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**Research paper**

# Axial dynamic impedance of soil and rod systems considering three-dimensional wave effects

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**Abstract:** In order to reveal the influence of vertical and radial deformation and improve the accuracy of the calculation model of pile-soil dynamic interaction, an analytical solution of vertical dynamic impedance considering the three-dimensional wave influence of pile-soil is proposed. First, the Biot three-dimensional porous elastic medium governing equation is used to describe the dynamic behavior of saturated soil, and the pile is regarded as a three-dimensional rod with radial and vertical deformation, and its dynamic behavior is described by Navier motion equation. Then the motion equations of pile-soil are solved by the method of separation variables, and the dynamic impedance expression of pile is given. The accuracy of the proposed solution is verified by comparing with the FEM results and existing solutions. Finally, the three-dimensional solution is compared with the plane strain solution, the radial simplified solution and the one-dimensional solution. The results show that the radial deformation of the pile under the three-dimensional fluctuation effect has a significant effect on the dynamic impedance. Ignoring the radial deformation of pile will lead to overestimate the static stiffness of pile-soil system and underestimate the peak dynamic impedance of pile-soil system. When the excitation frequency $f \leq 5$ Hz, the utilization of the three-dimensional strict solution is more advantageous to obtain the dynamic impedance of the pile. In the range of excitation frequency, the radial deformation of single-phase soil can be ignored.

**Keywords:** dynamic impedance, saturated poroelastic soil, three-dimensional pile, Biot's theory, radial deformation

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# 1. Introduction

The mechanical properties of soil in soft soil area are poor, and the load transfer mechanism in pile-soil composite foundation is complicated [1]. The precise evaluation of the dynamic behavior for the pile foundations under vertical loads is intimately associated with the pile-soil interaction, so it is very important to build a reasonable mathematical-physical model of the pile-soil dynamic interaction in geotechnical engineering, seismic engineering and structural engineering [2–7]. In the continuous medium model of the pile-soil interaction, the problem of pile-soil interaction is usually solved by the conditions of boundary and continuity of the pile-soil system. The dynamic response of pile-soil system is usually expressed by the pile top impedance function. It is available for the separate design of superstructure and the completeness testing of pile foundation.

After Tajimi made an original research on vibration response of the single pile in elastic foundation in 1969 [8], many researchers have extensively studied the pile-soil interaction problem [9–13]. Zheng et al. [14], Dai et al. [15], Wu Peng and Ren Weixin [16] researched the lengthways vibration response of the pile foundation in single-phase soil. Considering soil as a complex saturated porous medium, Zheng et al. [17], Liu et al. [18], Zhang et al. [19] and Wang et al. [20] studied the dynamic interaction of pile foundation in saturated soils. However, the above studies consider pile as one-dimensional bar, and the influence of radial deformation and bottom reaction of the pile on the dynamic interaction of surrounding soil medium is ignored. In the process of pile installation, the soil around the pile will produce stress and deformation, which will affect the settlement and bearing capacity of the pile [21]. Zheng et al. [22] and Zhang et al. [23] studied the vibrating response characteristics of pile foundations based on plane strain method and radial simplification method for vertical dynamic loads, respectively. However, since the two methods initially presented for single-phase soil conditions, ignoring the influence of soil radial deformation. So considering the three-dimensional wave effect, the vertical dynamic response of pile foundation in saturated soil is less studied.

In this article, aiming at the influence of pile and soil radial deformation and pile bottom reaction, based on Biot three-dimensional porous media model, an analytical solution of vertical dynamic impedance of pile-soil system in saturated soil with three-dimensional wave effect is proposed in this paper. And the correctness of proposed solution is verified by comparing with the numerical results of finite element and the existing solutions.

# 2. Mechanical model and assumptions

The mechanical model of 3D pile foundation established is shown in Figure 1. The pile elastic modulus is $E_p$, the pile radius $r_0$, the Poisson's ratio $v_p$, the length $L$ and the density $\rho_p$. The top center of pile ($r = 0$, $z = 0$) is loaded by vertical time-harmonic loads $p(t) = P_0 e^{i\omega t}$ ($i = \sqrt{-1}$; $P_0$ is the load amplitude; $\omega = 2\pi f$ is circular frequency).

The pile-soil system uses the following assumptions:

– The soil is a uniform, isotropic and two-phase saturated medium;
– The viscosity and weight of pore fluid are ignored;

– The deformation and strain of pile-soil system during vibration are infinitely small;
– In the process of vibration, complete contact between the pile and the surrounding soil without disengagement or relative slip.

