WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2025.157490

ARCHIVES OF CIVIL ENGINEERING

Vol. LXXII ISSUE 1 2026

pp. 535 –550

Research paper

Modeling of root reinforcement of European hornbeam considering the use different fiber bundle models

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Abstract: This study comprises measurements of root systems of European hornbeam in forests located near Winiary, in the vicinity of Gdów (Wiśnickie Foothills, Poland) and near the town of Kanice (Moravia, Czech Republic). Field research included root measurements using the trench wall method conducted at a distance of approximately 1 m from the tree trunks. Tensile strength of the roots collected from both study sites and the geotechnical properties of the soil were determined in the laboratory. Based on the results of field measurements and laboratory tests, the root reinforcement was calculated using selected fibre bundle models. This study was aimed to compare root systems and mechanical properties of roots of the European hornbeam from the two sites and to compare the root reinforcement results obtained using different bundle models, taking into account different approaches to load distribution. The paper also discusses the impact of the tensile strength-strain characteristics of roots and their orientation relative to the direction of slip plane (zone) on the root reinforcement. The results of laboratory tests showed that tensile strength and elasticity of roots collected from both sites are very similar. Calculations showed that the values of root reinforcement computed using fiber bundle models are even 2.5 times smaller than those obtained from the classic Wu–Waldron model. On the other hand, the calculation results between fiber bundle models can differ by more than 2 times. It was also shown that the force-strain relationship used for calculation of root reinforcement has a significant impact on the results of the analyses, and the use of a bilinear force-strain relationship provides the results higher by approximately 10–13%.

Keywords: root reinforcement, fiber bundle model, European hornbeam

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1. Introduction

Plant roots are characterized by the ability to transfer tensile stresses in the ground, similarly to reinforcement strips or bars used in engineering structures made using reinforced soil technology. Therefore, the influence of plant root systems on the increase of shear strength of the soil, and consequently on the stability of slopes, is commonly considered positive. However, determining the value of root reinforcement is difficult, due to the large diversity of plant species, variability of the geometry and strength properties of roots, and the very limited possibility of performing tests on a real scale. Therefore, computational models are the primary main tool for estimating root reinforcement.

The basic and widely used model was developed by Waldron [1] and Wu et al. [2], which is commonly called the Wu–Waldron model (WW). This model has the following form:

$$c_{r(\mathrm{WWM})} = [\sin\theta + \cos\theta \cdot \tan\phi] \cdot \sum_{i=1}^{N}(T_{ri} \cdot RAR_i) = k' \cdot \sum_{i=1}^{N}(T_{ri} \cdot RAR_i) \tag{1.1}$$

where: c_r – root reinforcement, φ – the soil angle of internal friction, θ – the angle of root orientation in relation to the vertical plane, T_{ri} – the tensile strength of i-th root, RAR_i – the root area ratio of the i-th root (ratio of cross section area of a root crossing the shearing plane within the soil), k' – factor accounting the effect of root orientation at its failure ($k' = 1.0$ was recommended by Thomas and Pollen–Bankhead [3]).

In the Wu–Waldron model, it is assumed that the amount of reinforcement is proportional to the root area ratio and their tensile strength. Research (including [4, 5]) indicate that the model described above overestimates the effect of root reinforcement. Hence, an additional coefficient k'' was proposed in equation (1.1) [6, 7], which takes into account progressive failure of roots in the bundle:

$$c_{r(\mathrm{WWM})} = k' \cdot k'' \cdot \sum_{i=1}^{N}(T_{ri} \cdot RAR_i) \tag{1.2}$$

The proposed solution is a subjective criterion and does not take into account the distribution of number and diameters of roots in the bundle. This approach allows to obtain conservative results of the root reinforcement, but on the other hand it prevents insight into the mechanism of root bundle failure.

As an alternative to the model described by the Eq. (1.2), fiber bundle models (FBM), proposed by Daniels [8], were used which introduced the influence of plant root systems on soil reinforcement by Pollen and Simon [5]. The model takes into account the fact that the progressive root breaking occurs during shear failure; the weakest roots are ruptured first, and as they are ruptured, the tensile force is distributed to the remaining roots in the bundle. Later, the proposed solution was analyzed in terms of the method of distributing the external force on the root bundle (e.g. [3, 9, 10]) showing that the calculated values of the root reinforcement are clearly smaller than those obtained using the traditional Wu–Waldron model. The proposed solution basically required knowledge of the diameter and number of roots in the bundle as well as their tensile strength. During this period, another type of fiber bundle model (RBM) was also proposed [11, 12], in which stress-strain behaviour for stepwise increase of root bundle displacement was considered. In this model, it is assumed that the length of the roots and their elasticity (or stiffness) depends on their diameter. In another work, Schwarz et al. [13] proposed a modification of the model by using the Weibull survival function, which takes into account the probability of root rupture/pulling out depending on its relative displacement. The authors analyzed the sensitivity of the model and compared the results of calculations from the pull-out test, indicating the potential possibility of using the model for root systems characterized by a narrow range of diameters and/or root systems with high variability of mechanical properties. In recent years, Ji et al. [14] proposed a slightly different approach to the calculation of root reinforcement (FBM-W), assuming that the force distribution by roots depends on the amount of energy (work) transferred by individual roots in the bundle. The results of this work indicate that this model provides more conservative results than other FBM models.

