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Research paper

Elevation error prediction of continuous beam cantilever construction phase based on LS-SVM

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Abstract: According to the construction of long-link continuous beam bridge, I choose field measured data as research samples and establish an elevation error prediction model by dealing samples, selecting the kernel function, selecting parameters, training, and predicting. I compare the Least Squares-Support Vector Machines (LS-SVM) prediction value with the measured value, the SVM model predictions, the BP neural network model predictions and the dimensionality reduction model predictions, so that predict elevation errors during cantilever construction phases by established models. According to the results of the comparison, the elevation error prediction model is highly accurate and has more high efficiency, good stability, and strong learning ability. Under the verification of the elevation control results in the cantilever stage, LS-SVM elevation error prediction is used for controlling the elevation of the bridge and solves the problem–predictive control successfully which is caused for few beam blocks in the cantilever phase of a continuous girder bridge.

Keywords: LS-SVM model, Cantilever pouring, Elevation prediction, Continuous beams

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1. Introduction

The study of bridge construction began very early in foreign countries [1]. Until end of the last century, bridge workers used the evolving computer technology to intelligent construction control and propose the construction control process for suspension bridges and cable-stayed bridges in foreign countries. Foreign bridge workers used a combination of predictive control and other methods by developing construction control system [2–4].

The advent of Support Vector Machines (SVM) led a global boom in machine learning. In the early days of SVM theory, most of the researchers were mathematicians, who carried out research, because it provided a new method for the study of finite sample problems. The research mainly focused on the improvement of the theory, such as function regression, approximation, density estimation, data mining, and chaotic time series prediction [5–8]. By the beginning of the century, SVM theory began to enter the practical application stage, such as face detection, handwritten digit recognition, automatic text classification, optimization control, computer vision, gene classification and target recognition [9, 10]. Until recent years, SVM begun to be used in civil engineering, but it was rarely used in bridge construction control applications [11].

After less than 10 years of development, the theory of construction control has taken shape, from the concept of construction control to open-loop control, closed-loop control and adaptive control, each control theory has been verified in practice [12]. Through several scientific research projects, a systematic control system of construction, monitoring, identification, correction, control, and prediction has been basically formed. At the same time, in the process of realizing prediction, Chinese scholars mainly use Kalman filter method, gray theory and BP neural network method to control the elevation of bridge construction [13, 14]. In addition, various predictive control methods have been applied in cable-stayed bridge cable replacement, bridge health monitoring and other projects [15].

At present, SVM has been applied in many directions in civil engineering, such as bridge seismic assessment, bridge damage identification, dam safety prediction and tunnel safety control. In the bridge construction control, it is a trial stag. The least-squares support vector machine is a more accurate and faster algorithm that combines the Sam theory with the least-squares algorithm. Based on the actual engineering background, according to the characteristics of the long-connected long-span continuous girder bridge, with the help of construction control and finite element model analysis, the least squares support vector machine (LS-SVM) is used to predict the elevation error in the cantilever stage.

2. Project overview

A bridge is a 13-span variable cross-section continuous box girder bridge, with a span diameter of 59.7 m +11 × 100 m + 59.7 m. The superstructure adopts a single-box, single-chamber variable-section prestressed continuous box girder. The pier is a solid pier, wherein piers 56 and 69 are transitional piers, and the junction with the cushion cap is provided with an ice-breaking body, and the pier cap size is 1060 × 360 cm, and the pier body size is 600 × 300 cm. The foundation is made of bored piles with a diameter of 180 cm. The advantages of the long continuous beam bridge include many stress points, good structural stiffness, simple maintenance, strong earthquake resistance, smooth and comfortable driving and favorable for high-speed driving.

The beam height at the pier is 6.09 m, the middle span and the beam end are 2.59 m, the top surface is set according to the two-way 1.5% cross slope, and the beam height changes according to 1.6 parabolas in the range from block 0 to 49 m in the middle of the span. The top width of the box girder is 12 m, the width of the bottom plate is 6.0 m, and the cantilever length of the wing plate is 3.0 m. The edge of the flange is 18 cm thick, and the flange support is 60 cm thick. The web is straight, the thickness of the web is 70 cm for the No. 0–5 block, the No. 6 block is 70 × 50 cm, and the rest is 50 cm. The thickness of the middle span bottom plate is 32 cm, the thickness of the root of the pier top section is 110 cm, and the middle changes according to the parabola. The main beam adopts a three-way prestressed system, the longitudinal beam is OVM15-12, OVM15-15 steel strands, the transverse is BM15-4 prestressed steel strands, and the vertical is made of 25 mm high-strength finely tied threaded coarse steel bar, which is arranged in the web. Block 0 is 12 m long, Block 1–2 is 3.5 m long, Block 3–11 is 4 m long, the middle span closure section is 2 m long, and the side span closure section is 2 m long. The elevation and cross-sectional view of the bridge are shown in Fig. 1 and Fig. 2. In order to reduce the influence of temperature change on the continuous beam bridge, the fixed support is set at the position of pier 62#. Under the action of temperature load, the expansion of the bridge is minimized.

