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Research paper

Effect of intermediate principal stress on the stability of slopes

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Abstract: The intermediate principal stress (σ_2) effect is a mechanical property inherent in many rock and soil materials. However, the effect of σ_2 is often ignored in slope stability analyses, and its impact on slope stability is seldom investigated. The primary purpose of this study is to thoroughly investigate the impact of the σ_2 effect on slope stability via numerical method. A detailed numerical analysis using the Finite Difference Code Fast Lagrangian Analysis of Continua (FLAC) and Unified Strength Theory (UST) is conducted. The numerical analysis evaluates the values of the Factor of Safety (FOS) for two types of slopes (plane strain and axisymmetric) using the Strength Reduction Method (SRM), and the impact of the σ_2 effect on slope stability is analysed. The study found that the influence of the σ_2 effect on slope stability is not sensitive to the values of friction angle φ, cohesion c, and slope height H, but increases with increasing slope angle β values. Furthermore, the stability of the plane strain slope is more affected by the σ_2 effect than the axisymmetric slope. The impact of the σ_2 effect on the footing capacity and slope stability were compared, and the impact of the σ_2 effect on slope stability is significantly lower than its effect on the bearing capacity of footings. The innovation of this paper is to systematically analyse the effect of σ_2 on slope with different geometric shapes and soil parameters and reveal the inherent characteristics of σ_2 effect on the stability of soil slope.

Keywords: slope stability, intermediate principal stress effect, safety factor, strength theory, soil mechanics, numerical analyses

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1. Introduction

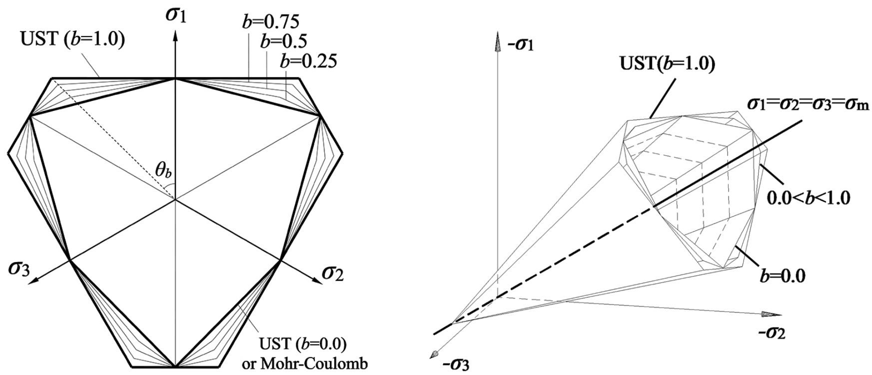
1.1. Intermediate principal stress effect of geomaterials

Numerous complex stress strength tests, including true triaxial and plane strain tests, have definitively demonstrated that the intermediate principal stress (σ_2) significantly impacts the strength and mechanical characteristics of geomaterials. Research conducted by Mogi [1] has confirmed that rocks' mechanical behavior and strength are specifically influenced by σ_2 as tested by the true triaxial tests on dolomite and limestone. Additionally, You [2] has compiled a summary of several types of rocks' mechanical behavior and strength characteristics under complex stress states. Sutherland and Mesdary [3] conducted true triaxial tests on sand and observed that σ_2 significantly impacts sand's strength. The Mohr–Coulomb strength criterion (M–C criterion) commonly used in geotechnical engineering neglects the σ_2 effect. Ignoring the σ_2 effect when analyzing geotechnical problems using the Mohr–Coulomb criterion often leads to an underestimation of geomaterial strength. Lade has provided a comprehensive overview of the strength of sand under complex stress states [4]. A remarkable σ_2 effect will be yielded for geomaterials under the plane strain, axisymmetric, or three-dimensional condition [5]. The σ_2 effect is an inherent property of most geomaterials. Hence, the strength of geomaterials will be overestimated or under-estimated if the influence of the σ_2 effect has not been reasonably assessed.