The RBMw model considers the elasticity of the roots, which is used for computation of force mobilized within the roots at their deformation. Usually, the elastic modulus represents only the initial portion of tensile stress-strain curve. The use of one value of elasticity modulus for computation of root force mobilization means stress-strain response is linear, which is not always consistent with the results of tensile strength tests [15–17]. Therefore, Meijer et al. [18] proposed to use of bilinear stress-strain relationship that includes elastic as well as plastic behavior of root subjected to loading. Such approach seems to be universal and flexible solution for root reinforcement computing than was originally proposed in RBMw or FBM models.

The aim of this study was to compare the fiber bundle models used to determine the root reinforcement as well as to determine the effect of the stress-strain characteristics of roots on the results of calculations. To achieve the research objectives, a number of numerical analyzes were performed based on studies of the root system of the European hornbeam from two sites located in the foothill regions characterized with different geotechnical conditions. This tree species is widespread in central Europe, prefers damp site conditions, usually has a highly developed heart-shaped root system [20, 21], so it can potentially enhance slopes in landslide susceptible areas. We also compared the impact of site conditions on properties of European hornbeam root system, and thus its potential impact on improving soil shear strength.

2. Methodology

As part of field research, measurements of root systems of the common hornbeam (*Carpinus betulus* L.) were conducted in forest areas that differed in the geotechnical conditions. The common hornbeam is a tree from the hazel family (Corylaceae) growing to a height of 20–25 m and with a diameter of 60 cm at breast height This species is commonly found in central Europe. It grows best in moist soils, but avoids swampy soils. Hornbeam occurs in periodically flooded areas (riparian forests). It has average requirements in terms of the content of mineral compounds in the soil. It grows well on loamy soils, but it can also occur on loamy sands and compact formations [20]. The hornbeam has a highly developed heart-shaped root system with roots radiating in all directions. In clay and loamy soil and with a high content of rock fragments, it roots flatten, but individual vertical roots have a high ability to penetrate deep into the compact substrate. In turn, on sandy soils it develops a long tap root [21].

The research sites were located in the vicinity of the town of Winiary near Gdów (Poland), and the results from this site are partially presented in Zydroń and Gruchot [22], and in the vicinity of Kanice near Brno (Czech Republic). Winiary is located in the Wielickie Foothills in the Raba River valley, and the natural substrate in the area is loess. In May and June 2010, long-lasting and intense rainfalls occurred in the area, which triggered numerous mass movements. The town of Kanice is located in Moravia, close to the Moravian Karst, and the ground at the research site is silty sand, product of weathered limestone rocks.

The trench wall method [23] (Fig. 1) was used to determine the number of roots. This method is widely used to quantitatively measure plant root systems, especially trees, despite being tedious, invasive and time-consuming. Ground-penetrating radar (GPR) [24, 25] may be an alternative to the trenching method in the future, but this method still needs work to develop a suitable measurement methodology and calibration using the results of the trench method.

Research pits were made at a distance of 1.0 m from the trees. The depth and width of the trenches in Winiary were a maximum of 1.0 m, while in Kanice the trenches reached up to 0.4 m below ground level and their width was approximately 0.5 m. The wall of the trench on the side of the trees was loosened at a depth of approximately 5 cm to expose the roots. Then, root diameters were measured for 10 cm high layers on the wall. Roots with a diameter smaller than 1 mm and larger than 10 mm were excluded from the analysis. Fine roots are difficult to recognize. Large roots, on the other hand, are usually excluded due to questionable tensile strength values. As noted by Bischetti et al. [6], large roots significantly affect the RAR values, and on the other hand, they are so stiff that they do not fit to the assumptions of the original Wu–Waldron model. Six excavations were carried out in Winiary, and eight in Kanice. The average value of the diameter at breast height of trees in Winiary was 23.8 cm (with standard deviaton of 3.7 cm), and in Kanice 21.6 cm (with standard deviaton of 4.8 cm).

Root samples were taken from the research trenches and their tensile strength was determined. Before testing, the samples were placed in water for at least 24 hours to saturate them. After removing from the water, the diameter of each sample was measured in 3 places, and the mean value was taken as representative diameter [26]. The tests were carried out in a Hounsfield tensile test machine at a strain rate of 10 mm $\cdot$ min^{-1} and the effective length of the root sample was 10 cm. In order to avoid root damage at clamps of the device, the roots were covered with stripes of a rubber. In further analyses, the results of tests on samples broken in the clamps of the device were excluded. The value of tensile strength was determined as the quotient of the maximum tensile force and the cross-sectional area of the root, assuming that it has a circular shape. The elasticity of the root was backward calculated as a ratio of the tensile strength value and the relative displacement of the root at the moment of rupture.