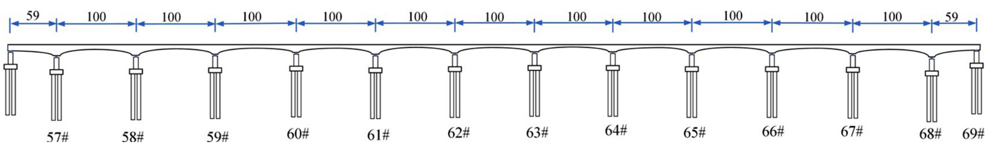

Fig. 1. Bridge elevation (unit: m)

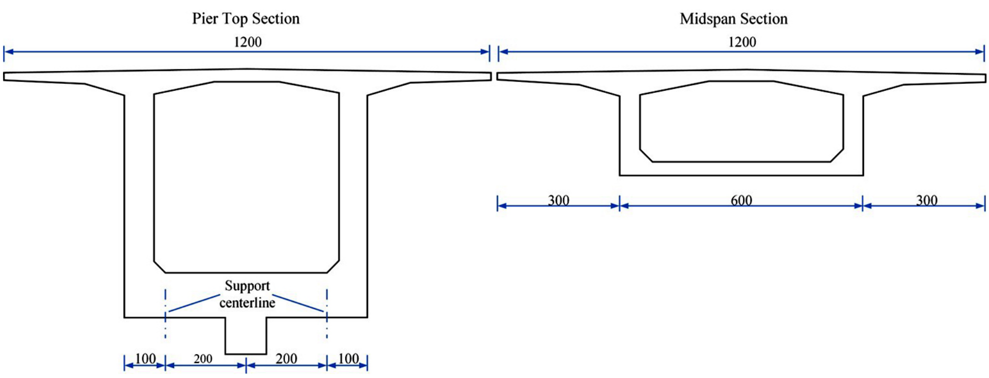

Fig. 2. Surface view (unit: cm)

3. LS-SVM basic principle

SVM is a method of machine learning and machine learning is based on the existing training samples to mine the connection between input and output and seek an optimal function to minimize the empirical risk of a learning process, any machine learning is inseparable from statistical learning, but SVM has its own uniqueness.

The LS-SVM algorithm cleverly uses the kernel function of the original space to replace the point product operation in the high-dimensional feature space, and the main parameters affecting the algorithm are only the penalty coefficient γ and the influence parameters of the kernel function, and the traditional SVM quadratic programming method is replaced by the least squares linear system, different loss functions are taken, and the equation constraint is used to transform the solving quadratic programming problem into a linear equation problem, which obviously greatly reduces the complexity of the algorithm and improves the anti-interference ability and speed of the operation process.

4. Establishment of a Le-Sam elevation error prediction model

4.1. Predictive model building process and methodology

Before building a predictive model, you first need to determine the process of model building and choose a correct method to achieve a target. Based on the principles of LS-SVM prediction introduced in the previous section, the process of determining the establishment of a prediction model is shown in Fig. 3.

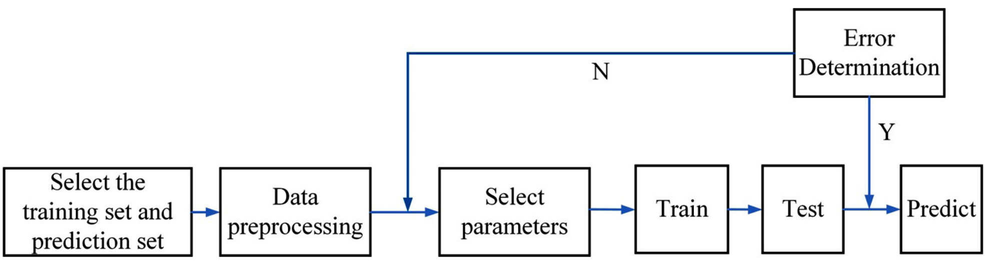

Fig. 3. Predictive model building flowchart

To build a prediction model, the mathematical software Matlab is used to realize the linguization of the algorithm, that is, the software calculates in the form of input commands and obtains the calculation results. However, it is a cumbersome process to linguize the calculation principle of LS-SVM, so many scholars have developed a toolbox in combination with Matlab for program calling, among which the most famous are the Libsvm toolbox developed by Dr. Lin Zhiren of Taiwan. The LS-SVM toolbox developed by K. Pelckmans and Suykens J.A.K, which is more targeted and more algorithmic, will also be used in this paper for model building.