Strength theory is one of the most critical factors in constitutive relations. Also, it plays a vital role in nonlinear analysis for geomechanics, e.g., slope stability analysis and bearing capacity evaluation. The σ_2 effect is not considered by the failure criterion of Mohr–Coulomb. Several strength criteria with nonlinear yield surface for geomaterials strength estimation take the σ_2 effect into account, i.e., the Zienkiewicz-Pande criterion [6], Lade-Duncan criterion [7] and Matsuoka–Nakai criterion [8]. During the 20th century, multiple theories were developed to clarify how materials behave when subjected to complex stress states. Yu [9] presented an overview of these theories, and subsequently, he proposed a strength criterion called the Unified Strength Theory (UST). This theory utilizes a bilinear yield surface and considers the σ_2 effect in a reasonable manner. Notably, the principal stress equation of the UST can be expressed as follows:

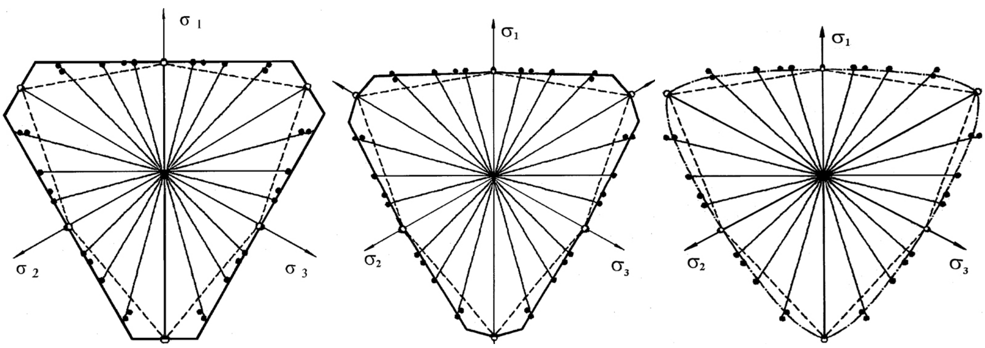
$$
\begin{cases} f = \dfrac{(b\sigma_2 + \sigma_3)(1-\sin\phi)}{(1+b)(1+\sin\phi)} - \sigma_1 + 2c\sqrt{\dfrac{(1-\sin\phi)}{(1+\sin\phi)}}, & \text{if } \sigma_2 \le \dfrac{1+\sin\phi}{2}\sigma_1 + \dfrac{1-\sin\phi}{2}\sigma_3 \\ f' = \dfrac{\sigma_3(1-\sin\phi)}{(1+\sin\phi)} - \dfrac{\sigma_1 + b\sigma_2}{1+b} + 2c\sqrt{\dfrac{(1-\sin\phi)}{(1+\sin\phi)}}, & \text{if } \sigma_2 \ge \dfrac{1+\sin\phi}{2}\sigma_1 + \dfrac{1-\sin\phi}{2}\sigma_3 \end{cases} \tag{1.1}
$$

where b is a parameter that reflects the influence degree of σ_2, c and φ are the cohesion and friction angle of soil respectively. The magnitudes of principal stresses are ranked as $\sigma_1 \ge \sigma_2 \ge \sigma_3$, where the positive direction represents tension and the negative direction represents compression. UST can reflect the influence degree of σ_2 on the strength of materials by adjusting the value of parameter b. The formula of UST will shift to the M–C criterion if the parameter b equals zero, i.e., M–C criterion is a particular case of UST. The limit loci surfaces of UST in principal stress space or projection on the deviatoric plane are shown in Figs. 1a and 1b. θ_b depends only on the friction angle of the material, and can be determined by the intersection of two limit loci surfaces. The limit locus surfaces of UST cover the region between the lower and upper bounds of convex theory, i.e., between the M–C criterion and UST (b = 1.0). Fig. 2 compares the data obtained from true triaxial experiments on Toyoura sand from the limit loci of UST and Matsuoka–Nakai criterion [10]. As shown in Fig. 2, the σ_2 effect and the strength characteristics of sand under the complex stress state can be reasonably predicted by UST via the bilinear limit loci surface. The impact of the σ_2 effect on fracturing and strength of rock besides the excavation boundaries in deep underground was investigated by Cai [11]. The impact of the σ_2 effect on the strip and circular footings' bearing capacity was investigated by Ma et al. [12] using the numerical method, the results show that the σ_2 effect has various impact degrees on plane strain, axisymmetric, and three-dimensional bearing capacity problems. The discrete element method and explicit finite element method were used to simulate and analyze dynamic compaction of gravel soil and seismic failure of soil slope by Ma et al. [13, 14] respectively.