The Wu–Waldron (WW) model and seven versions of fiber bundle models taking into account the progressive failure of the root bundle were used to calculate the increase in shear strength of the soil provided by the presence of roots. In the model proposed by Pollen and Simon [5] and described in detail by Thomas and Pollen [3] and Mao et al. [10], the force transmitted by the roots may be proportional to cross-sectional area of the roots in the bundle (FBM-S), root diameter (FBM-D) or number of roots in the bundle (FBM-N).

This group of models mainly take into account the diameter of the roots and their tensile (or pull-out) strength, which allows to determine the peak value of the root reinforcement, but in original version does not take into account the bundle displacement or the influence of the root orientation.

Another fiber bundle model used in the calculations was the RBMw model proposed by Schwarz et al. [13], in which the variability of the tensile or pull-out strength of individual roots is described by the Weibull survival function. In this model, the basic parameters of the roots (length, elastic modulus, maximum tensile strength) in the bundle are described by the following equations:

$$L(d) = L_0 \cdot \left(\frac{d}{d_0}\right)^{\gamma} \tag{2.1}$$

$$E(d) = E_0 \cdot r \cdot \left(\frac{d}{d_0}\right)^{\beta} \tag{2.2}$$

$$F_{\max}(d) = F_0 \cdot \left(\frac{d}{d_0}\right)^{\xi} \tag{2.3}$$

where: d – diameter of a root , d_0 – reference diameter of a root (commonly assumed 1.0 mm to simplify Eqs. (2.1)–(2.3)), L_0 – empirical characteristic length of the root of d_0 diameter, E_0 – empirical characteristic elastic modulus of the root of d_0 diameter, F_0 – empirical characteristic maximum tensile force of the root of d_0 diameter, r – dimensionless coefficient which considers the effects of root tortuosity on the tensile behavior of a root (0.3–0.5 are recommended for laboratory tests), γ, β, ξ – empirical exponents determined by tests.

The E_0, β, F_0 and ξ parameters were determined on the basis of tensile tests, and only the values of parameters L_0 and $\gamma = 0.285$ and 0.7, respectively, were taken from the literature [11].

The rearrangement of the above equations enables to determine the value of mobilized force that is transferred to a single root at the deformation Δx:

$$F(d, \Delta x) = \frac{\pi \cdot E_0 \cdot r}{4 \cdot L_0} \cdot \Delta x \cdot d^{2+\beta-\gamma} \tag{2.4}$$

Considering large variability of the tensile (or pull-out) strength of the roots, the value of the force transmitted by the root bundle (F_{tot}) is calculated as follows:

$$F_{tot}(\Delta x) = \sum F(d, \Delta x) \cdot S(\Delta x^*) \tag{2.5}$$

where: $S(\Delta x^*)$ – two-parameter survival function defined as follows:

$$S(\Delta x^*) = \exp\left[-\left(\frac{\Delta x^*}{\lambda}\right)^{\omega}\right] \tag{2.6}$$

where: Δx^* – normalized displacement of a root, λ – the scaling parameter, ω – shape factor of the Weibull function.

Normalized displacement is defined as follows:

$$\Delta x^*(d) = \frac{\Delta x}{\Delta x_{\max}(d)} \tag{2.7}$$

where:

$$\Delta x_{\max}(d) = \frac{4 \cdot F_0 \cdot L_0}{\pi \cdot r \cdot E_0} \cdot d^{\xi+\gamma-\beta+2} \tag{2.8}$$

The values of λ and ω were determined based on the results of laboratory tensile tests by fitting the function to the values of normalized root strain observed at its rupture. Another model used in the calculations was the FBM-W model proposed by Ji et al. [14], in which the amount of root deformation and the force mobilized by the roots is a derivative of the work performed thereof, and the amount of energy transferred by individual roots is, as in the FBM model, proportional to root cross-sectional area in the bundle (FBM-WS) – root diameter (FBM-WD) and number of roots in the bundle (FBM-WN).

As noted by Meijer [18], all models are part of the same "family", and the method of dividing the tensile force into the roots in the bundle can be reduced to the following formula:

$$\frac{F_i}{F_j} = \left(\frac{d_i}{d_j}\right)^{\beta_F} \tag{2.9}$$

where: F – tensile force in unbroken roots "i" and "j", β_F – load sharing parameter.

For the FBM models group, the values of β_F are 0, 1 and 2 for the FBM-N, FBM-D and FBM-S models, respectively. On the other hand, in the case of RBMw models, the parameter β_F depends on the elastic modulus of the roots in the bundle and their length ($\beta_F = 2+\beta-\gamma$) [19,27]. In the case of a root bundle consisting of roots of identical length and elasticity, the distribution of force transmitted by the roots will be the same as in the FBM-S model. On the other hand, for models from the FBM-W group, for the FBM-WN, FBM-WD and FBM-WS models the values are as follows: $2+0.5\beta$, $2.5+0.5\beta$ and $3+0.5\beta$. Meijer [19] points out that the parameter values may be in the range $(-\infty; \infty)$. The detailed analysis of the values of β_F obtained for the RBM model and the FBM-W model group was also carried out in this paper.