4.2. Selection of samples

There are 8 influencing factors that cause the elevation error of the cantilever construction beam block, but in order to give the elevation of the vertical mold before the construction of the next section, and the elevation error value of the next beam block is predicted, some parameters need to be adjusted, and finally selected: the self-weight of the beam block (G), the length of the beam block (l), the cantilever length (L), the concrete construction age of the previous beam block (D), the initial measurement temperature of the beam block construction (T), the theoretical deflection value (ΔH_1) caused by pouring concrete, The theoretical deflection value (ΔH_2) caused by the prestress tension and the elevation error value (ΔH_3) of the previous beam block are taken as the input values, and the elevation error value (ΔH) of each section is taken as the output value.

Table 1. Input and output sample raw data tables (60# north)

Block Number	***G* (t)**	***l* (m)**	***L* (m)**	***D* (d)**	***T* (°C)**	**ΔH_1 (mm)**	**ΔH_2 (mm)**	**ΔH_3 (mm)**	**ΔH (mm)**
2# block	139.3	3.5	13	10	15	–0.3	0.4	2.1	3.3
3# block	148.4	4	17	9	15	–0.6	1.2	3.3	4.1
4# block	138.9	4	21	7	17	–1.2	3.4	4.1	3.2
5# block	129.4	4	25	7.5	19	–1.9	4.4	3.2	3.5
6# block	115.8	4	29	7	16	–2.9	6.5	3.5	4.6
7# block	103.3	4	33	8	19	–4.6	9.9	4.6	3.9
8# block	96.2	4	37	8.5	20	–6.5	10.5	3.9	5.5
9# block	91.7	4	41	8	21	–9.9	10.3	5.5	5.3
10# block	87.5	4	45	9	21	–14.4	15.3	5.3	4.5
11# block	83.3	4	49	8	21	–21.6	23	4.5	4.1

In the cantilever pouring construction, the structural stiffness of the 0# block and the 1# block is large, the cantilever length is short, and the deformation is small, so when selecting the sample set, it is considered from the 2# block to the 11# block. Due to the large number of beam blocks in bridge construction, the more representative data of each beam block on the north side of the 60# pier and the south side of the 61# pier are selected for analysis, and the original data are shown in Table 1 and Table 2.

According to the original data, the data of 2# blocks – 7# blocks of each pier are selected as the training set, and the data of 8# blocks – 11# blocks are the prediction set, and the previous value is also used as the training set when predicting, to predict one by one.

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Table 2. Sample original data table (61# south)

Block Number	***G*** **(t)**	***l*** **(m)**	***L*** **(m)**	***D*** **(d)**	***T*** **(°C)**	**ΔH_1 (mm)**	**ΔH_2 (mm)**	**ΔH_3 (mm)**	**ΔH (mm)**
2# block	139.1	3.5	13	9	13	–0.3	0.4	1.4	3.8
3# block	148.0	4	17	9	14	–0.6	1.2	3.8	4.5
4# block	138.7	4	21	8	16	–1.2	3.4	4.5	4.6
5# block	129.2	4	25	8	18	–1.9	4.4	4.6	5.2
6# block	115.5	4	29	7	17	–2.9	6.5	5.2	4.3
7# block	102.9	4	33	7	19	–4.6	9.9	4.3	4.3
8# block	95.8	4	37	9	18	–6.5	10.5	4.3	6.2
9# block	91.3	4	41	10	20	–9.9	10.3	6.2	6.3
10# block	87.1	4	45	8	18	–14.4	15.3	6.3	6.1
11# block	83.0	4	49	7	20	–21.6	23	6.1	5.4

4.3. Preprocessing of data

To reduce the data processing error and improve the calculation speed, it is necessary to normalize the input samples. The normalized mapping is as follows. The visualization of samples before and after normalization is shown in Fig. 4– 7.