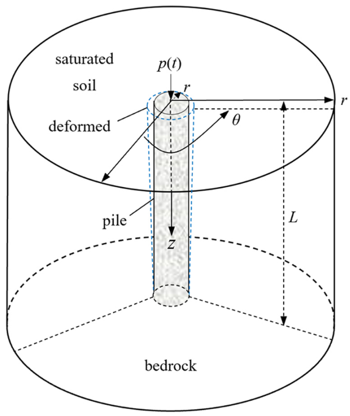


Fig. 1. Three-dimensional piles embedded in saturated soil layers under vertical loading

# 3. Governing equations and solutions of the pile-soil system

## 3.1. Dynamic response of saturated soils

Based on the Biot consolidation theory for fully saturated media and considering the axisymmetric response of saturated soil, the governing equations can be given by:

$$\mu^s\nabla^2\mathbf{u}_s + \nabla[(\lambda^s+\mu^s)e_s + Qe_f] = \frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u}_s + \rho_{12}\mathbf{u}_f) + b\frac{\partial}{\partial t}(\mathbf{u}_s - \mathbf{u}_f) \tag{3.1}$$

$$-n^f\nabla p^f = \frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u}_s + \rho_{22}\mathbf{u}_f) - b\frac{\partial}{\partial t}\left(\mathbf{u}_s - \mathbf{u}_f\right) \tag{3.2}$$

$$Qe_s + Re_f = -n^f p^f \tag{3.3}$$

where: $\mathbf{u}_s$ and $\mathbf{u}_f$ – the displacement of soil skeleton and interstitial liquid; $p^f$ – pore pressure; $\lambda^s$, $\mu^s$ – the Lame constants of soil skeleton; $\rho_{11}$and $\rho_{22}$ – the total effective mass of the soil skeleton and pore fluid in motion, and the symbol is positive; $\rho_{12}$ – additional surface mass, the symbol is negative; $n^s$ and $n^f$ – the volume fraction of the soil skeleton and the pore fluid; $e_s$ – the volume strain of soil skeleton, $e_s = \frac{u_s}{r} + \frac{\partial u_s}{\partial r} + \frac{\partial w_s}{\partial z}$, $w_s$ – the vertical displacement of soil skeleton; $e_f$ – the volume strain of the pore fluid, $e_f = \frac{u_f}{r} + \frac{\partial u_f}{\partial r} + \frac{\partial w_f}{\partial z}$, $w_f$ – the vertical displacement of pore fluid.; $\nabla^2$ – Laplacian operator, $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$; $b$ – the liquid-solid coupling coefficient; $Q$ and $R$ – the coupling parameters for volume changes between the soil skeleton and pore fluid.

Applying the separation of variables and the operator decomposition theory [24] $e_s = e_{s1} + e_{s2}$, setting $e_{s1} = R_1(r)W_1(z)e^{i\omega t}$, the volumetric strain of soil skeleton can be given by:

$$e_s = [A_1 I_0(b_2 r) + A_2 K_0(b_2 r)][B_1 e^{b_1 z} + B_2 e^{-b_1 z}] + [A_3 I_0(b_4 r) + A_4 K_0(b_4 r)][B_3 e^{b_3 z} + B_4 e^{-b_3 z}] \tag{3.4}$$

By the same token, the pore fluid pressure can be obtained by:

$$p^f = [A_5 I_0(b_2 r) + A_6 K_0(b_2 r)][B_5 e^{b_1 z} + B_6 e^{-b_1 z}] + [A_7 I_0(b_4 r) + A_8 K_0(b_4 r)][B_7 e^{b_3 z} + B_8 e^{-b_3 z}] \tag{3.5}$$

where: $A_1 \sim A_8$, $B_1 \sim B_8$ – unknown constants; $I_0(\cdot)$ and $K_0(\cdot)$ – the first and second type modified Bessel's functions of the zero order, respectively; $b_1$, $b_2$, $b_3$ and $b_4$ – unknown constants.

The stress and displacement components of saturated porous elastic soil can be obtained by solving $e_s$ and $p^f$.

The boundary and continuity conditions of the pile-soil system are as follows:

At infinity $r \to \infty$, the soil field variable will decay to disappear, i.e.,

$$\begin{cases} u_s|_{r\to\infty} = 0, \; w_s|_{r\to\infty} = 0, \; u_f|_{r\to\infty} = 0, \; w_f|_{r\to\infty} = 0, \\ \sigma_r^s|_{r\to\infty} = 0, \; \sigma_z^s|_{r\to\infty} = 0, \; \tau_{rz}^s|_{r\to\infty} = 0, \; p^f|_{r\to\infty} = 0 \end{cases} \tag{3.6}$$

The normal stress and pore fluid pressure at the top of the soil are zero, and the shear stress of soil is equal to that of the pile, i.e.,

$$\sigma_z^s|_{z=0} = 0, \; p^f|_{z=0} = 0, \; \tau_{rz}^s|_{r=r_0} = \tau_{rz}^p|_{r=r_0} \tag{3.7}$$