Another important factor affecting the results of calculations of the increase in shear strength of root-reinforced soil is the method by which mobilization of the tensile force in a root is calculated. Usually the calculations of root reinforcement are made using the constant value of elastic modulus, which usually considers linear elastic portion of stress-strain curve. If only such value of elastic modulus is considered, the tensile force mobilization described by equation (2.4) is also linear and does not realistically describe root behavior as the yielding strain is exceeded. Thus, to include non-linear stress-strain behavior, Schwarz et al. [12] proposed a function in which elastic modulus was a function of strain. On the other hand, Meijer [18], considering the tensile tests of Loades [16], proposed a bilinear stress-strain function, which, after converting tensile force and strain to normalized values, is shown in Fig. 1. In the present study, a modification was introduced into the RBMw and FBM-W models, which allows root reinforcement calculations using both a linear and a bilinear relationship to be applied between the value of the tensile force mobilized by the root and its deformation. A grid search method was used to determine the values of the strain and mobilized force at yield point (x_{yield} and F_{yield}, respectively) and the criterion for selecting both parameters was the value of the mean squared error.

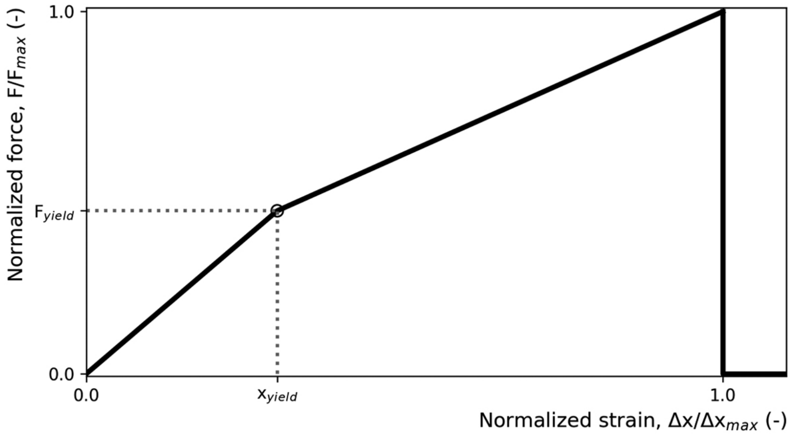


Fig. 1. The scheme of bilinear relation between mobilized tensile force and root strain

It should be noted that the effect of root systems on slope stability can be considered as basal and lateral reinforcement. In general, most roots are found close to the ground surface (up to 0.5 m) (e.g. [6,9,28]), so often a small number thereof penetrate to a depth corresponding to the position of the sliding surface. In contrast, a significant proportion of roots cross the lateral surface of the sliding earth mass, so these roots will generally play the dominant role in the slope reinforcement. Schwarz et al. [29] stated that the lateral reinforcement is the most effective in the case of small and shallow mass movements, and as the dimensions of landslide increase, the role of lateral reinforcement decreases. On the other hand, the presence of roots at the depth of 1 m was scarce so only the lateral root reinforcement was considered in the paper.

The calculations were performed in the Google Colaboratory program in the Python programming language. To data analyse the *numpy* [30], *pandas* [31] packages were used: SciPy [32] and scikit-learn [33] were used to calculate equation and selected metrics. Visualization of results were performed with the use of matplotlib [34] library.

3. Results and discussion

3.1. Parameters of roots systems

Measurements of European hornbeam root systems from Winiary and Kanice sites (Fig. 2) revealed variability of root area ratio values both between and within each site. The average RAR value was 0.12% for hornbeam trees in Winiary and 0.17% for Kanice. There is a clear tendency that the value of the root area ratio decreases with depth. At a depth of 0.0–0.1 m below the ground, RAR values for trees from Winiary were on average 0.23%, and 0.4% in Kanice. The obtained RAR values are within the range of values given in the literature for various tree species or plant groups (e.g. [6, 26, 27, 35–38]). The RAR values for European hornbeam given in the work by Abdi et. al. [35] are clearly larger than those obtained in the present study, but in the cited work the authors carried out measurements for trees with a larger breast diameter heights and it is not clear what range of root diameters they took into account in the RAR calculations. In turn, the results of RAR measurements of common hornbeam presented in the work of Deljouei et al. [36] are very similar to those obtained in this study.