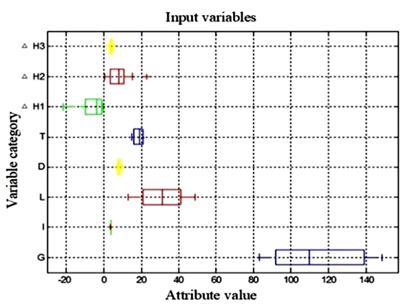

Fig. 4. Sample visualization (60# north)

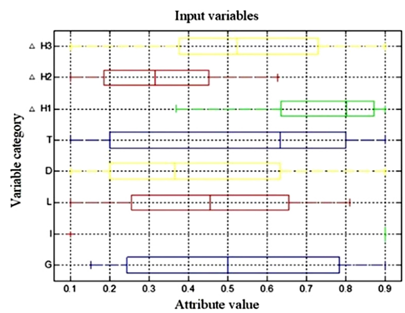

Fig. 5. Sample visualization (60# north)

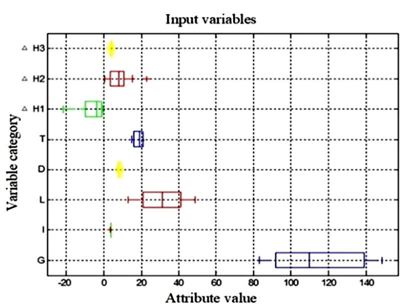

Fig. 6. Sample visualization (61# south)

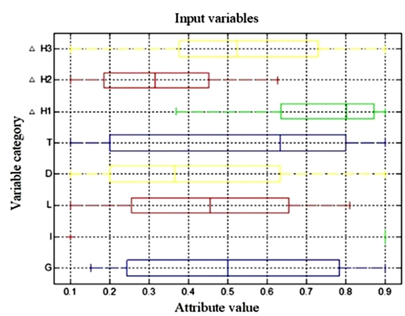

Fig. 7. Sample visualization (61# south)

4.4. Selection of parameters

In practical application, the combination of “grid search method” and “cross-validation method” is used to select the optimal parameters, and the specific steps are as follows: first, determine the approximate range of the two parameters $\gamma \in [1,10000]$ and $\sigma^2 \in [0.01,10]$, and then perform K aliquots to cross-verify the accuracy of the test within the selected range.

The contour plots selected for the two sample data parameters are shown in Fig. 8 and Fig. 9. After predicting the error of a beam block, the data of this section is used as the training set to predict the error of the next section, then there will be some optimal parameter combinations, and the optimal parameter combination is not a specific set of values, as long as it can meet the requirements of the model’s calculation accuracy, it is considered “optimal”, and there is no need to demand absolute optimality.

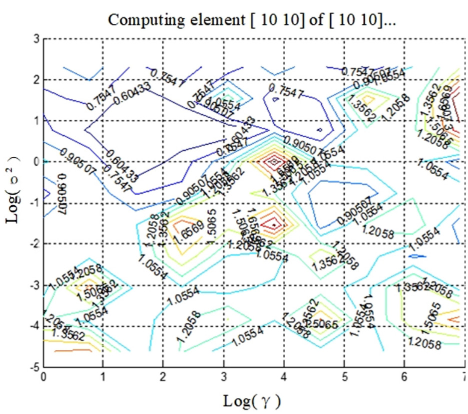

Fig. 8. Contour map (60# north)

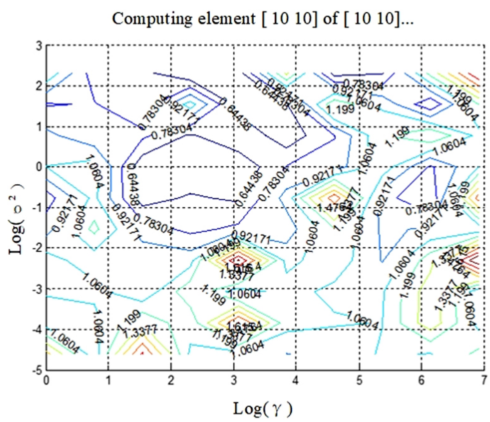

Fig. 9. Contour map (61# south)

4.5. Training and prediction of samples

The determined parameters can be used to train and predict the samples accordingly, and when the training accuracy of the training samples meets the requirements, the prediction samples can be used to predict the elevation error of the next section. In the establishment of the bridge model studied in this article, the Mean Square Error (MSE) is used as the standard for the determination of training error, and the accuracy of the model is considered appropriate when the error is less than 0.001. In the establishment of the bridge model studied in this article, the Mean Square Error (MSE) is used as the standard for the determination of training error, and the accuracy of the model is considered appropriate when the error is less than 0.001. Training result diagrams are shown in Fig. 10–13. Parameters and result are shown in Table 3.