The vertical displacement of the soil bottom and the pore fluid are zero, i.e.,

$$w_s|_{z=L} = 0, \; w_f|_{z=L} = 0 \tag{3.8}$$

The pile-soil contact surface is impervious, water adheres to the surface of the pile, and the radial displacement of pore fluid is equal to that of the pile, i.e.,

$$u_f|_{r=r_0} = u_p|_{r=r_0} \tag{3.9}$$

Pile and soil are in complete contact, the radial and vertical displacement of the soil is equal to that of the pile,

$$u_s|_{r=r_0} = u_p|_{r=r_0}, \; w_s|_{r=r_0} = w_p|_{r=r_0} \tag{3.10}$$

By substituting the vertical displacement of the soil into the boundary condition Eq. (3.8), a characteristic equation of the form $e^{b_n L} + e^{-b_n L} = 0$ can be obtained. Combined with the soil skeleton volume strain equation $e_s = \dfrac{u_s}{r} + \dfrac{\partial u_s}{\partial r} + \dfrac{\partial w_s}{\partial z}$ and the boundary condition Eqs. (3.6) and (3.7), the field variable can be rewritten as

$$p^f = \sum_{n=1}^{\infty}\left[\frac{(1-C_{3n}C_{1n})}{C_{5n}C_{1n}}A_{2n}K_0(b_{2n}r) + \frac{(1-C_{4n}C_{2n})}{C_{6n}C_{2n}}A_{4n}K_0(b_{4n}r)\right](e^{b_n z} - e^{-b_n z}) \tag{3.11}$$

$$u_s = \sum_{n=1}^{\infty}\left[-\frac{1}{C_{1n}}A_{2n}b_{2n}K_1(b_{2n}r) - \frac{1}{C_{2n}}A_{4n}b_{4n}K_1(b_{4n}r) + B_{9n}K_1(b_{6n}r)\right] \times (e^{b_n z} - e^{-b_n z}) \tag{3.12}$$

$$w_s = \sum_{n=1}^{\infty}\left[\frac{1}{C_{1n}}A_{2n}b_nK_0(b_{2n}r) + \frac{1}{C_{2n}}A_{4n}b_nK_0(b_{4n}r) + B_{9n}\frac{b_{6n}}{b_n}K_0(b_{6n}r)\right] \times (e^{b_n z} + e^{-b_n z}) \tag{3.13}$$

$$u_f = \sum_{n=1}^{\infty}D_1\left\{\begin{array}{l}-\left[\dfrac{1}{C_{1n}} + \dfrac{n^f}{(\rho_{12}\omega^2 + bi\omega)}\dfrac{(1-C_{3n}C_{1n})}{C_{5n}C_{1n}}\right]A_{2n}b_{2n}K_1(b_{2n}r)\\ -\left[\dfrac{1}{C_{2n}} + \dfrac{n^f}{(\rho_{12}\omega^2 + bi\omega)}\dfrac{(1-C_{4n}C_{2n})}{C_{6n}C_{2n}}\right]A_{4n}b_{4n}K_1(b_{4n}r)\\ + B_{9n}K_1(b_{6n}r)\end{array}\right\} \times (e^{b_n z} - e^{-b_n z}) \tag{3.14}$$

$$w_f = \sum_{n=1}^{\infty}D_1\left\{\begin{array}{l}\left[\dfrac{1}{C_{1n}} + \dfrac{n^f}{(\rho_{12}\omega^2 + bi\omega)}\dfrac{(1-C_{3n}C_{1n})}{C_{5n}C_{1n}}\right]A_{2n}b_nK_0(b_{2n}r)\\ + \left[\dfrac{1}{C_{2n}} + \dfrac{n^f}{(\rho_{12}\omega^2 + bi\omega)}\dfrac{(1-C_{4n}C_{2n})}{C_{6n}C_{2n}}\right]A_{4n}b_nK_0(b_{4n}r)\\ + B_{9n}\dfrac{b_{6n}}{b_n}K_0(b_{6n}r)\end{array}\right\}(e^{b_n z} + e^{-b_n z}) \tag{3.15}$$

Then the stress of saturated soil is obtained as

$$\sigma_r^s = \sum_{n=1}^{\infty}\left\{\begin{array}{l}A_{2n}\left[\left(\lambda^s + 2\mu^s\dfrac{1}{C_{1n}}b_{2n}^2\right)K_0(b_{2n}r) - \left(2C_{3n} - \dfrac{1}{C_{1n}}\right)\dfrac{1}{r}2\mu^s b_{2n}K_1(b_{2n}r)\right]\\ +A_{4n}\left[\left(\lambda^s + 2\mu^s\dfrac{1}{C_{2n}}b_{4n}^2\right)K_0(b_{4n}r) - \left(2C_{4n} - \dfrac{1}{C_{2n}}\right)\dfrac{1}{r}2\mu^s b_{4n}K_1(b_{4n}r)\right]\\ -2\mu^s B_{9n}\left[b_{6n}K_0(b_{6n}r) + \dfrac{1}{r}K_1(b_{6n}r)\right]\end{array}\right\} \times (e^{b_n z} - e^{-b_n z}) \tag{3.16}$$