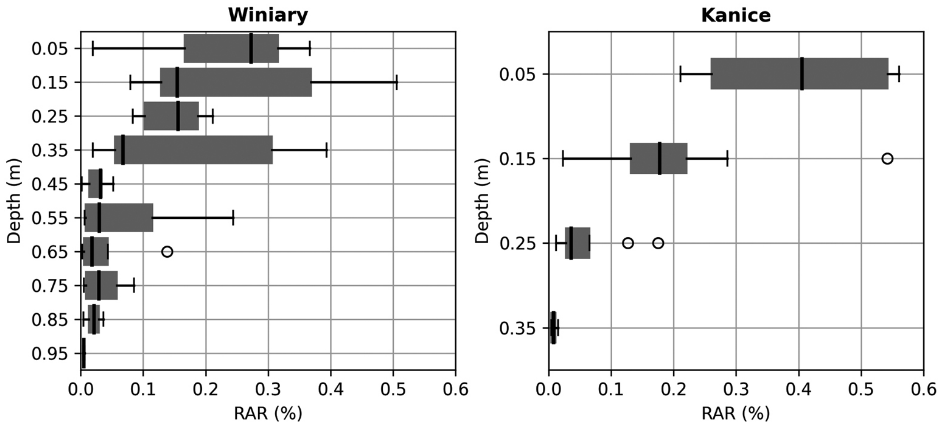
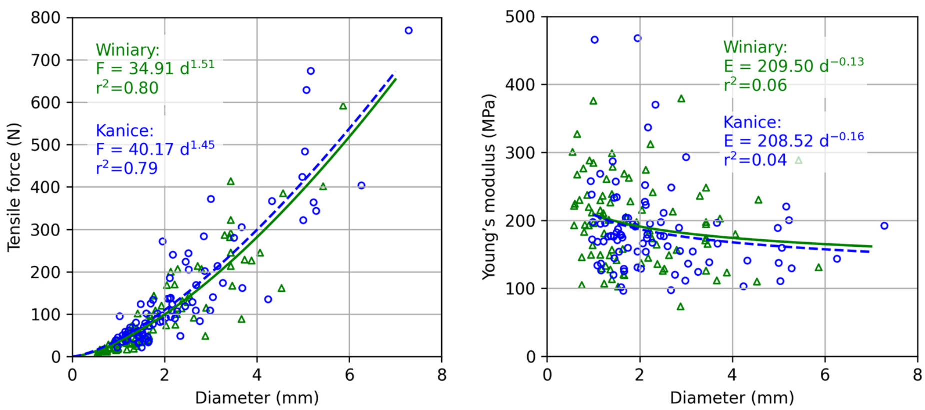


Fig. 2. Results of root area ratio measurements

The tests revealed that the roots collected at both sites are similar in terms of tensile strength and tensile force (Fig. 3). The mean tensile strength values of hornbeam roots from Winiary was 30.6 MPa (with standard deviation of 11.4 MPa), whereas the mean tensile strength of roots from Kanice was 33 kPa (with 13.9 MPa of standard deviation). The results also showed, that roots of the European hornbeam are relatively strong, which is also confirmed by other research [37, 38].



(a) (b)

Fig. 3. Tensile force (a) and elasticity (b) of roots from the both sites

The values of elastic modulus were determined on the basis of tensile tests, considering the value of the tensile force and the deformation of the samples at the moment of rupture. The tests showed a relatively weak correlation with the root diameter, which is typical in this type of research [39, 40]. The adopted method of elasticity calculation simplifies the interpretation of root elasticity, makes the stress-strain relationship linear and enable avoiding the problem of interpretation of elastic modulus, where stress-strain curve can be both sigmoid and hyperbolic [15].

Generally, the literature data providing values of elastic modulus of roots are scarce and vary considerably. The value of this parameter provided for spruce is 24.8 MPa [13]. In contrast, Commandeur and Pyles [15] obtained the mean value of modulus for Douglas fir equal to 503 MPa. Boldrin et al. [39] presented this value in the range from 150 to over 300 kPa for woody European shrubs and small trees. With the same criteria for determination of the elastic modulus, i.e. considering the initial linear part of stress-strain curve, mean values of modulus of tested roots of hornbeam were 350 MPa (with standard deviation of 147 MPa) and 309 MPa (with standard deviaton of 128 MPa) for roots from Winiary and Kanice, respectively.

The results of root tensile tests indicate a significant variability of the measured values of the root tensile force and the dependence of the mobilized tensile force on deformation (strain) of the root sample. For the purposes of further analysis, the parameters of the Weibull survival function were determined and the behaviour of roots during tensile tests was analyzed (Fig. 4). The analysis showed that the values of the Weibull function parameters obtained for the roots from both places (Winiary: $\lambda = 1.08$, $\omega = 3.58$ and Kanice: $\lambda = 1.11$, $\omega = 3.81$) are very similar, while the roots from Winiary are slightly stiffer, as evidenced by a smaller value of strain at which roots exhibit plastic behaviour.

