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Research paper

Application of topology optimisation in the strut-and-tie method: a review

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Abstract: The paper delivers an overview of the literature concerning the adaption of Topology Optimisation (TO) to the Strut-and-Tie Method (STM). In the beginning, the foundations and basics of STM are briefly summarised. STM is a practical implementation of the lower bound theory of plasticity for reinforced concrete (RC). It is generally used to design so-called D-regions (i.e. Discontinuity caused by irregular geometry or concentrated load) working under the complex stress state. These regions are modelled with the equivalent truss consisting of struts (representing the flow of compressive forces carried by concrete), ties (representing rebar) and nodes. The STM algorithm's most demanding part is determining the layout of the truss, which correctly reflects force flow in a specific D-region. During this stage, TO methods can eliminate the designer's arbitrary decisions. Analysed literature sources are divided into two groups differing in the adopted TO algorithms: the former uses layout optimisation procedures for trusses, whereas the latter uses TO methods for continuum domains. In the first approach, the equivalent truss is obtained explicitly as an outcome of the TO phase. In the second approach, the material continuum material layout is an inspiration for the ST model or is post-processed with image analysis methods and possibly shape optimisation methods to obtain bending-free bar structures. The advantages and limitations of both approaches are put forward in the conclusion section. Further development in this field is very likely, so future prospects are also anticipated.

Keywords: strut-and-tie method, reinforced concrete, topology optimisation, reinforcement layout optimisation

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1. Introduction

Reinforced concrete (RC) can be classified as a composite material consisting mainly of cement matrix, aggregates and rebar (most often made of steel). Its behaviour under mechanical load is highly non-linear, essentially due to complicated interactions between the abovementioned components [1] and the cracking phenomenon, which needs enhanced mechanical theories to be correctly captured, e.g. a non-local approach [2]. Problems with the mechanical characterisation of RC arise in regions under the complex stress state, for instance, support zones or sudden geometry changes. Therefore, methods of substituting such parts of RC with a simplified truss model were introduced at the end of the nineteenth century [3] and significantly developed in the preceding years at the beginning of the twentieth century [4, 5].

The theoretical foundation of such an approach constitutes the lower-bound theory of plasticity, which can be briefly summarised as: "*If an equilibrium distribution of stress can be found which balances applied loads, and is everywhere below yield or at yield, the structure will not collapse or will be just at the point of collapse*" [6]. It is worth mentioning that strain compatibility conditions do not need to be fulfilled, so many stress distributions can be found for the same load and support arrangements. The lower-bound theorem is valid for perfectly plastic materials and the small deformation range. When the latter assumption can be applied to RC, the former usually is not true since there is a limited possibility of developing plastic strains for concrete [7]. On the contrary, the modern concrete of higher classes behaves in a rather brittle manner in tension as well as in compression [8]. Therefore, the additional conditions, formulated mainly based on experimental outcomes, must be fulfilled to ensure the safe design [9, 10]. The practical application of lower-bound theory for RC was established with the name Strut- and Tie Method (STM).

The most demanding part of the STM algorithm is finding the shape of an equivalent truss, which accurately reflects the force flow in the structure's analysed region. Initially, the force path method [9] was proposed, which, however, requires some engineering sense and is easily adapted to areas free from highly non-linear strain distributions, in particular, many zones of stress concentration. Moreover, reinforcement layouts designed using models created in such a manner are often not the most efficient in terms of steel usage. Therefore, Topology Optimisation (TO) methods applied in STM have recently become increasingly popular. They eliminate most arbitrary human decisions from the design process and, consequently, make it more automatised and reliable. It is worth mentioning that they seem to be the shortest way of bridging the gap between TO and structural designing of RC structures [11].

The present paper aims to summarise and assess recent developments in the field of application of TO in STM. The author believes that further extensions of TO-based STM are very likely in the near future, so possible enhancement paths are also anticipated. The paper is structured as follows: the second section presents the basics of STM and its terminology. The third section summarises applications of truss layout optimisation algorithms to STM and the fourth presents examples of STM based on TO methods for the continuum. The last section contains the main conclusions drawn from the literature survey and possible future prospects.

2. Basics of strut and tie method

It is believed that the first consistent and comprehensive rules for STM were given by Schlaich et al. [9]. Later, STM was implemented in many technical codes and guidelines, for instance, Eurocode 2 [12], ModelCode 2010 [13], AASHTO LFRD [14] or ACI 318-19 [15].

The first step of STM is to perform global static analysis with an assumed static scheme and to divide a structure into Bernoulli and Discontinuity regions (B- and D-regions), see Fig. 1. It is assumed that within B-regions, the Bernoulli hypothesis of linear strain distribution through the section height is valid, and consequently, the stress state can be considered as a uni-axial one. On the contrary, in D-regions, the strain profile through the height is non-linear, and the state stress is complex. Obviously, the distinction between B- and D-regions is not clearly specified, and the entire structural elements can also be classified as D-regions, e.g., deep beams [10, 16]. After finding D-regions, load sets acting on the D-regions are determined based on results from the global analysis.