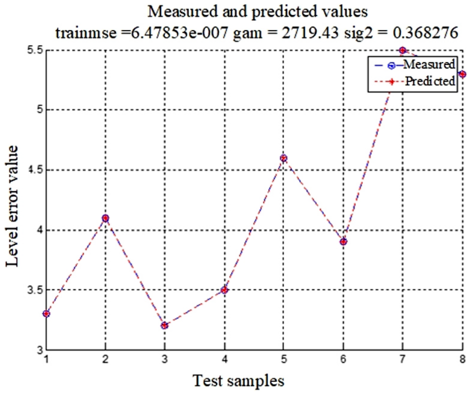

Fig. 10. Training result diagram (60# north)

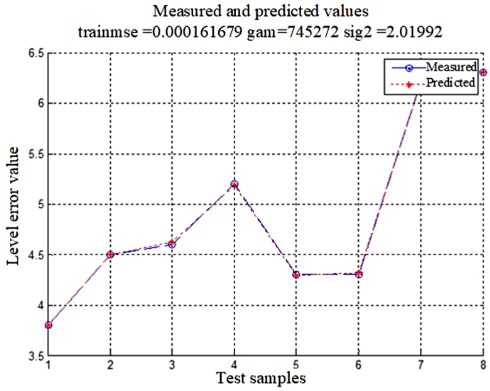

Fig. 11. Training result diagram (61# south)

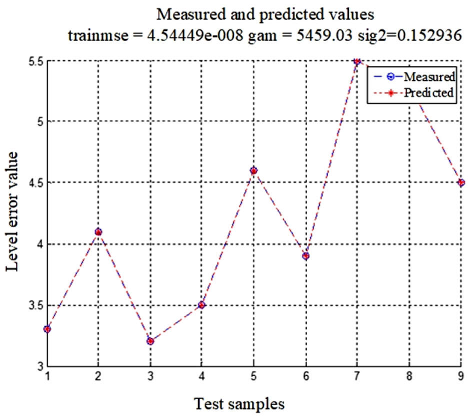

Fig. 12. This training result diagram (60# north)

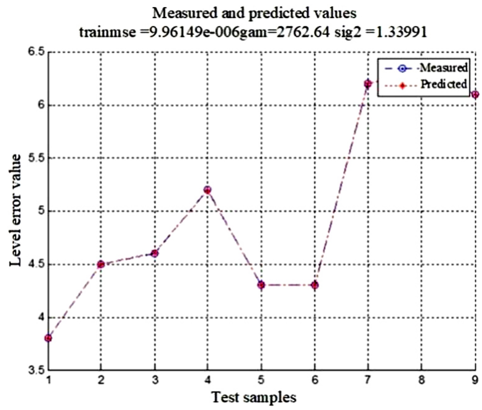

Fig. 13. Training result diagram (61# south)

5. Prediction results analysis

5.1. Predictive model simulation capability analysis

The established elevation error prediction model was used to train and predict the same group of samples for multiple times, and the simulation learning ability and generalization ability of the model were analyzed by comparing the training process and prediction results. The sample data on the north side of the 60# pier are selected to predict the elevation error of the 11# block for many times, and the results obtained are shown in Table 4. Prediction outcome diagram is shown in Fig. 14.

From the data in Table 4 and Fig. 14, LS-SVM has strong learning ability, and the training accuracy and prediction accuracy can be high by selecting appropriate parameters.

LS-SVM has a strong generalization ability, although the mean square error MSE and absolute percentage error APE of each training and prediction are different, but the prediction

Table 3. Parameters and result tables

Pier number	Training samples	γ	σ^2	MSE
60# north	2# block – 7# block	154.457	0.0109348	9.88×10^{-6}
	2# block – 8# block	138.558	0.121351	4.28×10^{-5}
	2# block – 9# block	2719.43	0.368276	6.48×10^{-7}
	2# block – 10# block	5459.03	0.152936	4.54×10^{-8}
61# south	2# block – 7# block	440.068	2.22441	2.62×10^{-4}
	2# block – 8# block	250.792	2.3501	9.51×10^{-4}
	2# block – 9# block	745.272	2.01992	0.62×10^{-4}
	2# block – 10# block	2762.64	1.33991	9.96×10^{-6}

error is relatively stable, if the training accuracy meets the requirements, the prediction accuracy is generally not much different, and the two parameters can adapt to each other to adjust the value size to ensure the generalization ability of the model.