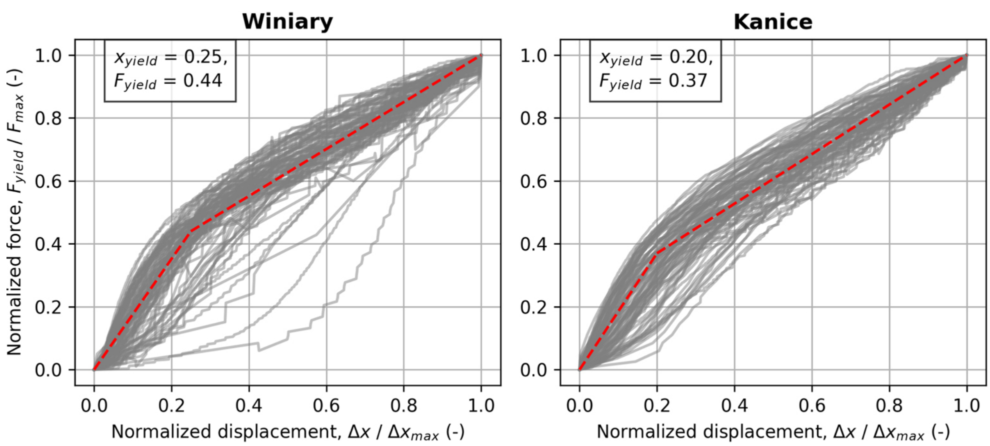


Fig. 4. Root normalized displacement versus normalized tensile force response

3.2. Comparison of fiber bundle models

To calculate the value of soil reinforcement, eight models were used (Wu–Waldron model and seven fiber bundle ones). The obtained results confirm the significant impact of the adopted method and load sharing on the values of root reinforcement (Fig. 5). The highest root reinforcement values were obtained using the WW model and, in the extreme case, they were on average 2.5 times higher than those obtained from the other models. The smallest values, representing 0.29–0.40 of the values of root reinforcement determined by the WW model, were obtained using the FBM-N model. Similar relationships are shown by Meijer [19] or Mao [10]. Root reinforcement values determined by FBM-N model are slightly lower than obtained using the FBM-WN and RBMw models. Greater variation in root reinforcement values was obtained in the case of the root systems of trees from Winiary, which was most likely related to the larger number of roots in the bundle.

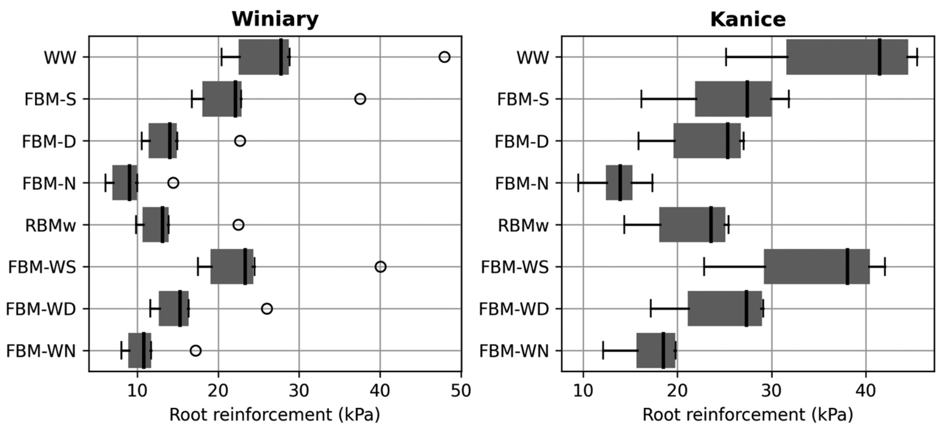


Fig. 5. Root reinforcement results obtained using different models

The obtained values of root reinforcement were in a relatively wide range, i.e. from 6.1 to 48.0 kPa, while excluding the WW model, they ranged from 6.1 to 42.0 kPa. In the case of the most conservative model, the root reinforcement ranged from 6.1 to 17.4 kPa, and slightly higher values were obtained for the root systems from Kanice. This relationship is a result of the smaller depth of research excavations at this site. The values of soil reinforcement in layers located near the ground surface, calculated using fiber bundle models, reported in the literature may exceed 50 kPa. On the other hand, the increase in shear strength of soil provided by roots from direct tests (most often the direct shear method) given in the literature usually do not exceed 20 kPa (e.g. [4, 40, 41]), which is consistent with the results of the backward analysis described in [13].

The obtained calculation results clearly indicate that the values of root reinforcement obtained using all models varied significantly. These differences, as shown earlier, are related to the different distribution of the tensile force into the roots, which can be described by the values of parameter β_F (Eq. (2.7)). In the case of the FBM-S, FBM-D and FBM-N models, the values of this parameter are constant and independent of the root properties (Eqs. (2.1)–(2.3)).

In turn, in the case of the RBMw model, the values of β_F change with the increase in bundle displacement. In the initial range of deformations, when the force-displacement relationship of the root bundle is linear, the β_F values for the roots from Winiary are approximately 1.17, and for the roots from Kanice β_F values are approximately 1.14. As the strain increases, the β_F values increase geometrically, especially when the displacement exceeds the value corresponding to the maximum force mobilized in the bundle. When the maximum value of the tensile force of one of the roots is exceeded, the β_F value exceeds 2. In the case of the FBM-WS, FBM-WD, FBM-WN models, the β_F values are constant during the displacement of the root bundle (Table 1). It can be noted that the value of the parameter characterizing the distribution of tensile forces according to the FBM-D and FBM-WD models differ slightly.