(a) (b)

Fig. 1. Limit surfaces of UST: (a) Deviatoric plane, (b) Principal stress



(a) (b) (c)

Fig. 2. Limit surface of Toyoura sand (Matsuoka and Nakai 1974) compared with different criteria: (a) UST (b = 1), (b) UST (b = 0.75), (c) Matsuoka–Nakai

1.2. Problem presentation

Slope stability has traditionally been assessed using the factor of safety (FOS) coefficient. In recent years, various methods, such as analytical and numerical, have been suggested for determining FOS. The Limit Equilibrium Method (LEM) is a widely used analytical method for slope stability analysis. The slices method for computing the factor of safety (FOS) using a circular slip surface according to the principle of limit equilibrium, as proposed by Bishop, Bishop and Morgenstern, and Spencer [15–17]. The general slices methods with the polyline sliding surface were developed and suggested by Janbu [18], Morgenstern and Price [19], and Sarma [20]. The LEM method for slope stability analysis is reviewed by Fredlund and Krahn [21]. The limit analysis method used in plasticity has been recently applied to slope stability problems. The three-dimensional slope's stability was analyzed by several researchers using the numerical limit analysis method [22, 23]. The upper bound finite element method is used by Yang and Chi [24] to analyze the slope stability with the nonlinear failure criterion.

The numerical methods also can be used to calculate a slope's factor of safety (FOS), e.g., the finite element method (FEM) or finite difference method (FDM) combine with the strength reduction method (SRM). The principle of SRM involves dividing the soil's initial shear strength parameters (cohesion c and friction angle φ) by the strength reduction factor (SRF). This gradual reduction in strength causes the slope to approach instability. The value of SRF at the point of failure represents the slope's FOS. This topic has been discussed by various researchers [25–28]. The gravity increase method is another numerical method for FOS evaluations. The gravity loading method for slope stability analysis involves increasing the gravity loading until the slope fails without changing the soil mass strength parameters. This method, as explained by Swan and Seo [29] in 1999. The numerical method offers several advantages over the slice and limit analysis methods, such as FEM or FDM, which can automatically determine the critical failure surface, making it highly efficient. Additionally, Griffiths and Lane [30] have discussed the differences and connections between LEM and SRM in slope stability analysis. The soil mass's strength prediction in slope stability analyze mostly follows the M–C criterion or its nonlinear criterion [31, 32]. The σ_2 effect also cannot be accurately predicted by the Drucker–Prager (D–P) criteria, and this problem has been emphasized by Zienkiewicz and Pande [6]. Slope stability analysis must consider the effect of σ_2 on geomaterials, as the current research falls short in this regard [33]. In this study, an elastoplastic constitutive model according to UST has been proposed to evaluate the soil slope's FOS by using an FDM code FLAC. FLAC is a two-dimensional explicit Lagrangian computation scheme that can easily simulate the behavior of soil, rock, and concrete that undergo plastic flow when their strength limits are exceeded [34]. In the past, many researchers such as Dawson et al. [26], Cheng et al. [28], and Zhang et al [35]. have used FLAC to analyze slope stability. In this study, the impact of the σ_2 effect on slope stability has been analyzed through various numerical solutions. The purpose of this paper is to reveal the influence regularity and inherent characteristics of σ_2 effect on the stability of soil slope.

2. Theory and method

UST's formula that incorporates the shear strength parameters c and φ is utilized to derive the elastoplastic constitutive relationship, which is an important aspect of slope stability analysis. The potential function g can be expressed as below:

$$\begin{cases} g = \dfrac{(b\sigma_2 + \sigma_3)(1 - \sin\psi)}{(1+b)(1+\sin\psi)} - \sigma_1, & \text{if } \sigma_2 \le \dfrac{1+\sin\phi}{2}\sigma_1 + \dfrac{1-\sin\phi}{2}\sigma_3 \\ g' = \dfrac{\sigma_3(1-\sin\psi)}{(1+\sin\psi)} - \dfrac{\sigma_1 + b\sigma_2}{1+b}, & \text{if } \sigma_2 \ge \dfrac{1+\sin\phi}{2}\sigma_1 + \dfrac{1-\sin\phi}{2}\sigma_3 \end{cases} \tag{2.1}$$

where ψ is the soil dilation angle. In plasticity, the plastic flow rule represented by the plastic potential function is considered non-associated when the value of ψ is less than φ. Conversely, the associated flow rule works when ψ is equal to φ. The FLAC code uses a dynamic-link library (DLL) file written in C++ to load the elastoplastic constitutive model based on UST as the user-defined subroutine [12, 33]. In this study, FOS values for slope were calculated via SRM and applied the same values of the factor SRF to the shear strength parameters. The strength parameters after reduction are c_f and φ_f, respectively, and their formulas are given as follows:

$$c_f = \frac{c}{\text{SRF}}, \quad \phi_f = \arctan\left(\frac{\tan\phi}{\text{SRF}}\right) \tag{2.2}$$

To determine the slope's FOS, it is essential to note that the non-convergence of calculations indicates slope failure. In FLAC, the nodal unbalanced force in the finite difference mesh is used as the criterion for convergence. It is important to note that convergence is achieved when the normalized nodal unbalanced force is lower than 1×10^{-3} in a two-dimensional problem analysis.