Table 4. Simulation capability analysis table

Numbering	The value of the parameter		Training results	Predict the outcome
	γ	σ^2	MSE	APE
1	1691.940	0.192	6.85×10^{-7}	1.20%
2	5459.030	0.153	4.54×10^{-8}	1.29%
3	1480.080	0.221	1.17×10^{-6}	1.29%
4	62.416	0.216	5.74×10^{-4}	1.32%
5	82.931	0.157	1.95×10^{-4}	1.32%
6	78.500	0.243	4.28×10^{-4}	1.37%
7	27.736	0.164	6.67×10^{-4}	1.41%
8	41.860	0.222	1.28×10^{-5}	1.41%
9	10.498	0.193	7.11×10^{-4}	1.71%
10	6.397	0.208	9.70×10^{-4}	1.85%

The accuracy of the prediction results APE basically improves with the improvement of training accuracy, and the optimal parameters are not a specific set of parameters in the parameter selection, as long as the parameters that meet the set accuracy can be understood as "optimal parameters", in the actual modeling selection, there is no need to one-sidedly pursue the highest set of parameters with the highest training accuracy, so that the model training time can be reduced in practical applications and the work efficiency can be increased.

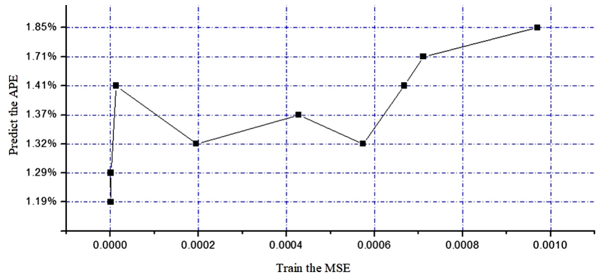

Fig. 14. Prediction outcome diagram

5.2. Forecasting results and analysis

The established LS-SVM continuous beam bridge elevation error prediction model is adopted, and the parameters with relatively high training accuracy are selected to predict the elevation error of the beam section, and the results obtained are shown in Fig. 15–17.

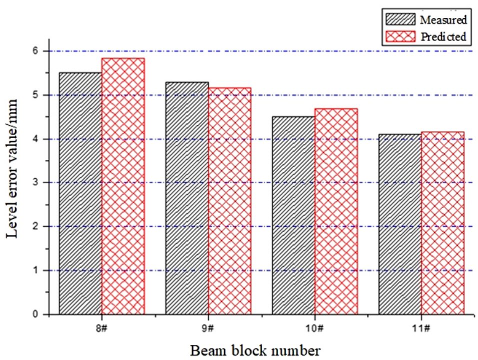

Fig. 15. Comparison chart of the predicted value and the measured value

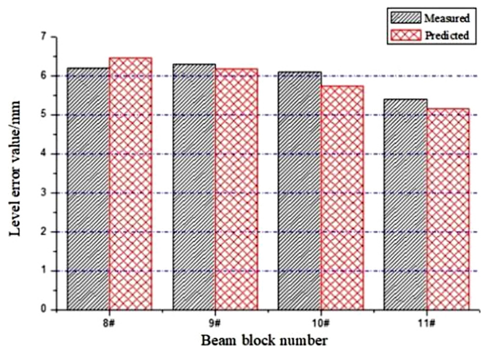

Fig. 16. Comparison chart of the predicted value and the measured value on the south side

It can be seen from Fig. 15 and Fig. 16 that the absolute percentage error APE between the predicted value of the elevation error of each beam block and the measured value is within 10%, which meets the requirements of bridge construction control in terms of accuracy. The prediction mean square error (MSE) was 0.04 and 0.07, and the prediction error was relatively uniform, and there was no extreme or small condition, indicating that the LS-SVM prediction model was relatively stable.

As can be seen from Fig. 17, the absolute percentage error APE of prediction does not vary greatly with the increase of the number of training samples, because different parameters are used by different training sample models to ensure the accuracy of small-sample prediction, which further indicates that LS-SVM has a strong ability to solve small-sample problems.