Table 1. Values of load sharing parameter β_F

Location	RBMw	FBM-WS	FBM-WD	FBM-WN
Winiary	≥ 1.17	1.58	1.08	0.58
Kanice	≥ 1.14	1.57	1.07	0.58

The values of the β_F parameter obtained from the calculations differ significantly from those given in the work of Meijer [19] or Cohen et al. [27], who used a model from the RBM group. These differences result from different characteristics of roots of the tested plants. In the case of Meijer's [19] study, root elasticity or tensile strength typically increased with increasing diameter. In the case of the analysis performed by Cohen et al. [27], the value of the β coefficient was greater than γ, which meant that the β_F values were usually greater than 2. Meijer [19], when using the same models as in this study, obtained load sharing parameter values ranging from −0.55 to 2.36, and he suggested the use the coefficient $\beta_F = 1.5 + 0.5\beta$, which was given by Waldron [1] and validated by the experiments of Bull et al. [42].

To determine the correct force distribution in the root bundle, it is helpful to know which the order of roots breakage. As noted by Thomas and Pollen-Bankhead [8], and also indicated by the results of the analyses of Mao et al. [10], Ji et al. [14], the use of models in which the distribution of forces on the roots is related to the cross-sectional area (FBM-S model) leads to a situation in which thick roots are ruptured (or pulled out) first, which, according to Thomas and Pollen-Bankhead [3], is not representative and even opposed to the results of the research by Giadrossich et al. [43]. On the other hand, assuming that the tensile/pull-out force is distributed evenly over all roots usually gives a very conservative estimate of the root reinforcement. The RBM model assumes that when the tension is mobilized in the bundle, all roots undergo the same displacements, which, as emphasized by Cohen et al. [27] has local applications for roots that are located at short distances from each other. The validation of the models analyzed in this work carried out by Meijer [19] does not indicate which model performs best. However, the research results of Meijer et al. [18] indicate that the RBMw model gives more reliable peak root reinforcement results than the FBM models. Therefore, for the purposes of preliminary assessment of increase in soil shear strength provided by roots, it seems reasonable to use the RBMw model, which is relatively simple to use and, unlike the methods from the FBM-W group, does not require tedious iterative calculations.

3.3. The impact of root stress-strain relationship on root reinforcement

The results of tensile tests showed that stress-strain relationship is not linear. Therefore, calculations of reinforcement were performed using linear and bilinear relation between mobilized force within the root and its displacement. These calculations showed that the use of bilinear root tensile characteristics clearly affects the peak values of root reinforcement (Fig. 6), and, in the case of the RBMw model, also the value of displacement at which the maximum bundle resistance is achieved. The calculation results obtained for the RBMw and FBM-WN models showed that the root reinforcement values calculated using the bilinear root stress-strain characteristic provide values of peak root reinforcement higher on average by 13% and 10%, in the case of Winiary and Kanice, respectively, than when using the linear function $F = f(\Delta x)$. On the other hand, the root reinforcement values calculated with the use of nonlinear stress-strain characteristics are lower on average 47 and 53% (for RBMw and FBM-WN, respectively) than those determined by the WW model.

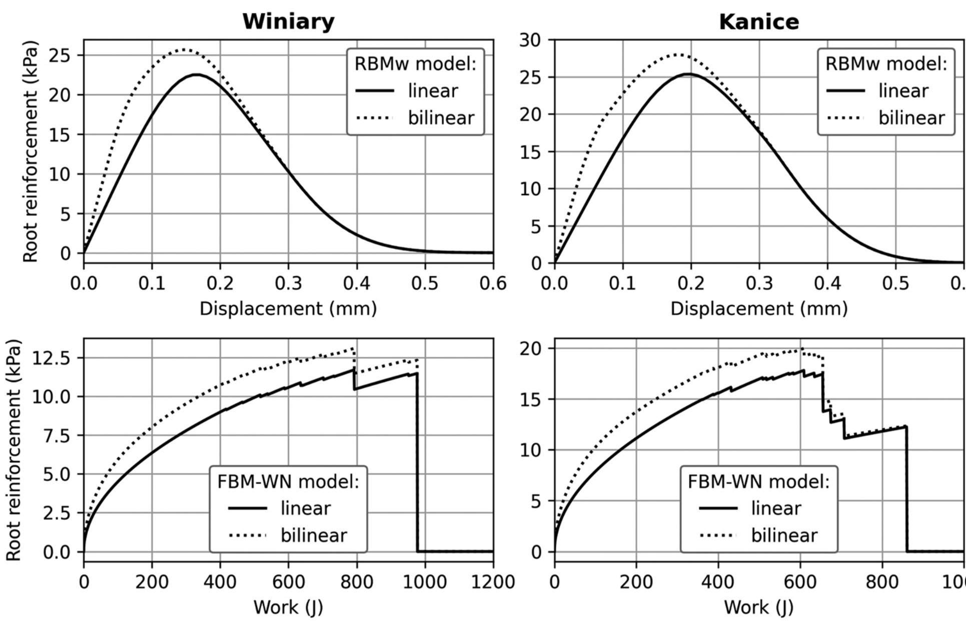


Fig. 6. The influence of force-strain behaviour on mobilization of root reinforcement for RMBw and FBM-WN models

Calculation results revealed that the use of the bilinear $F - \Delta x$ characteristic increases the β_F coefficient in the case of the RBMw model, but does not have a significant impact on changes in force distribution in the case of the FBN-WN model.