3. Slope stability analyses

3.1. Plastic flow rule

The values of FOS of a soil slope with plane-strain and associated or non-associated flow rule was calculated using FLAC code. In this study, the acceleration of gravity g was set to 10.0 m/s^2. The soil's parameters of the slope are as follows: the slope angle (β) is 26.565° (slope ratio is 1:2), the height of the slope (H) is 10 meters, Young's modulus E is 100.0 MPa, Poisson's ratio ν is 0.3, unit weight γ is 20.0 kN/m^3, cohesion $c/\gamma H$ is 0.05, and friction angle φ is 20.0°. Fig. 3 displays the boundary conditions and meshes utilized in the slope stability analysis. The initial gravity field was set up before conducting the nonlinear computation. Griffiths and Lane [30] analyzed this issue using combined FEM and SRM. In their analysis, the non-associated flow rule was utilized and the dilatation angle ψ was assumed to be zero in the plastic potential function. The result yield by FEM with the associated flow rule can be compared with the analytical methods (e.g., slices method and limit analysis method) directly. The values of the slope's FOS are calculate using D–P, M–C, and UST criterion. Three types of yield surfaces for D–P criteria were used: compression cone, Extension cone, and incircle cone, respectively [6].

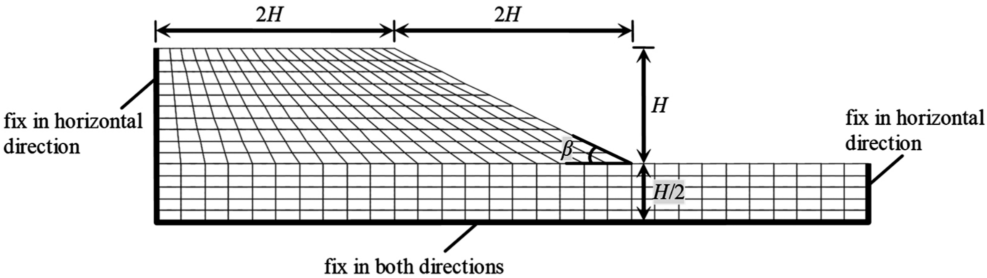


Fig. 3. Mesh and boundary conditions for slope stability analysis

Figure 4 depicts the maximum shear strain contours, deformed mesh, and displacement vector plots obtained from the UST constitutive model and the associative flow rule. A higher plastic dilatation than the actual case will be yielded when the associative flow rule is applied. However, the non-associative flow rule can effectively reduce the plastic dilatation. In the case of the non-associative flow rule, the dilatation angle ψ is smaller than the friction angle φ, and the stress and velocity fields are non-orthogonal. The non-associative flow rule also produces different failure mechanisms. Fig. 5 presents the calculation results of the UST constitutive model and the non-associated flow rule (with $\psi = 0$). The mesh deformations decrease as parameter b values increase. There are significant differences in the mesh deformation yielded by the associative and non-associative flow rules. The associative flow rule calculation significantly upwarps the toe of the slope. When other conditions remain the same, the values of FOS obtained by the associative flow rule are greater than those yielded by the non-associative flow rule. Fig. 6 illustrates the values of FOS for a plane strain slope (with $\varphi = 20°$) obtained with several yield criteria. The values of FOS yielded by UST are between the values of FOS yielded by M–C and D–P (compression cone), respectively. The values of FOS yielded by the non-associative flow rule are slightly smaller than those obtained by the associative flow rule. In this study, a factor ξ was used to evaluate the impact of the σ_2 effect on the slope stability, and the factor ξ is formulated as follows:

$$\xi = \frac{\text{FOS}(\text{UST } b > 0 \text{ or other criteria})}{\text{FOS}(\text{M–C criterion})} \tag{3.1}$$

where the factor ξ equals the values of FOS yielded by the criteria considering the influence of the σ_2 effect divided by the results yielded by M–C criterion. Fig. 7 shows the comparison of the factor ξ for footing capacity and slope stability (this study) using the associated flow rule ($\varphi = 20°$) [12]. The factor ξ determines the significance of the impact of σ_2 effect on slope stability, with higher values indicating greater impact. It is noteworthy that the impact of the σ_2 effect on the of footing's bearing capacity is significantly higher than that of slope stability. Additionally, Table 1 displays the values of FOS for slope yield using different methods and criteria when $c = 10.0$ kPa and $\varphi = 20°$.