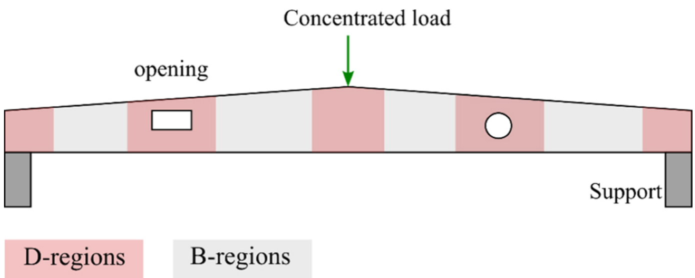


Fig. 1. Illustrative division of a roof girder with openings into B- and D-regions

The next step is to find the equivalent truss that correctly reflects the force flow in the D-region. The equivalent truss consists of members subjected to compression (struts), tension (ties) and nodes. Struts mimic the flow of compressive stress in concrete, whereas ties represent tensile forces carried out by rebar. As mentioned above, this stage is the most demanding and can be supported by TO methods to avoid the subjective decisions of the designer. It should be noted, though, that the experienced designers are able to propose a reasonable scheme for the truss based on their professional experience. Examples of Strut and Tie models are shown in Fig. 2.

Then, stresses in members are checked. In the case of struts, the stresses cannot exceed $\sigma_{\mathrm{Rd,max}}$, which, according to Eurocode 2 [12], can be calculated as follows:

(2.1) $\sigma_{\mathrm{Rd,max}} = f_{\mathrm{cd}}$ for struts not subjected to transversal tension,

(2.2) $\sigma_{\mathrm{Rd,max}} = 0.6\nu' f_{\mathrm{cd}}$ for cracked areas with transversal tensile stresses,

where: f_{cd} – the design compressive strength, $\nu' = 1 - \frac{f_{\mathrm{ck}}}{250}$, f_{ck} – the characteristic compressive strength. It should be noted that other non-prismatic types of struts can be found in the literature [14], namely fan- and bottle-shaped ones. The stresses in reinforcement should be below the design yield strength.

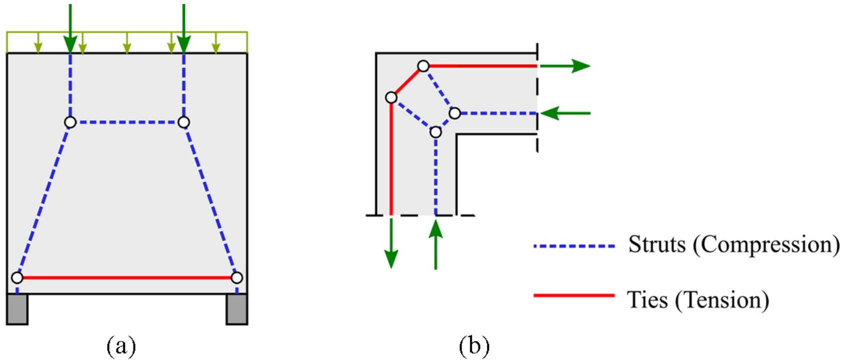


Fig. 2. Examples of ST models; (a) for a deep beam (based on [9]), (b) for a frame corner subjected to the opening moment (based on [12])

The STM algorithm's last step is verifying nodes' resistance. In general, there are three types of planar nodes distinguished in Eurocode 2 [12] differing forces signs configuration (C means a compressive force and T a tensile one): C-C-C node with three compressive forces, C-C-T with two compressive forces, and T-T-C with two tensile forces and one compressive, cf. Fig. 3. During the verification of nodes, the condition of allowable stresses in concrete should be checked, and the proper anchorage length for ties should be provided. A more comprehensive review of the load-bearing capacity of struts and nodes according to various design codes can be found in [17].

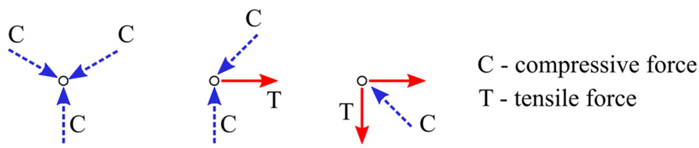


Fig. 3. Basic types of nodes according to Eurocode 2 [12]

3. Methods based on truss optimisation

One of the first programs allowing the automatic finding of STM using TO methods of trusses was presented in a paper by Ali and White [18]. The authors used the ground-structure approach and put much effort into explaining that trusses generated with optimisation codes not adjusted to STM do not always ensure a safe design due to the limited possibility of plastic stress redistribution for concrete. Using a simple example of a short rectangular cantilever plate, they showed that the optimisation criteria should be a compromise between elastic and plastic approaches. In Fig. 4, three trusses used to design reinforcement layouts are presented. Results of Non-Linear Finite Element Analysis (NLFEA) showed that plates designed according to schemes a) and b) fail prematurely due to severe cracking since their layout is too far from the directions from the trajectories of principal stresses obtained for a solution with assumptions of isotropic, elastic material. Only the c) model sustained the assumed load level. In general, three approaches were analysed in this paper: 1) EL – based on elastic criterion, which uses the relative error of complementary potential energy (calculated in advance with FEM), 2) MRV – Minimum Reinforcement Volume, 3) COMP – composite criterion mixing the two abovementioned ones.