$$\sigma_z^s = \sum_{n=1}^{\infty}\left[\left(\lambda^s + 2\mu^s\frac{1}{C_{1n}}b_n^2\right)A_{2n}K_0(b_{2n}r) + \left(\lambda^s + 2\mu^s\frac{1}{C_{2n}}b_n^2\right)A_{4n}K_0(b_{4n}r) + 2\mu^s B_{9n}b_{6n}K_0(b_{6n}r)\right] \times (e^{b_n z} - e^{-b_n z}) \tag{3.17}$$

$$\tau_{rz}^{s} = \tag{3.18}$$
$$= \sum_{n=1}^{\infty} \mu^{s} \left\{ -\frac{2}{C_{1n}} A_{2n} b_n b_{2n} K_1(b_{2n} r) - \frac{2}{C_{2n}} A_{4n} b_n b_{4n} K_1(b_{4n} r) + \left( b_n - \frac{b_{6n}^2}{b_n} \right) B_{9n} K_1(b_{6n} r) \right\} \times \left(e^{b_n z} + e^{-b_n z}\right)$$

where: $b_n = \frac{(2n-1)\pi i}{2L}$, $n = 1, 2, 3, \ldots$; $b_{2n} = \sqrt{C_1^2 - b_n^2}$; $b_{4n} = \sqrt{C_2^2 - b_n^2}$; $b_{6n} = \sqrt{D_6^2 - b_n^2}$; $A_{2n}$, $A_{4n}$, $B_{9n}$ and $C_{1n} \sim C_{6n}$ are unknown constants that will be determined by the boundary and continuity conditions.

## 3.2. Vertical vibration analysis of pile

According to elasticity, the governing equation of pile is

$$\mu^{p} \nabla^2 w_p + (\lambda^p + \mu^p) \frac{\partial e_p}{\partial z} = \rho_p \ddot{w}_p \tag{3.19}$$

$$\mu^{p} \left( \nabla^2 - \frac{1}{r^2} \right) u_p + (\lambda^p + \mu^p) \frac{\partial e_p}{\partial r} = \rho_p \ddot{u}_p \tag{3.20}$$

where: $u_p$ and $w_p$ – the radial and vertical displacement of the pile; $\lambda^p$, $\mu^p$ – the Lame constants of pile, in which $\lambda^p = \nu_p E_p / [(1 + \nu_p)(1 - 2\nu_p)]$, $\mu^p = E_p / [2(1 + \nu_p)]$; $e_p$ – volumetric strain of the pile.

According to the Saint–Venant's principle, the boundary conditions of pile can be expressed as

$$\iint_A \sigma_z^p r \,\mathrm{d}r\,\mathrm{d}\theta = P_0 e^{iwt}, \qquad \iint_A \tau_{rz}^p r \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta = \iint_A \tau_{rz}^p r \cos\theta \,\mathrm{d}r\,\mathrm{d}\theta = 0 \tag{3.21}$$

$$\iint_A \tau_{rz}^p r \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta = \iint_A \tau_{rz}^p r \cos\theta \,\mathrm{d}r\,\mathrm{d}\theta = 0, \qquad w_p|_{z=L} = 0 \tag{3.22}$$

By the method of separation of variables, let $e_p = R_2(r) W_2(z)$, and by the solution of the homogeneous of Eqs. (3.19) and (3.20) plus the particular solution, the displacement component of the pile can be obtained as

$$\begin{aligned} u_p = {} & C_{11} b_9 [A_{13} I_1(b_9 r) - A_{14} K_1(b_9 r)][B_{13} e^{b_{10} z} + B_{14} e^{-b_{10} z}] \\ & + [A_{15} I_1(b_{11} r) + A_{16} K_1(b_{11} r)][B_{15} e^{b_{12} z} + B_{16} e^{-b_{12} z}] \end{aligned} \tag{3.23}$$

$$\begin{aligned} w_p = {} & C_{12} b_{10} [A_{13} I_0(b_9 r) + A_{14} K_0(b_9 r)][B_{13} e^{b_{10} z} - B_{14} e^{-b_{10} z}] \\ & + [A_{17} K_0(b_{13} r) + A_{18} I_0(b_{13} r)][B_{17} e^{b_{14} z} + B_{18} e^{-b_{14} z}] \end{aligned} \tag{3.24}$$

where: $A_{15}$, $A_{16}$ and $B_{15}$, $B_{16}$ – unknown constants; $b_{11}^2 + b_{12}^2 = -\frac{\rho_p \omega^2}{\mu^p}$, $b_{13}^2 + b_{14}^2 = -\frac{\rho_p \omega^2}{\mu^p}$.