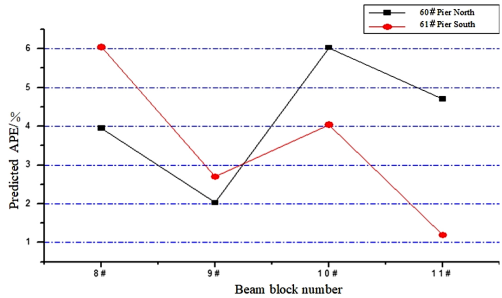

Fig. 17. Variation of error with the increase of beam segments

5.3. Comparative analysis of other predictive models

The SVM model is like LS-SVM, the radial basis kernel function is also used, and the model is established using the LIBSVM toolbox developed by Professor Lin Zhiren of Taiwan, and the parameter selection and error judgment are the same as those of LS-SVM, which will not be repeated here. The data on the north side of the 60# pier was selected as the comparison sample, and the APE, MSE and calculation time predicted by the three models were compared, and the comparison results are shown in Table 5 and Fig. 18– 20.

From the above comparison, the three prediction models can be used in the prediction of the construction control elevation error of continuous beam bridges, and the prediction accuracy is high, which can meet the actual engineering needs.

Table 5. Comparison table between the predicted and measured values of the three models

Beam block number	LS-SVM			SVM			BP neural network		
	APE	MSE	Time	APE	MSE	Time	APE	MSE	Time
8#	6.05%	0.04	12.33	8.47%	0.1	35.16	9.50%	0.12	88.64
9#	2.70%		20.18	5.51%		51.22	3.56%		104.4
10#	4.04%		29.32	4.79%		67.28	7.71%		123.13
11#	1.20%		40.14	5.77%		80.35	6.45%		152.36

The LS-SVM prediction model has the lowest absolute percentage error APE and mean square error MSE, indicating that it had the highest accuracy and the best stability, followed by the SVM prediction model, and the BP neural network prediction model is the worst. The reason for the analysis is that the SVM has a large deviation e in the calculation, which leads to a larger structural risk term than the processed LS-SVM. As can be seen from Fig. 19 and Fig. 20, the error change of BP neural network is not smooth in the process of reaching the model training goal, and the gradient descent method used by BP may fall into the local minimum point during the training process, resulting in large prediction accuracy.

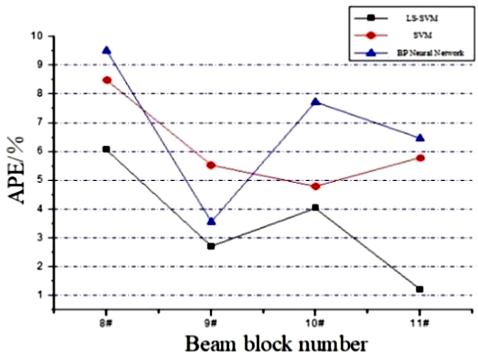

Fig. 18. Comparison chart of the prediction percentage error of the three models

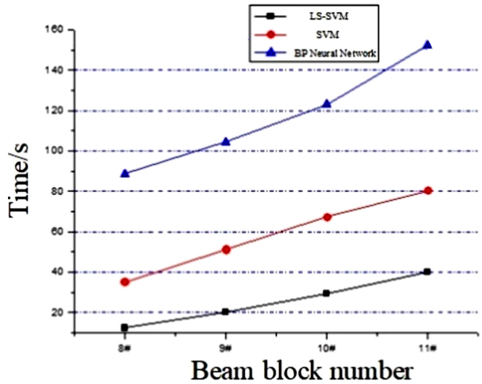

Fig. 19. Comparison of the calculation time of the three models

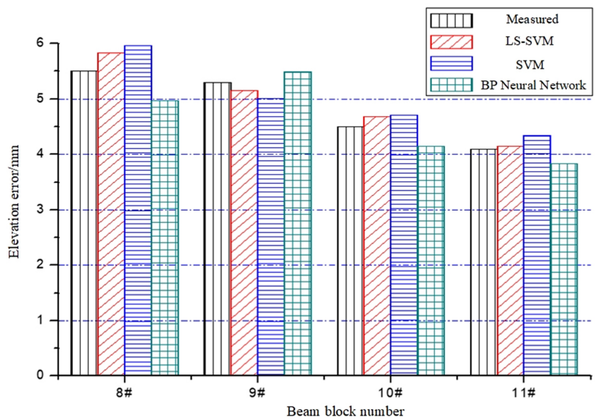

Fig. 20. Comparison of model predicted and measured values

5.4. Dimensionality reduction prediction model results and analysis

In the actual engineering application, it is found that there are inevitable emergencies in the whole construction process, the changes of influencing factors are too large, or the errors caused by the control personnel are too large, so that the elevation error prediction model cannot be completed according to the established steps, and the prediction model needs to be adjusted.