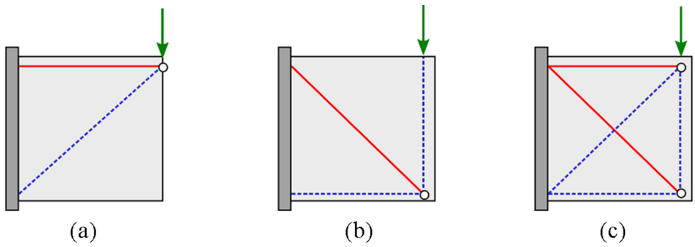


Fig. 4. Introductory example of different ST models from [18], description in the text

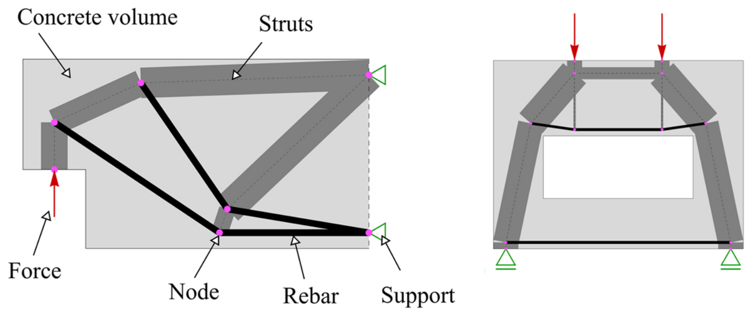
Moreover, the paper [18] pays attention to the ease of reinforcement layout design. The authors assumed that the orthogonal rebar nets are desirable, so they propose the introduction of the practicality factor β. Lengths of vertical and horizontal rebars can be multiplied by β with assumed value, decreasing the cost of such rebars in the objective function. Obviously, such treatment increases the obtained weight of reinforcement. Different objective functions and possibilities of ST modelling automatisation were also analysed in Kostic's thesis [19].

Another application of truss layout optimisation to STM was published by Biondini et al. [20]. Their algorithm is also based on layout truss optimisation with a ground-structure approach. In this code, the standard minimum volume criterion was modified to include a deviation between the directions of the bars and the principal stresses in the optimisation procedure. The principal directions can be obtained with a linear or nonlinear analysis for the continuum model of the analysed D-region. The authors put forward only one example of a rectangular deep beam and presented ST models for linear and non-linear stages. Particularly interesting are the results for NLFEA with the increasing load levels.

The adaptation of the truss optimisation algorithm written by Sokół [21] in Mathematica software to STM was made by Bołbotowski and partially published in papers [22, 23]. Firstly, the algorithm of ground structure generation was generalised to an area in the shape of any polygon, including openings. Then, the layout optimisation procedure was modified to take into account issues typical for STM. The optimisation task was formulated in the following manner:

$$V = \min_{\mathbf{T},\mathbf{C}} \left\{ \frac{\mathbf{L}^{\mathrm{T}}}{\sigma_T} \;\middle|\; \mathbf{T},\ \mathbf{C} \text{ such that } \mathbf{B}^{\mathrm{T}}(\mathbf{T}-\mathbf{C}) = \mathbf{P},\ T_i \geq 0,\ C_i \geq 0,\ i = 1, \ldots, N \right\}, \tag{3.1}$$

where: V – the volume of rebar, σ_T – the allowable stress for ties (design yield strength of reinforcement), $\mathbf{T}$, $\mathbf{C}$ – vectors of tensile and compressive forces (respectively), $\mathbf{L}$ – the vector of ties' length, $\mathbf{B}$ – the truss geometrical matrix, cf. [24], N – the number of members. Consequently, the reinforcement volume was minimized. Such a condition is consistent with the engineering sense since the dimensions of the D regions are usually fixed, and structural designers aim to use as little steel as possible. The algorithm had also the following features: the filtering of the layout optimisation outcomes to remove members close to each other, the ensuring proper value of rebar concrete cover, and the enforcing positioning of sturts within the design domain done iteratively. The practicality of ST layout was controlled with a parameter p chosen by the user added to the lengths of ties, i.e. $\hat{L}_i = L_i + p$. The higher the value of this parameter, the simpler truss layout is obtained since every tie has the increased cost. The illustrative layouts obtained with this code for support zone of a simply supported beam with an undercut and a plate with opening are shown in Fig. 5.



