	150.00	150.00	0.00
22	150.00	150.00	150.00	0.00	0.00
23	150.00	150.00	150.00	150.00	150.00
24	150.00	150.00	150.00	150.00	146.87
25	150.00	150.00	144.27	142.87	137.23
26	150.00	150.00	150.00	150.00	144.16
27	150.00	150.00	150.00	150.00	150.00
number of pumping wells (nos.)	21	20	19	18	17
running time (minutes)	11.81	16.51	23.46	39.85	825.53
pumping capacity (m^3/d)	3024.90	2932.80	2829.76	2685.58	2481.00
cost of precipitation USD Million)	10.82	10.48	10.10	9.58	8.87
curtain costs (USD Million)	0	9.27	10.67	12.07	13.48
total cost (USD Million)	10.82	19.74	20.77	21.66	22.35

The following main conclusions can be drawn from Table 3:

1. The foundation pit gradually increases from no curtain to the depth of the foundation pit inserted into the curtain, the number of 27 standby pumping wells gradually decreases, when the curtain is not inserted, the number of pumping wells is 21, the curtain insertion depth is from 33 m to 48 m, the number of pumping wells is reduced from 20 to 17 in turn, from the simulation results, it can be seen that the more effective precipitation auxiliary measures when the curtain is inserted in the process of foundation pit precipitation, and the number of pumping wells can be effectively controlled.
2. With no curtain inserted in the pit, the pit pumped 3042.9 m^3 per day; As the depth of the pit dewatering curtain increased, the amount of water pumped per day for pit dewatering decreased from 2932.80 m^3 at a curtain depth of 33 m to 2481.00 m^3 at a curtain depth of 48 m, it is shown that the insertion of curtain can effectively reduce the pumping volume of pit dewatering, and then effectively control the problem of ground settlement of pit dewatering.
3. The cost of pit dewatering is 108,220.32 USD when the pit is not inserted into the curtain, and decreases from 104,780.52 USD to 88,676.64 USD as the depth of the curtain is inserted progressively; The cost of unit depth curtain is 2,808.00 USD/m, and the total cost of pit dewatering increases from 205,405.20 USD at a curtain depth of 33 m to 223,460.64 USD at a curtain depth of 48 m, but the insertion of the curtain can significantly improve

the safety of pit construction. For different engineering geological conditions, the actual construction safety requirements of the pit and the constraints of engineering economy and other factors, you can reasonably choose to insert the curtain depth.

4. Comparing the optimisation results under different curtain depths, the total cost change of the precipitation scheme is small, and Considering the differences in the coupled runtimes of the optimised models for insertion of curtain simulation, the scheme with a curtain insertion depth of 43 m is selected.

Figures 4 shows the groundwater flow field under the optimization result of curtain depth of 43 m, it can be seen that the water level in the range of the pit is reduced to below 54 m in elevation, which indicates that under the condition of inserting curtain, the optimization result obtained by GWM-2005 can meet the requirements of pit descending, and it can effectively reduce the number of pumping wells and the total amount of water pumped in pit descending project, and reasonably optimize the location of the pumping wells, which can significantly improve the safety of pit construction, and effectively reduce the impact of pit descending project on the surrounding geological environment, and then effectively control the ground subsidence problem in the process of descending water.

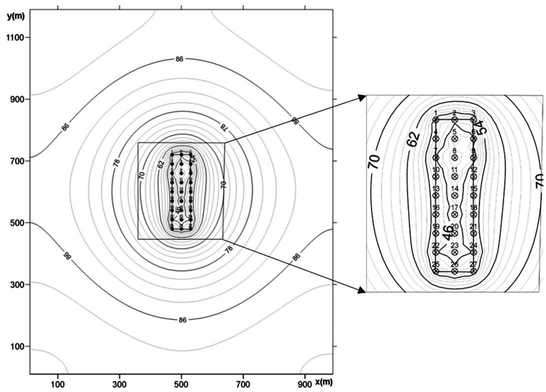

Fig. 4. Groundwater flow field for optimisation results at curtain depth 43 m

4. Conclusions

1. The pit precipitation optimisation problem is a groundwater simulation optimisation problem and can be solved using simulation optimisation methods. The GWM-2005 program based on the response matrix method and combined with the grid encryption technology of MODFLOW's LGR2 can effectively solve the optimisation problem of pit dewatering under the condition of considering the insertion of different drapery depths, and reduce the amount of engineering precipitation as much as possible under the premise of considering the cost of pit dewatering and the safety of pit construction, which is conducive to the reduction of the impact of pit dewatering engineering on the surrounding geological environment, and the effective control of the Ground subsidence problem.

2. As the depth of curtain insertion increases, the GWM-2005 model based on the response matrix method can reasonably optimise the number of pumping wells for pit dewatering, the total amount of pit dewatering, and the total cost of pit dewatering.
3. In the actual project, the optimisation results of GWM-2005 combined with the local refinement model will be more accurate as the depth of curtain insertion increases; However, it should be noted that as the curtain depth increases so that the simulation optimisation model coupling run time increases, while considering the total cost of precipitation change is not large, so you can combine the actual choice of reasonable curtain insertion depth of the project.

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