The displacement of pile is obtained as

$$u_p = \sum_{n=1}^{\infty} \left[ C_{11n} b_{9n} A_{13n} I_1(b_{9n} r) - \frac{b_n}{b_{11n}} A_{18n} I_1(b_{11n} r) \right] \left(e^{b_n z} - e^{-b_n z}\right) \tag{3.25}$$

$$w_p = \sum_{n=1}^{\infty}[C_{12n} b_n A_{13n} I_0(b_{9n} r) + A_{18n} I_0(b_{13n} r)](e^{b_n z} + e^{-b_n z}) + (B_{19} e^{b_{15} z} + B_{20} e^{-b_{15} z}) \tag{3.26}$$

Therefore, the stress of pile is obtained as

$$\sigma_z^p = \sum_{n=1}^{\infty}\left\{\begin{array}{l} C_{11n}[\lambda^p \left(b_{9n}^2 + b_n^2\right) + 2\mu^p b_n^2] A_{13n} I_0(b_{9n} r) - \lambda^p b_{11n} \frac{b_n}{b_{11n}} A_{18n} I_0(b_{11n} r) \\ +(\lambda^p + 2\mu^p) b_n A_{18n} I_0(b_{13n} r) \end{array}\right\} \times [e^{b_n z} - e^{-b_n z}] + (\lambda^p + 2\mu^p) b_{15}(B_{19} e^{b_{15} z} - B_{20} e^{-b_{15} z}) \tag{3.27}$$

$$\sigma_r^p = \sum_{n=1}^{\infty}\left\{\begin{array}{l} [\lambda^p C_{11n}(b_{9n}^2 + b_n^2) + 2\mu^p C_{11n} b_{9n}^2] A_{13n} I_0(b_{9n} r) - (\lambda^p + 2\mu^p) b_n A_{18n} I_0(b_{11n} r) \\ +\lambda^p b_n A_{18n} I_0(b_{13n} r) - \frac{2}{r}\mu^p C_{11n} b_{9n} A_{13n} I_1(b_{9n} r) + \frac{2}{r}\mu^p \frac{b_n}{b_{11n}} A_{18n} I_1(b_{11n} r) \end{array}\right\} \times [e^{b_n z} - e^{-b_n z}] + \lambda^p b_{15}(B_{19} e^{b_{15} z} - B_{20} e^{-b_{15} z}) \tag{3.28}$$

$$\tau_{rz}^p = \sum_{n=1}^{\infty}\mu^p\left\{2C_{11n} b_{9n} b_n A_{13n} I_1\ (b_{9n} r) - \frac{b_n^2}{b_{11n}} A_{18n} I_1(b_{11n} r) + b_{13n} A_{18n} I_1(b_{13n} r)\right\} \times [e^{b_n z} + e^{-b_n z}] \tag{3.29}$$

where: $b_n = \frac{(2n-1)\pi i}{2L}$, $n = 1, 2, 3, \ldots$; $b_{9n} = \sqrt{-b_n^2 - \frac{\rho_p \omega^2}{(\lambda^p + 2\mu^p)}}$; $b_{11n} = \sqrt{-b_n^2 - \frac{\rho_p \omega^2}{\mu^p}}$; $b_{13n} = \sqrt{-b_n^2 - \frac{\rho_p \omega^2}{\mu^p}}$; $B_{19} = B_{20} + \frac{P_0}{\pi r_0^2(\lambda^p + 2\mu^p) b_{15}}$; $B_{20} = -\frac{P_0 e^{b_{15} L}}{\pi r_0^2(\lambda^p + 2\mu^p)(e^{b_{15} L} + e^{-b_{15} L}) b_{15}}$; $P_0$ – load amplitude of the pile top, $P_0 = \iint_A \sigma_z^p r\,\mathrm{d}r\,\mathrm{d}\theta\ (z = 0)$, $C_{11n}$, $C_{12n}$, $A_{13n}$, $A_{18n}$ – unknown constants.

The dynamic impedance of pile is defined as

$$K_d = K_R + iC_I = \frac{p(t)}{w_p(z = 0)} \tag{3.30}$$

where: $K_R$ – the actual dynamic stiffness of pile-soil system against axial strain, $K_R = real(K_d)$; $C_I$ – the equivalent damping caused by pile-soil material damping, wave radiation in the pile-soil system, and the relative motion of pore liquid and soil skeleton, reflecting the energy dissipation characteristics of the pile-soil system, $C_I = imag(K_d)$.

## 4. Numerical results and discussion

Finite element (FEM) values are used to confirm the accuracy of the solutions proposed in this paper and to explore the dynamic properties of the pile-soil system. Unless explicitly stated otherwise, the finite element numerical calculations in this section will be verified according to the parameter values in Table 1.