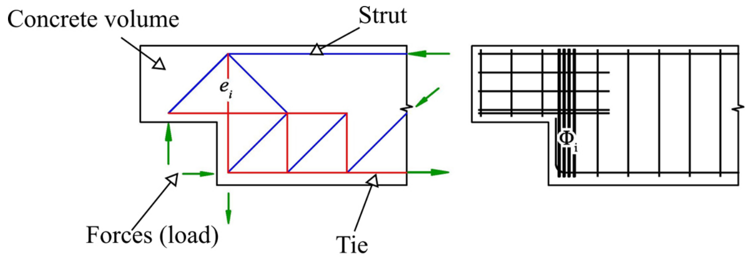
(a) (b)

Fig. 5. ST layouts obtained with Bołbotowski's code; (a) support zone of a simply supported beam with an undercut, (b) a plate with an opening, source [25]

Mozaffari et al. [26] proposed the combination of truss layout optimisation and graphic statics. The layout optimisation procedures were used to generate an initial shape of the ST model. In this formulation, the design variables were truss forces and cross-sectional areas. An additional constraint was to eliminate non-orthogonal ties since the authors believe that such reinforcement is less practical. The graphic static methods are used to modify outcome trusses from layout optimisation in order to eliminate compression fields overlapping or to add new members representing additional stirrups, see Fig. 6. Moreover, this integrated approach enables users to analyse statically indeterminate trusses (ordinary layout optimisation procedures return statically determinate ones). The thesis [27] shows that this approach is efficient in the case of spatial ST models as well.

The paper by Zhao et al. [28] concerns the optimisation procedure of planar ST models for short beams under three-point bending prone to shear failure and assessing the efficiency of equivalent trusses generated using different constraints. As an objective function, the negative potential energy of the system in an equilibrium state is assumed, and the design variables are cross-sectional areas of members. In the proposed algorithm, constraints can be related to the position and length of struts or ties. To evaluate the performance of reinforcement layouts obtained with different ST models, the Michell number Z (sometimes called the load path, see Baker et al. [29]) is used, which is defined as follows:

$$Z = \sum_{k=1}^{N} |F_k| L_k, \tag{3.2}$$



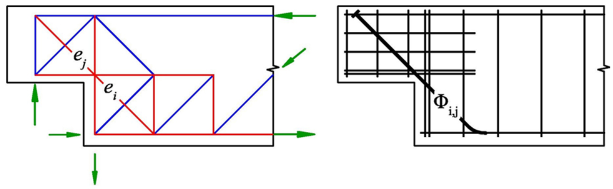


Fig. 6. ST models and corresponding reinforcement layouts obtained with Mozaffari's code for the support zone of a simply supported beam with an undercut; (a) reference solution, (b) modified solution with an additional diagonal bar, source [26]

where: N – the number of members, F_k, L_k – the axial forces and lengths for member k. The experimental campaign was performed for reinforcement layouts designed according to different ST models. It proved that the lower Z for a ST model, the higher load capacity and stiffness of a designed beam is.

4. Methods based on continuum optimisation

Many papers concerning the application of TO algorithms for continuum to STM can be found in the recent literature. Bruggi [30, 31] proposed the framework for finding ST equivalent trusses using a commercial FE code Straus7 conjugated with an application programming interface. He used the Solid Isotropic Material with a Penalty (SIMP) optimisation scheme to find the material layouts, which were treated as an inspiration for the ST models. In this approach, the "density" distribution is a design variable, and the structure's compliance is minimised with constraints on the material volume. The special treatment has to be used to eliminate mesh dependency typical for this approach, i.e. an appropriate filtering technique. Bruggi [30, 31] formulated guidelines on choosing parameters for this whole method to obtain reasonable ST shapes. He analysed various examples, including planar (beams, plates with openings, corbels) and spatial ones (cantilevers with box cross-section subjected to torsion). In paper [31], he paid particular attention to the multi-load cases, for instance, interior beam-column connection, which can be subjected to significant horizontal forces during seismic actions. In paper [32], Bruggi extends his procedures to the hyperelastic no-tension material prescribed for concrete. This enables him to find optimal load paths in the plain materials, which can be useful for dimensioning masonry structures. Assuming reinforcement layouts, he found arrangements of compressive struts for the following RC elements: a cantilever beam, a short, simply-supported beam under three-point bending, a shear wall and a square deep beam loaded with the concentrated force.

He and Liu [33] used TO methods to find the ST model layout for anchorage diaphragms in an externally prestressed bridge. They showed the complete design procedure: from finding the shape of the truss using TO methods for continuum, through solving 3D truss inspired by these results and designing reinforcement according to code ACI 318 [14], up to the verification of design safety with experimental outcomes.