Table 1. Values of the slope's FOS obtained through different methods and using different criteria (c = 10.0 kPa, φ = 20°)

Methods and criteria	**Values of FOS**	
	Associated ($\psi = \varphi$)	**Non-associated (ψ = 0)**
FLAC, Drucker–Prager (Incircle Cone)	1.36	1.32
Slip circle method, Mohr–Coulomb (Griffiths 1999)	1.37	–
FEM, Mohr–Coulomb (Griffiths 1999)	1.40	–
FLAC, Drucker–Prager (Extension Cone)	1.44	1.39
FLAC, UST (b = 0.0)	1.415	1.39
FLAC, UST (b = 0.25)	1.58	1.54
FLAC, UST (b = 0.5)	1.70	1.67
FLAC, UST (b = 0.75)	1.81	1.77
FLAC, UST (b = 1.0)	1.895	1.855
FLAC, Matsuoka–Nakai	1.68	1.655
FLAC, Drucker–Prager (Compression Cone)	1.81	1.75

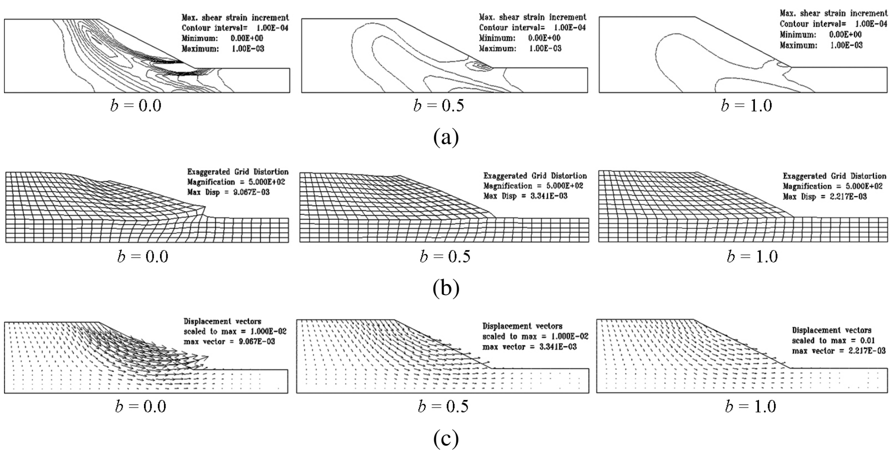


Fig. 4. Failure mechanisms of a plain strain slope (based on UST) with SRF = 1.40 and flow rule ($\psi = \varphi$) for different values of b (0.0, 0.5 and 1.0): (a) Maximum shear strain contours, (b) Deformed mesh, (c) Displacement vector

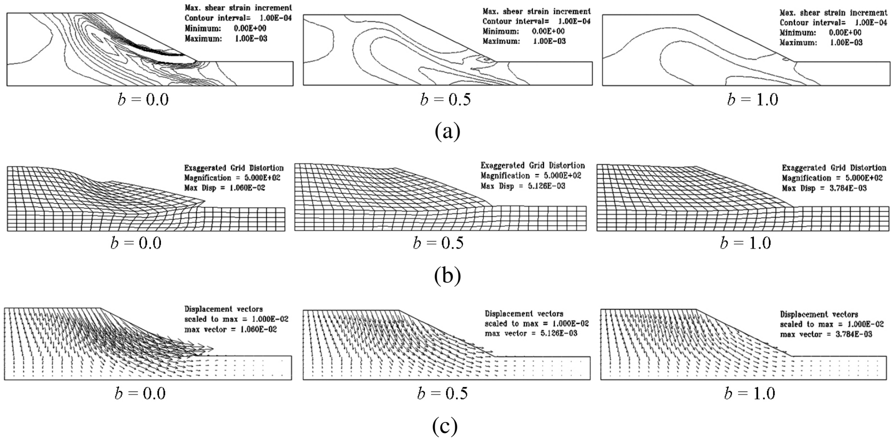


Fig. 5. Failure mechanisms of a plain strain slope (based on UST) with SRF = 1.38 and non-associated flow rule ($\psi = 0$) for different values of b (0.0, 0.5 and 1.0): (a) Maximum shear strain contours, (b) Deformed mesh, (c) Displacement vector