Table 1. Finite element calculation model parameters

| Model | Parameter | Value | Unit |
|---|---|---|---|
| Pile | Length $L$ | 10 | m |
| | Radius $r_0$ | 0.25 | m |
| | Young's modulus $E_p$ | 20 | GPa |
| | Density $\rho_p$ | 2500 | kg/m$^3$ |
| | Poisson's ratio $v_p$ | 0.1 | – |
| Soil | Shear modulus $G_s$ | 20 | MPa |
| | Poisson's ratio $v_s$ | 0.2 | - |
| | Actual density of soil skeleton $\rho^{sR}$ | 1800 | kg/m$^3$ |
| | Actual density of pore liquid $\rho^{fR}$ | 1000 | kg/m$^3$ |
| | Pore fluid volume fraction $n^f$ | 0.4 | - |
| | Darcy's permeability coefficient $k^f$ | 0.1 | mm/s |
| Loads | Load amplitude of pile top $P_0$ | 1250 | kN |
| | Excitation frequency $f$ | 20 | Hz |

## 4.1. Verification of solution

To prove the reliability of the proposed solution, the axisymmetric finite element model of the pile-soil system is built by ADINA. From Figure 2, the finite element model uses 9-node porous medium material rectangular element to simulate soil and 9-node homogeneous material rectangular element to simulate the pile. The left side of the model is set as an axisymmetric boundary, the right side and the bottom of the model is a fixed boundary, the soil region on the upper surface of the model is a free boundary, the top of the pile bears uniform vertical harmonic external load. A comparison of the computational results of the finite element and the solution of this paper is shown in Figure 3. The solution in this paper is basically accordance with the calculation results of the FEM, thus confirming the credibility of the solution in this paper.

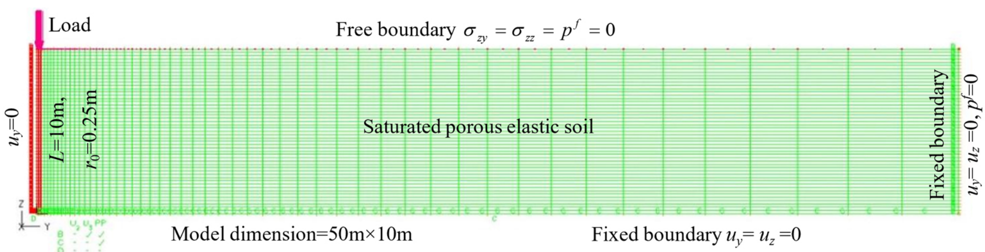


Fig. 2. Axisymmetric finite element model based on pile-soil system

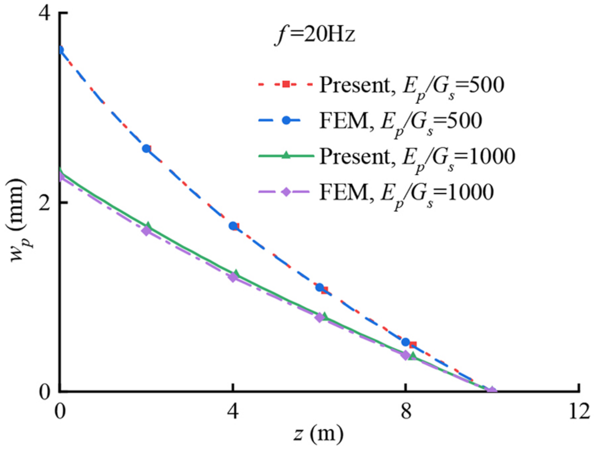


Fig. 3. Comparison between the solution of this paper and the results of finite element calculation

## 4.2. Comparison of solutions

The superiority of the three-dimensional solution proposed in this paper is showed by the comparison with the plane strain solution [4] and the radial simplified solution [5]. From Figure 4, the changes of pile dynamic impedance under different pile length-diameter ratio. When the exciting frequency $f > 5$ Hz, the dynamic impedance of three-dimensional solution in saturated porous elastic soil is very consistent with the plane strain solution, but the radial simplified solution is significantly different. When the excitation frequency $f \leq 5$ Hz, the results of comparison between this solution and the plane strain solution in saturated soil are similar to those in single-phase soil, and the solution is basically consistent with the radial simplified solution in single-phase soil.

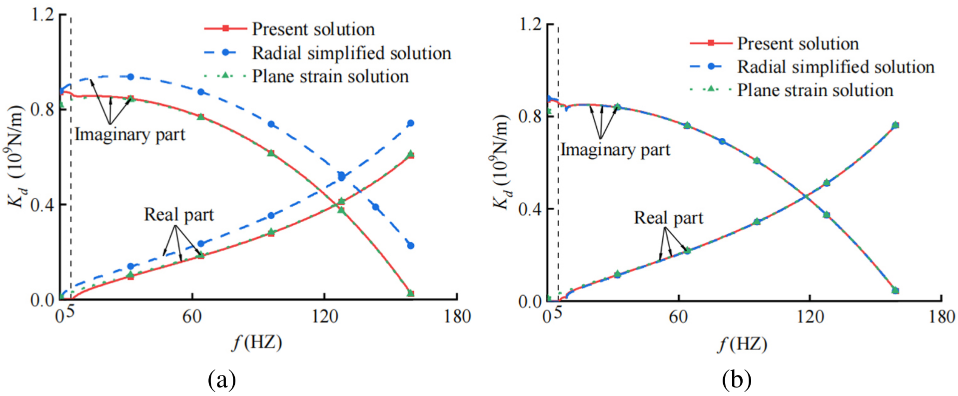


Fig. 4. [cont.]