Many important contributions in the field discussed within this paper can be found in papers by Xia, Langelaar and Hendriks [16, 34, 35, 37]. In the paper [34], the authors presented a comprehensive approach for assessing different ST models, particularly the results of TO. This issue is of great importance since different objective functions and constraints are used in the literature, which results in sets of other solutions for the same D-regions. They used image analysis algorithms to determine "truss-like structures", or more precisely, frames based on TO results. Then, they calculated three indices: suitable truss structure index (STS), tensile region similarity index (TRS) and steel reinforcement ratio (SR). STS measures the bending state's contribution to the bar structure's equilibrium (for STM, bar structures free from bending are needed). TRS was calculated by comparing the stress state in ties and the continuum FE model of analysed regions. Local averaging techniques of stresses in the areas subjected predominantly to tension were used. SR indicate the steel usage for the specific D-regions. There are two of the most important findings from this paper. Firstly, the modification of the TO phase, for instance, by the enhancement of material models or introducing the non-standard objective function, does not lead to a better ST model. Secondly, ST models created based on TO results generally enable us to reduce steel usage.

In the paper [35], the authors described their strategy for TO-based STM. It consists of three stages: TO phase, topology extraction phase, and shape optimisation phase, cf. Fig. 7. During the first one, the SIMP scheme is used without any substantial modifications (compliance minimisation with a volume constraint). In the second one, frame structures are generated using image analysis methods. Finally, the bending state is eliminated using shape optimisation methods, and geometric requirements (e.g., proper concrete cover) are enforced. Moreover, two illustrative D-regions are analysed (dapped-end beam and irregular deep beam). NLFEA of these two structures with reinforcement layouts designed according to ST models found with TO and in the literature is performed to assess the efficiency and reliability of the proposed approach.

Paper [36] presents a generalisation of these procedures to 3D cases. The applicability of the described methods is proved with examples of a corbel, cf. Fig. 8, a cap, and a box girder. As previously mentioned, the performance of structures designed according to optimised ST models is assessed using NLFEA.

The last paper from this article series [37] concerns adapting the proposed methodology to multi-load cases. The authors confronted three approaches to this issue: 1) based on manual construction of ST model, 2) separate ST model for each load case, 3) one multi-load TO optimisation within the SIMP phase, 4) decomposing load combinations under consideration to set basis vectors. The main conclusions from this study are as follows: approach 2) leads

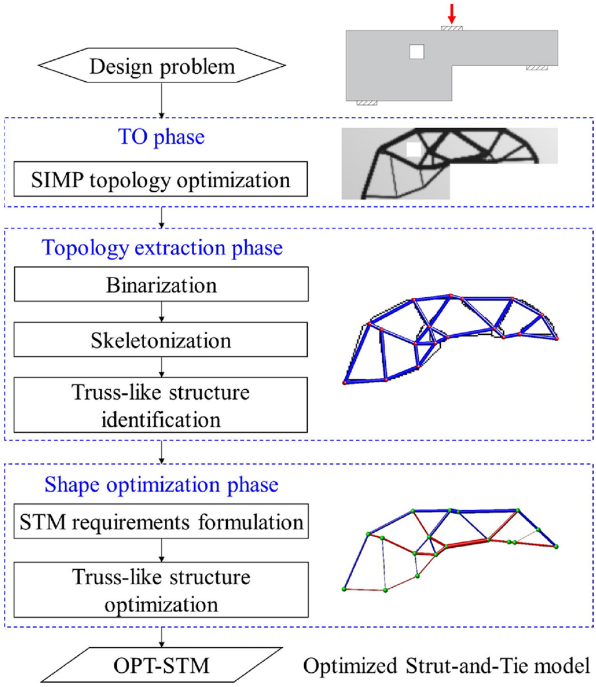


Fig. 7. Flowchart of generating truss-like structure based on TO, source: [35]

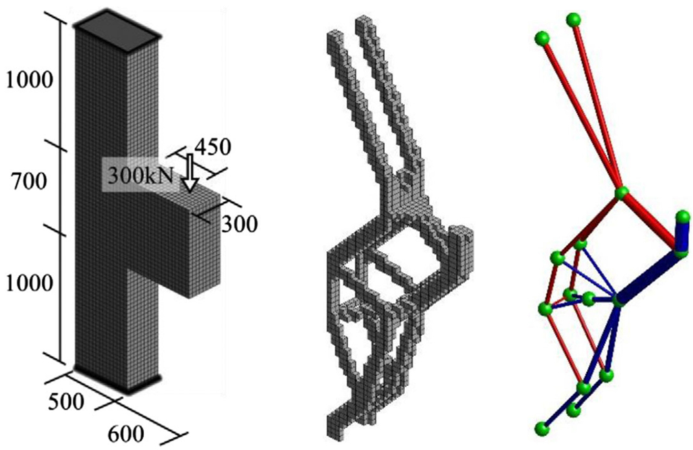


Fig. 8. 3D ST model for corbel generated with TO methods, source [36]

to uneconomical design; approach 3) often fails in finding bending-free frames; approach 4) leads to economical and safe designs. However, in the case of load combinations, the layout of reinforcement designed according to optimised ST models is complex and, consequently, seems impractical for execution.