When a change is wrong and needs to be eliminated, the data in this column is deleted from the input sample of the prediction model, that is, the dimensionality reduction process is carried out on the input sample. To study the ability of dimensionality reduction to process the prediction model and explore the relationship between the output and the input samples, the samples on the north side of the 60# pier are selected to reduce one parameter in turn, and the calculation results are shown in Fig. 21.

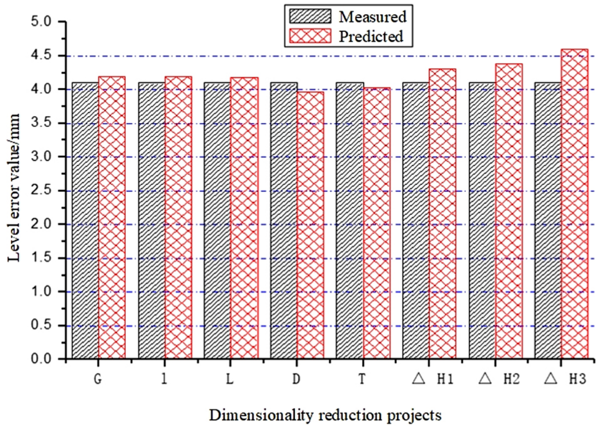

Fig. 21. Dimensionality reduction prediction result graph

As can be seen from Fig. 21, reducing the input sample dimension of the prediction model will lead to a decrease in the prediction accuracy of the prediction model. Each input sample has a different impact on the output results, and the last three have a significant effect. When the pouring time (D) of the beam segment (D) is not considered and the influence of temperature (T) is not considered, the prediction result will be smaller, and the missing data of the rest of the items will lead to the prediction result being larger. Among the many influencing factors, the elevation error value (ΔH_3) of the previous beam block has the greatest impact on the prediction results, which needs to be accurately measured and taken seriously in practical engineering applications.

5.5. The result of the elevation error control in the cantilever phase

The predictive control method based on LS-SVM is used to control the elevation of the cantilever construction stage, and the elevation error of each beam block is gradually adjusted until it is close to or consistent with the target elevation in the error adjustment, and the elevation error of each "T" structure cantilever end of the whole bridge is shown in Fig. 22.

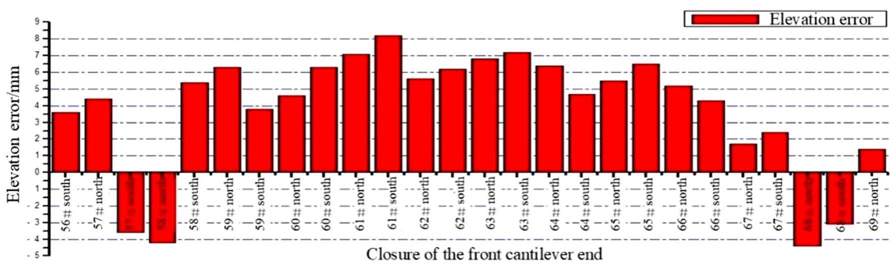

Fig. 22. Elevation error diagram of each cantilever end before closing

It can be seen from Fig. 22 that the maximum elevation error of the cantilever end of each "T" structure is 8.2mm, and the height difference between the two sides of the closure section is 6.8 mm, which realizes the goal of ±20 mm of the elevation error of the construction control closure dragon, which shows that the elevation error prediction based on LS-SVM is feasible to control the elevation of the bridge, and successfully solves the prediction control problem caused by the lack of beam blocks in the cantilever stage of the continuous beam bridge.

6. Conclusions

In this paper, the basic principle and prediction principle of LS-SVM are introduced, and the method and steps of the elevation error prediction model of continuous beam bridge are determined according to the calculation principle of LS- By comparing the predicted values of LS-SVM, SVM model, BP neural network model and LS-SVM dimensionality reduction model, it is found that the elevation error prediction model based on LS-SVM has high prediction accuracy, and has strong learning ability, stability and efficiency. It is verified by the elevation control results in the cantilever stage that the elevation error prediction based on LS-SVM is feasible for the elevation control of the bridge, and successfully solves the prediction control problem caused by the lack of beam blocks in the cantilever stage of the continuous beam bridge.

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