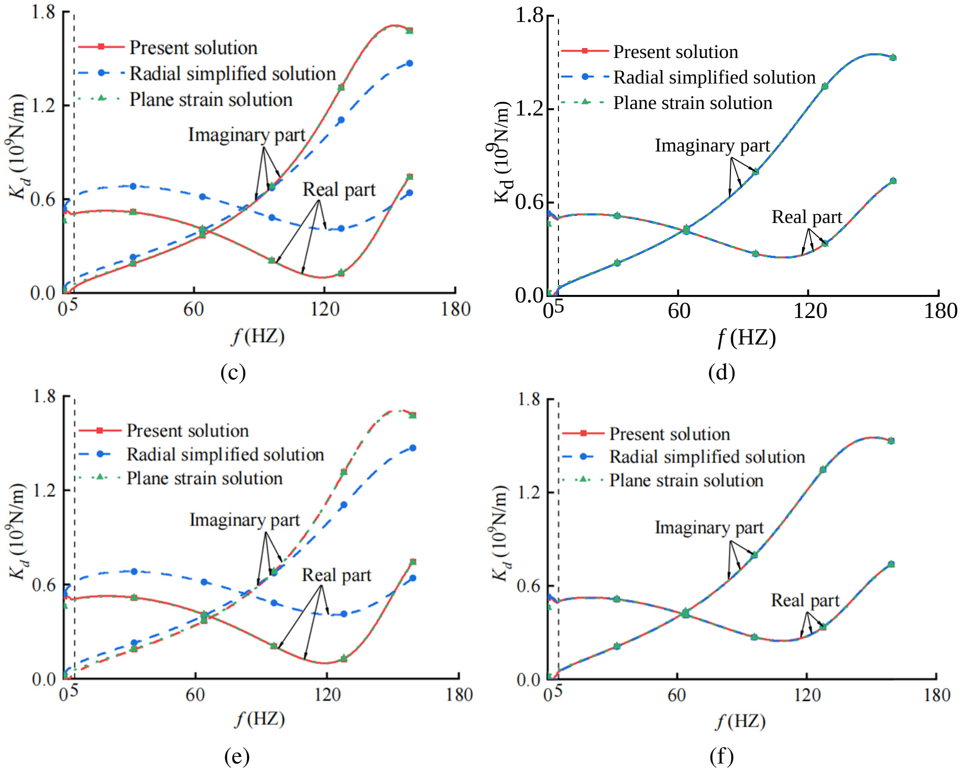


Fig. 4. Comparison of the dynamic impedance of the solution with the radial simplified solution and the plane strain solution: (a) Saturated porous elastic soil $L/r_0 = 20$; (b) Single-phase soil $L/r_0 = 20$; (c) Saturated porous elastic soil $L/r_0 = 40$; (d) Single-phase soil $L/r_0 = 40$; (e) Saturated porous elastic soil $L/r_0 = 80$; (f) Single-phase soil $L/r_0 = 80$

Meanwhile, as shown in Figure 5 that as the pile length-diameter ratio grows, the hydrostatic rigidity of pile eventually approaches a stable level, which reveals that the influence of pile length on hydrostatic resistance is limited. The variation between the plane strain solution and the proposed solution in this paper reduces with the growth of pile-soil modulus ratio and length-diameter ratio, which means that the soil around the pile exhibits a plane-strain mode when the pile becomes rigid.

Figure 6 shows a comparison between the dynamic impedance of the three-dimensional solution proposed in this paper and the one-dimensional solution obtained by Liu et al. [18] who set the internal radiu of tubular pile to zero. The results show that the dynamic impedance of pile-soil system is greatly affected by the radial displacement of pile body. Specifically, because the side of one-dimensional pile is fixed, the static rigidity of three-dimensional pile is smaller than one-dimensional pile. However, the peak value of dynamic impedance of three-dimensional pile is greater than one-dimensional pile, which means that one-dimensional pile hypothesis will underestimate the peak value of dynamic impedance of pile-soil system.