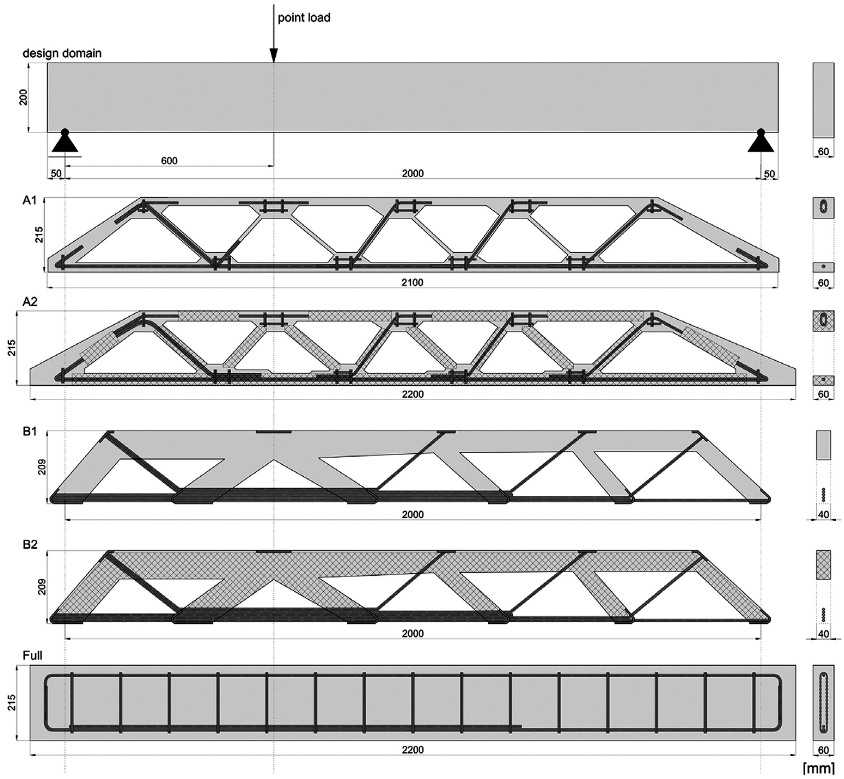


Fig. 9. Concrete girders designed with TO-based STM by two research groups, source: [11]

It is worth mentioning that these authors took part in a so-called blind competition described in [11], in which the task was to optimise concrete girder for the assumed value of the concentrated force. The paper [11] presents the results of experimental outcomes for two optimised RC girders loaded. Their geometrical features are shown in Fig. 9. Only one solution sustained the assumed load level. In the opinion of the present paper's author, the other failed prematurely due to problems with the proper anchorage of ties. This shows that fulfilling design codes' provisions is crucial for ensuring structural safety.

Qiao and Chen [38] used another optimisation scheme to obtain material distribution constituting the basis for ST models, namely Moving Morphable Components (MMC). They adapted code published by Zhang et al. [39]. In the beginning, the authors explained that the classic compliance minimisation subjected to volume constraints is sufficient for STM problems. As illustrative examples, they used deep beams loaded with one and two concentrated forces, a corbel and a double-corbel. They showed that the results of TO are independent of the initial material layout, which is intrinsic in this method. They compared their results with the ones obtained with the SIMP scheme. Moreover, they automatised the truss extraction phase in a way similar to that of Xia et al. [34]. Finally, they concluded that their approach can be more computationally efficient in problems of larger scale (e.g. 3D ones) due to the reduction of the number of variables and that it is applicable for multi-load cases.

Cedrim et al. [40] apply the Tosca Optimisation module for the popular FE code Abaqus to find ST models for two examples of short corbels (rectangular and trapezoid ones). This module is based on the SIMP optimisation scheme. Consequently, the compliance of the structure is minimised with the volume constraint. The authors give many practical tips on how to use this commercial software in STM concerning, among others, volume fraction or mesh size. They extracted the position of nodes and truss members, and the distribution of axial forces in the truss was found using code written in MATLAB.