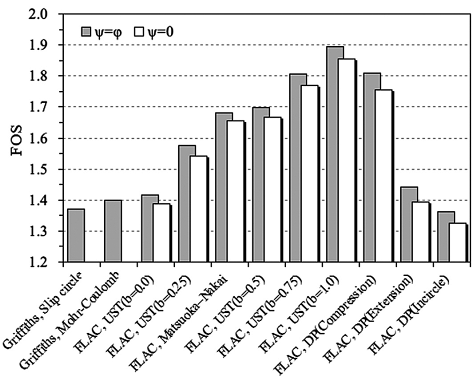


Fig. 6. Values of FOS for a plane strain slope obtained through numerical and analytical methods based on a different criterion, $\varphi = 20°$

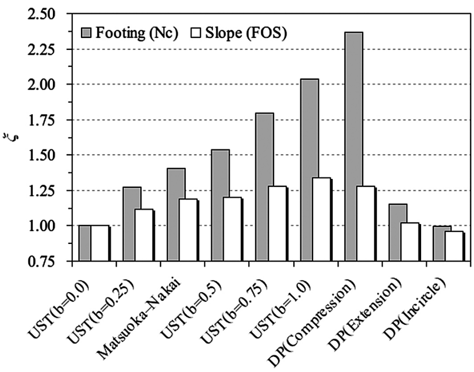
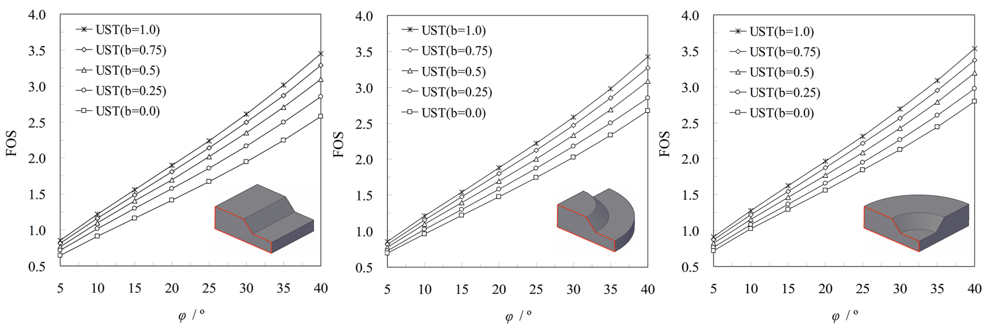


Fig. 7. Factor ξ for the plane strain slope and the smooth strip footing, $\varphi = \psi = 20°$

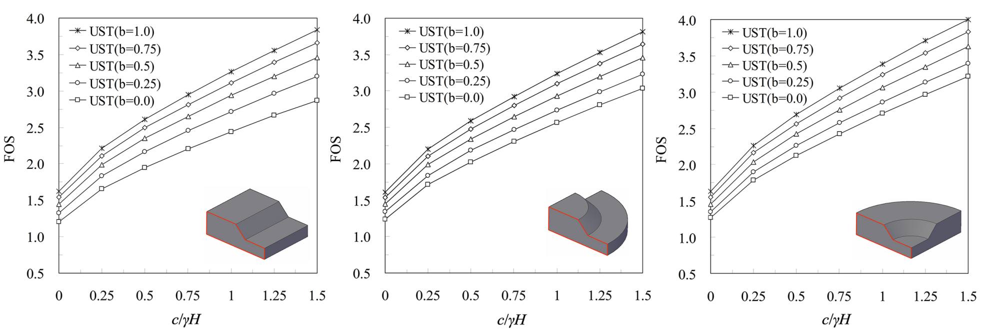
3.2. Strength parameter and slope geometry

The impact of the σ_2 effect on bearing capacity of footings is associated with the friction angle of the foundation soil [12, 36, 37]. The slope stability is affected by the geometry of the slope, as explained by Zhang et al. [35]. The stress state inside the slope varies with different slope geometries, and σ_2 has different effects on the stability of slopes. In this study, the impact of the σ_2 effect on slope stability is evaluated, considering different values of the strength and geometry parameters. The values of a single parameter were changed while keeping the other parameters fixed at their initial values. The analysis was conducted for three types of slopes, namely, plane strain slope, truncated cone slope, and open pit slope, with plane strain and axisymmetric conditions, respectively. The values of FOS for these slopes were obtained using FLAC with UST and different values of friction angle φ, cohesion c, slope angle β, and slope height H, as shown in Fig. 8 to Fig. 11. The FOS values for all three types of slopes increased with the value of the parameter b. Fig. 12 shows the factor ξ for three types of slopes with different values of friction angle φ, cohesion c, slope angle β, and slope height H. The σ_2 effect has a greater impact on plane strain slope stability than on axisymmetric slopes such as truncated cones and open pits. Additionally, the σ_2 effect on the stability of truncated cone slopes is equivalent to that of open pit slopes. The values of FOS for the plane strain slope obtained using UST (b = 1.0) were up to 1/3 higher than those obtained using UST (b = 0.0), i.e., M–C criterion. σ_2 also has a specific degree of impact on the stability of the axisymmetric slope, with an underprediction of 15% of the values of FOS due to using the M–C criterion. This means that, despite having axisymmetric conditions, the average stress state in these slopes of axial symmetry is far from the triaxial compression or extension conditions. The σ_2 effect on the stability of the plane strain or axisymmetric slope is consistent regardless of the variation