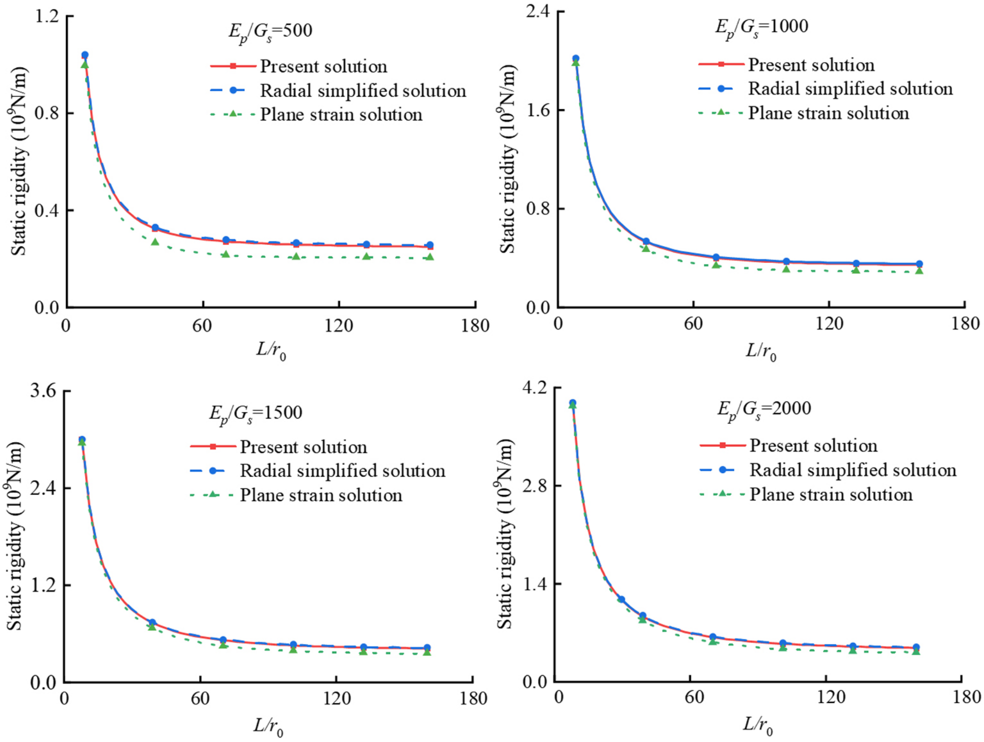


Fig. 5. Comparison of the static stiffness of the solution with the radial simplified solution and the plane strain solution

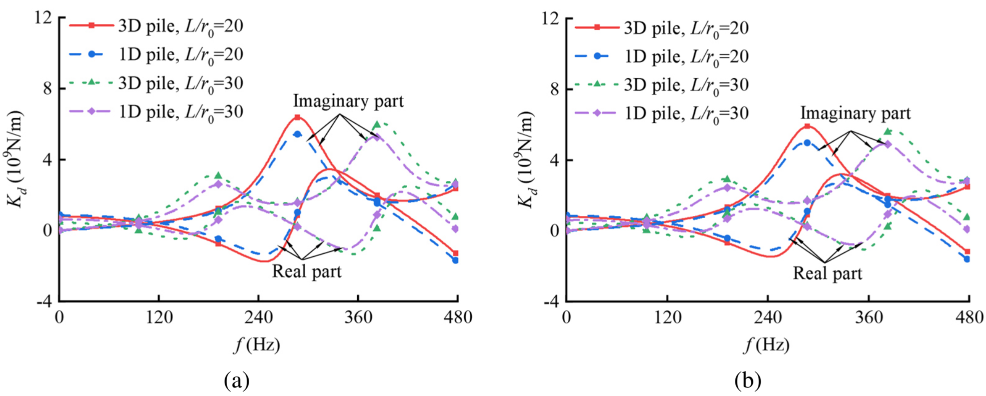


Fig. 6. Comparison of the dynamical impedance of the solution in this paper with that of the one-dimensional solution: (a) Saturated porous elastic soil; (b) Single-phase soil

## 5. Conclusions

This paper proposed an analytical solution of vertical dynamic impedance considering the three-dimensional wave effect of pile-soil based on Biot three-dimensional porous elastic medium theory and Navier equation of motion. Then verifies the rationality of the proposed solution by using finite element method. Finally, by comparing the three-dimensional solution presented in this paper with the plane strain solution, the radial simplified solution and the one-dimensional solution, the following conclusions are drawn:

1. In the range of $f > 5$ Hz, single-phase soil or saturated porous elastic soil can be considered as a plane strain model to acquire the dynamic impedance of the pile-soil system. But the 3D strict solving is more conducive to acquire the dynamic impedance in the range of $f \leq 5$ Hz. The radial displacement of soil can be ignored in the scope of excitation frequencies for the single-phase soil case. However, for saturated soils, this hypothesis is not valid.
2. There is a limit to the impact of pile length on impedance function. Along with the increase of pile-soil modulus ratio of pile and length-diameter ratio, the soil around pile shows plane strain mode.
3. The static rigidity of 3D pile is smaller than 1D pile, and the peak value of 3D pile is greater than 1D pile. It is shown that ignoring the radial deformation of pile will overestimate the static stiffness of pile and soil, and the peak value of dynamic impedance is underestimated.

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