Zhou and Wan [41] showed that the ordinary optimisation algorithms based on compliance minimisation are unable to find the bottle-shaped strut for a plate loaded with two equal concentrated forces, which is usually recommended by construction codes. However, this example is quite misleading since the design domain was not divided into B- and D- regions and the introduction of a bottle-shaped strut can be done during the dimensioning phase in the case of exceeding $\sigma_{Rd,max}$ for other types of struts. Then, the authors proposed the modified optimal criterion with a split of strain energy into a part associated with deformation parallel to the load direction and perpendicular to it. The optimisation procedure used the Evolutionary Structural Optimisation method (ESO) based on iterative processes with progressive elimination of finite elements with the lowest strain energies.

The Performance Index (PI) was used to assess the efficiency of the optimisation process:

$$PI = \frac{U_0 W_0}{U_i W_i} \tag{4.1}$$

where: U_0, W_0 –initial strain energy and weight of the design domain, U_i, W_i – strain energy and weight of the structure at iteration i. Using such the procedure, they obtained bottle-shaped strut for the introductory example. Then, they found ST models for a dapped beam with an opening and a dapped end of a beam. During the extraction truss models phase, they used two criteria: minimum total strain energy for ties and minimum compliance for struts.

Fang et al. [42] applied code written in MATLAB using the Bi-directional Evolutionary Structural Optimisation (BESO) scheme to determine material layout for ST problems. This approach uses the sensitivity number to check which elements should be removed for the current iteration. In the BESO optimisation scheme, special filtering techniques are also needed to obtain mesh-independent results. The truss extraction phase was algorithmised with image processing methods, and nodal zones were found by applying the proper condition for the stress state. The bar structure is initially analysed using frame elements, and shape optimisation is performed to eliminate shear forces (using the STS index from the paper [34]). The authors proposed the application of additional constraints to restrain kinematically variant trusses often obtained as an outcome of the optimisation process. They analysed the following examples: a deep beam and a deep beam with openings (the whole structures as a D-region) and vicinities of a small opening, a corner of a large opening and a lintel above a large-span opening (local D-regions). For the large deep beam with openings and reinforcement designed according to the proposed ST model, they performed NLFEA in the Abaqus code. They analysed two anchorage lengths of bars near the openings and concluded that this parameter is crucial for the structure's ductility.

5. Conclusions

Algorithms of automated generation of ST models using TO methods have been developed since the late 1990s, but over the last five years, this topic has gained much more attention in journals dedicated to mechanics as well as structural designers. On the other hand, there are still many challenges concerning the TO-based STM, which are clearly visible in the paper [11].

On the basis of the literature review presented in this paper, the following conclusions can be drawn:

– Two approaches are commonly used to find the topology of the equivalent truss for STM: based on layout truss optimisation and topology optimisation for the continuum domain. The latter needs extensive post-processing to obtain a bending-free bar structure representing the ST model. The post-processing most often consists of two stages: the bar structure's extraction using image analysis and the elimination of bending moments, for instance, using shape optimisation methods.
– In a ground-structure layout optimisation approach, including different practical demands, such as favouring orthogonal reinforcement nets [17] or specific regions for rebar [27], is easier. Moreover, various modifications of the objective function often lead to more efficient or safe designs.
– On the contrary, enhancing the objective functions or including more refined material models in the continuum approach does not seem to improve ST models' accuracy, which is shown in the in-depth review and evaluation paper [34]. Ordinary compliance-minimisation with volume constraint methods, e.g. SIMP, results in reasonable initial ST layouts. However, they need to be improved within the post-processing phase.
– ST models obtained with TO methods before the practical application should be verified against experimental tests (if possible) outcomes [28] or at least NLFEA [18, 35] due to the limited possibility of plastic redistribution in RC. Thanks to developments of different standards for NLFEA [43], this tool can be assumed to be a reliable way of verifying other calculation methods, e.g., STM.
– Taking into account the findings of the present review, the most complete methodology for STM using TO for continuum was presented in the series of papers [34–36]. The authors used firmly established in the literature, not mesh-dependent SIMP optimisation scheme and proved that their algorithm provides significant steel reduction and safe designs due to the verification with NLFEA. Their strategy seems to be exhaustive in terms of optimisation and RC design aspects, which was shown during the blind competition reported in the paper [11]. Surprisingly, it is hard to find in the literature such a comprehensive methodology for the approach based on the layout-optimisation of trusses. Usually, the papers in this trend refer to single or two planar cases and do not contain parallel comparisons with other authors, reinforcement layout designs and their verification with experimental tests or NLFEA. This indicates a knowledge gap worth filling. From all the analysed literature, the most advanced analyses based on layout optimisation can be found in the thesis [26].