(a) (b) (c)

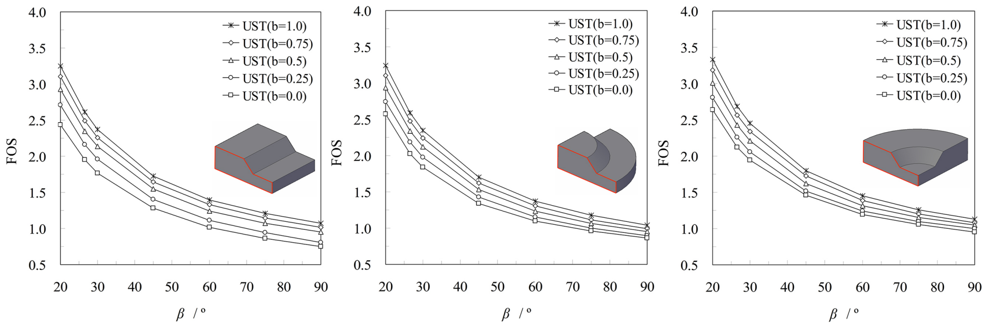
Fig. 8. Values of FOS for three types of slopes yield by FDM and UST with various values of friction angle cohesion: (a) plane strain, (b) truncated cone, (c) open pit



(a) (b) (c)

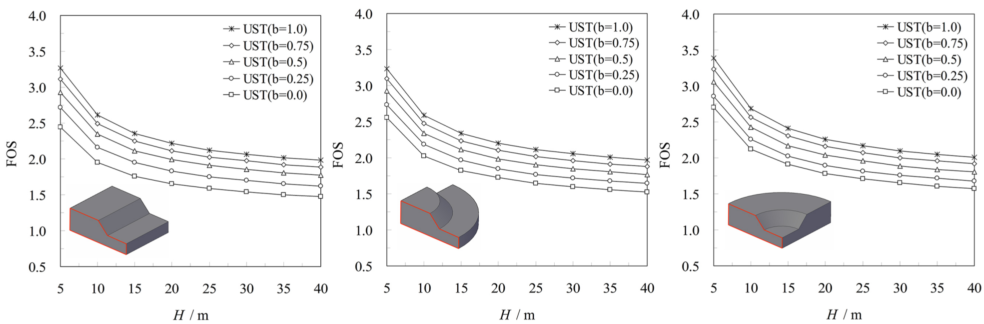
Fig. 9. Values of FOS for three types of slopes yield by FDM and UST with various values of cohesion: (a) plane strain, (b) truncated cone, (c) open pit

in friction angle φ, cohesion c, and slope height H. The impact of σ_2 on the stability of plane strain or axisymmetric slopes increases as the slope angle β increases. When comparing the calculation results of the of footing capacity with the slope stability (this study) [12], it was found that the impact of the σ_2 effect on slope stability is significantly weaker than on the problem of the footing capacity. The impact of σ_2 on the problem of the footing capacity increases with an increase in the friction angle of soil, but the σ_2 effect on slope stability is unrelated to the friction angle of soil. The highway cutting slope and excavation slope of mines are both in complex stress state, and these two types of slopes belong to plane strain and axisymmetric type problems. The influence of the σ_2 effect should be considered in these slopes' FOS calculation and stability analysis, which can give full play to the strength potential of geomaterials in engineering design and construction. In addition to the research methods in this paper, the effect of intermediate principal stress on slope stability can be further studied through in-situ monitoring [38].