4. Conclusions

1. The results of tests on the root system of the European hornbeam indicate that the site of sampling and type of soil did not have a significant impact on the tensile strength and elasticity of roots. The calculated root reinforcement values indicate that the hornbeam root system significantly increases the shear strength of the soils.
2. The assessment results of the root system impact on soil reinforcement depend on the adopted calculation model for the analysis and the related method of load sharing distribution on the roots in the bundle. The root reinforcement determined with fibre bundle models were on average 30% to almost 300% lower than those obtained with the classical Wu–Waldron model.
3. The use of bilinear approach for description stress-strain relationship increases the peak valued of root reinforcement up to 13%, and in the case of the RBMw it also causes a change in force sharing distribution between roots.
4. Considering the influence of root reinforcement on slope stability, the classic calculations of the safety factor should be extended with probabilistic calculations in which variability of root systems' mechanical parameters and geometry can be included. The values of root reinforcement by the European hornbeam presented in this study, correspond to the effect of lateral reinforcement. Thus, the obtained test results should be applied in the three-dimensional slope stabilization models.

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Obliczenia wzmocnienia gruntu przez system korzeniowy grabu pospolitego – implikacje stosowania różnych modeli wiązkowych

Słowa kluczowe: wzmocnienie gruntu, modele wiązkowe, wytrzymałość na ścinanie

Streszczenie:

Powszechnie przyjmuje się, że roślinność ma pozytywny wpływ na stateczność zboczy, a rolę te przypisuje się głównie korzeniom, które posiadają zdolność przenoszenia sił rozciągających. Zakres wzmocnienia gruntu przez system korzeniowy jest zróżnicowany i zależy m.in. od warunków siedliskowych, gatunku i wieku rośliny, gęstości i głębokości systemu korzeniowego, wytrzymałości korzeni na rozciąganie czy też orientacji względem płaszczyzny ścinania. Mnogość gatunków roślin oraz zmienność warunków siedliskowych powoduje, że wciąż istnieje potrzeba tego typu badań. Z drugiej strony do ilościowej oceny wpływu systemów korzeniowych na wzmocnienie gruntu opracowano szereg modeli wiązkowych (grupy modeli FBM, FBM-W oraz model RBMw), a otrzymane na ich podstawie wyniki mogą różnić się znacząco. Celem pracy było porównanie gęstości i parametrów mechanicznych systemów korzeniowych z obu miejsc, określenie wpływu zastosowanej metody obliczeń na potencjalne wartości wzmocnienia gruntu przez analizowane systemy korzeniowe. Zakres badań obejmował pomiary systemów korzeniowych grabu pospolitego w lasach położonych na zboczach osuwiskowych w pobliżu miejscowości Winiary k. Gdowa (Pogórze Wiśnickie, Polska) oraz w okolicach miejscowości Kanice

(Morawy, Republika Czeska). Badania terenowych obejmowały pomiary korzeni we wkopach badawczych w odległości ok. 1 m od drzew. Szerokość i głębokość wkopów badawczych uzależniona była od warunków geotechnicznych. W ramach prac laboratoryjnych wykonano pomiary wytrzymałości na rozciąganie korzeni oraz określono właściwości geotechniczne gruntów. Na podstawie wyników pomiarów terenowych oraz badań laboratoryjnych wykonano obliczenia wartości przyrostu wytrzymałości na ścinanie za pomocą wybranych modeli wiązkowych. W pracy analizie poddano również wpływ przyjętej w obliczeniach charakterystyki siła rozciągająca-odkształcenie korzeni. Otrzymane wyniki wykazały, że wartości przyrostu wytrzymałości na ścinanie gruntu przez korzenie obliczone na podstawie modeli wiązkowych są nawet 2.5-krotnie mniejsze niż uzyskuje się z klasycznego modelu Wu–Waldrona. Z kolei porównując wyniki obliczeń pomiędzy modelami wiązkowymi zakładającymi progresywne niszczenie wiązki korzeniowej różnice mogą dochodzić nawet do 60%. Analiza wyników obliczeń wskazuje, że rozsądnym podejściem w obliczeniach wpływu na korzeni na wzmocnienie gruntu będzie zastosowanie modelu RBMw. Obliczenia wykazały również, że znaczący wpływ na rezultaty analiz ma dobór metody opisu zależności pomiędzy mobilizowaną siłą przez korzeń a jego odkształceniem, a zastosowanie dwuliniowej zależności siła-odkształcenie zwiększa wyniki obliczeń o ok. 10–13%.

Received: 2024-09-24, Revised: 2024-10-08