Future prospects for ST methodology anticipated after the literature review are as follows:

- The application of STM supported by TO methods to the usual design practice seems to be closer than ever due to the increased availability of TO codes. A good example of such tendencies is paper [40], in which the add-on optimiser Tosca for the Abaqus code was used to find an initial layout of the ST model. In the case of an approach based on Layout Optimisation for Trusses, open source tools (e.g. LayOpt web app [44]) or more advanced commercial codes (e.g. Peregrine plugin for Grasshopper [45]) can be used. It is worth mentioning that using general-purpose codes not specifically dedicated to STM is possible since modification of objective function does not strongly influence the layout of the ST model, as the literature review results indicate.
- Until now, much more attention has been paid to planar problems due to limitations of computing power. However, this power constantly increases. Consequently, further progress in 3D ST models can be expected. Non-conventional optimisation approaches can be useful in this task, e.g., the application of graphic statics [24] or the force density method [46]. It is worth mentioning that many typical regions of RC structures under a triaxial stress state, for instance, column-slab connection (punching shear [47] or members subjected to torsion [48], cannot be reduced to planar tasks.
- In the actual design process, structural engineers always deal with load combinations. Therefore, further development for multi-load case problems is very likely [49]. Moreover, the existing research papers [37] indicate the possibility of a significant reduction in steel usage concerning the method of dimensioning reinforcement separately for each load case.
- The issue of the comprehensive assessment of various ST models‘ performance seems still open since recent papers propose new measures [28, 34]. Further blind competitions, as described in [11], would be helpful in choosing the most suitable ST model assessing measures. Another important aspect of a similar issue not studied in-depth in the literature was addressed in the paper [17]. The author highlighted that ST models obtained with TO methods usually are systems with poor reliability structure, so additional verification of the reliability index of structures designed with STM should be provided [50, 51].
- Building codes [12] allow structural designers to use ST models to verify serviceability limit states if their layouts reflect force flow in the D-region in an elastic regime. In many discussed codes, this demand is explicitly considered. However, the validation of the obtained ST model in all these papers is limited only to the load capacity. In the author’s opinion, the validation of crack widths predicted by ST models would be interesting since issues related to durability structures are constantly gaining attention [52].

Finally, it is worth emphasising that many papers on the optimal distribution of reinforcement within the RC girders can be found in the literature, for instance, [53–56]. In such works, the difference between concrete and steel behaviour is taken into account in a more refined way, e.g., tension-compression asymmetry, which is typical for concrete. However, this paper is focused on combining TO and STM. This path seems to be the shortest one to bridge these two issues [11] since STM is easy to apply during the design phase and firmly established in design codes.

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Zastosowanie optymalizacji topologicznej w metodzie strut-and-tie – przegląd literatury

Słowa kluczowe: metoda strut and tie, żelbet, optymalizacja topologiczna, optymalizacja układu zbrojenia

Streszczenie:

W artykule przedstawiono przegląd literaturowy dotyczący zastosowania optymalizacji topologicznej (TO) do metody Strut-and-Tie (STM). Na początku krótko przedstawiono podstawy teoretyczne i algorytm tej metody. STM jest inżynierską implementacją twierdzenia o dolnym oszacowaniu nośności plastycznej do projektowania konstrukcji żelbetowych. Warto podkreślić, że twierdzenie to zostało sformułowane dla materiałów sprężysto-idealnie plastycznych, a beton wykazuje ograniczoną zdolność do plastycznej redystrybucji naprężeń, więc właściwe oszacowanie nośności konstrukcji wymaga spełnienia dodatkowych warunków sformułowanych głównie w oparciu o wyniki badań eksperymentalnych. W ogólności STM jest stosowana do projektowania tzw. obszarów D (ang. *Discontinuity* – nieciągłość spowodowana zmianą geometrii lub obciążeniem skupionym) pracujących w złożonym stanie naprężenia. Obszary te modelowane są jako zastępcza kratownica składająca się z: zastrzałów (ang. *Struts*, odwzorowujących przepływ siłściskających przenoszonych przez beton), cięgien (ang. *Ties,* reprezentujących zbrojenie) i węzłów. Największym wyzwaniem dla użytkownika STM jest dobór schematu kratownicy, który najlepiej odzwierciedla przepływ sił w konkretnym obszarze D. Podczas tego etapu metody TO mogą wyeliminować arbitralną decyzję projektanta. Analizowane źródła literaturowe zostały podzielone na dwie grupy różniące się stosowanym podejściem do optymalizacji topologicznej. W pierwszej stosowano procedury optymalizacji dyskretnego układu kratowego, a w drugim metody optymalizacji opracowane dla continuum. W pierwszym podejściu kratownica zastępcza otrzymywana jest bezpośrednio w procesie optymalizacji topologii kratownicy metodą siatki bazowej. Natomiast w drugim wynik optymalizacji dla obszaru ciągłego stanowi inspirację do doboru kształtu kratownicy lub jest poddawany post-processingowi metodami obróbki obrazu i ewentualnie optymalizacji kształtu w celu uzyskania prętowych układów wolnych od zginania, Zalety i ograniczenia obu podejść podsumowane są we wnioskach. Podjęto również próbę przewidzenia kierunków dalszego rozwoju tego podejścia do STM.

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