(a) (b) (c)

Fig. 10. Values of FOS for three types of slopes yield by FDM and UST with various slope angles β: (a) plane strain, (b) truncated cone, (c) open pit



(a) (b) (c)

Fig. 11. Values of FOS for three types of slopes yield by FDM and UST with various values of slope heights H: (a) plane strain, (b) truncated cone, (c) open pit

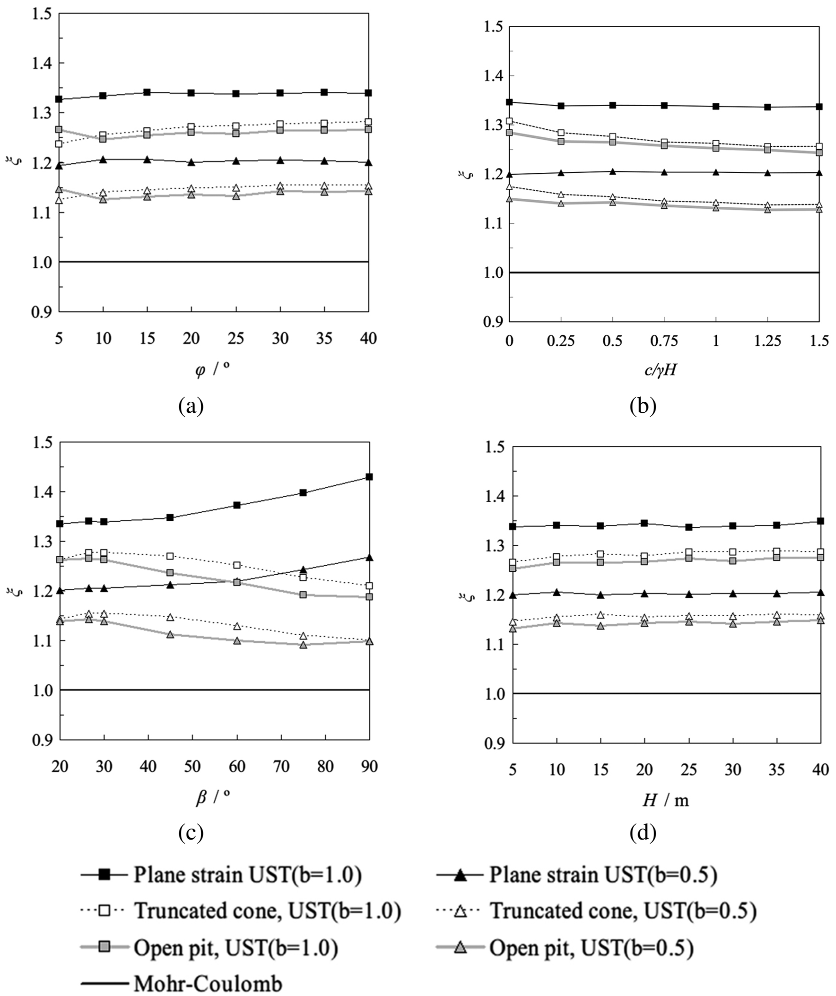


Fig. 12. The relationship between the factor ξ and the influence parameter for slope stability: (a) Friction angle, (b) Cohesion, (c) Slope angle, (d) Slope height, (e) Legend

4. Conclusions

The study used the FDM code FLAC and the UST elastoplastic constitutive model to compute the values of FOS for three types of slopes in plane strain and axisymmetric conditions. The study investigated the impact of the σ_2 effect on the slope stability problem with different geometry and plastic flow rules. The following conclusions can be deduced.

1. The impact of the σ_2 effect on the slope stability is not sensitive to the soil's strength parameters (friction angle and cohesion) and slope height. However, σ_2 significantly

impacts the two-dimensional slope stability problem (plane strain or axisymmetric) with the slope angle increased.

2. UST (b = 1.0) (upper bound surface of convex theory) yielded a FOS value for the plane strain slope that is 1/3 more than that of UST (b = 0.0) (lower bound surface of convex theory). UST (b = 1.0) also significantly influences the axisymmetric cases; the value of FOS for the axisymmetric slope increased by 15% of the value yielded by UST (b = 0.0).
3. σ_2 has a more significant influence on the stability of the plane strain slope than on the axisymmetric slope. The cutting slopes of highways and excavation slopes in mines are types of problems that fall under the plane strain and axisymmetric categories. Both of these slopes are subjected to a complex stress state, and it is imperative to consider the influence of the σ_2 effect when calculating the factor of safety, so as to fully utilize the strength potential of geomaterials in slope engineering design and construction.
4. The σ_2 effect has a greater impact on the footing capacity than on the slope stability problem. The σ_2 has more significant impact on the footing capacity with higher values of soil's friction angle. However, the σ_2 effect has no significant impact on slope stability, regardless of the soil's friction angle. Further research is needed to investigate the impact of the σ_2 effect on the seismic stability and failure process of slopes or landslides.

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