WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2025.155105

ARCHIVES OF CIVIL ENGINEERING

Vol. LXXI ISSUE 3 2025

pp. 369 – 384

Research paper

Methods of assessing concrete creep in prestressed bridge structures

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Abstract: The paper discusses the phenomenon of concrete creep, its mechanical models and simplified as well as more sophisticated methods of estimating creep effects applied in the design of bridge structures. The section on simplified methods describes the metod of substitutive concrete elasticity modulus and the method of estimating creep effects with the correction factor C_{creep} for spans with precast beams. Among the precise methods, it presents the modified effective modulus method (Trost 1967), age-adjusted effective modulus method (Bažant 1972) and the general incremental method according to the linear theory of elasticity. Methods for computationally accounting for creep according to current PN-EN standards, withdrawn Polish standards, and recommendations from foreign literature are characterized. The impact of creep on the redistribution of internal forces during the incremental erection of the structure was demonstrated using examples of a viaduct made of precast beams and a bridge constructed using balanced cantilever method. Attention was drawn to the possibilities of extending the description of creep phenomena in concrete bridge structures using the conceptual framework of fractional-order derivatives.

Keywords: rheology, viscoelasticity, concrete creep, internal forces redistribution, fractional-order derivatives

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1. Introduction

A reliable assessment of the effects of concrete creep in prestressed bridge structures is important for determining internal forces, stresses and displacements (particularly deflections) at various stages of construction and operation of the structure. One of the effects of creep can be a deterioration of the functional properties of the bridge, e.g. a change in the grade line, as well as an unacceptable increase in stresses in the concrete or the appearance of tensile stresses reducing the durability of the structure. In the case of large-span structures, underestimating the creep effects can even lead to damage or failure [1]. The properties of concrete are significantly influenced by additives commonly found in the concrete mix. Thes additives particularly affect creep deformations, which was analyzed and presented in [2]. An interesting aspect is the use of Self-Compacting Concrete (SCC) for bridge construction, due to its numerous advantages, as described in [3].

Concrete creep in building structures is interdependent with other rheological effects (e.g. shrinkage, relaxation of steel) and difficult to precisely formulate. The specificity of the structural analysis of bridge spans, related to the principle of superposition, allows for its separate analysis. Nowadays, numerical methods are routinely used in their design and the accuracy of calculations also depends on the sophistication of the commercial software used.

2. The phenomenon of concrete creep and its mechanical models

The creep phenomenon of concrete subjected to long-term loads of constant value is characterized by the increase of deformations over time [4]. To describe this, mechanical models such as the Maxwell model (series connection of spring and damper) or Kelvin–Voigt (parallel connection of spring and damper) are used.

The Kelvin–Voigt model is often employed in contemporary numerical models of building structures. It consists of two elements connected in parallel: elastic (Hooke's spring) and viscous (Newton's damper). The spring models the elastic properties of the material. It is characterized by its ability to accumulate elastic energy. It is responsible for the immediate response to a change in stress. The viscous element represents the ability of a material to dissipate energy. It is responsible for the delayed response to changes in stress. The differential equilibrium equation of the Kelvin–Voigt rheological model is expressed as:

$$\sigma(t) = E\varepsilon(t) + \eta\frac{d\varepsilon(t)}{dt} \tag{2.1}$$

where: $\sigma(t), \varepsilon(t)$ – stress and strain in the material, E – Young's modulus of elasticity, η – viscosity coefficient.

The Kelvin–Voigt model is relatively simple, facilitating its mathematical implementation. However, it does not always accurately reflect the actual behavior of materials under complex loading conditions. Therefore, more advanced rheological models are used in design practice depending on the specific material properties and operating conditions.

With the constant stress over time $\sigma = \sigma_0 = \text{const}$ and the assumed initial time condition: $t = 0 \rightarrow \varepsilon_0 = 0$ (creep test), the solution of equation (2.1) takes the form of formula [5]:

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\frac{t}{\lambda}}\right) \tag{2.2}$$

where: $\lambda := \dfrac{\eta}{E}$ is the creep constant, the so-called retardation time.

3. Simplified methods for estimating creep effects in design

To account for the effects of creep in bridge structures, many simplified ways of estimating creep have been developed over the years. The most popular one is the method of effective concrete elasticity modulus. Abroad, in the case of spans made of precast beams, the methods of the correction factor C_{creep} were used (e.g. PCA, CTL and Method-P [6]).

By using the method of substitutive concrete elasticity modulus, the impact of creep is accounted for by adjusting this parameter using the coefficient $\varphi(t, t_0)$ [7, 8]. This method is the least accurate, but the most commonly used in design. Non-specialized FEA systems for structural analysis are based on this approach. It requires calculations on two independent numerical models of the structure – separately in case of long-term loads with the correction of the E_c parameter and for short-term loads (without reduction), and then their superposition [7].

Using the substitutive concrete elasticity modulus method, it is difficult to consider the load history of the structure. It assumes complete reversibility of creep deformations (similarly to the Kelvin–Voigt rheological model). The constitutive relationship is then as follows:

$$\Delta\varepsilon_{ce} + \Delta\varepsilon_p(t) = \frac{1 + \varphi(t, t_0)}{E_{c0}} \Delta\sigma_c(t) = \frac{\Delta\sigma_c(t)}{E_{c0,ef}} \tag{3.1}$$

The effective modulus of elasticity is given by the formula:

$$E_{c0,ef} = \frac{E_{c0}}{1 + \varphi(t, t_0)} \tag{3.2}$$

Using the method of estimating creep effects with the correction factor C_{creep} allows for a relatively simple determination of internal forces in uncomplicated structures built in stages (e.g. from precast beams). The C_{creep} multiplier values given in Table 1 have the same physical and computational sense, but different accuracy of the creep effect.

The creep phenomenon in case of staged construction causes a redistribution of internal forces, which in engineering calculations is often simplified by using additional induced forces (so-called secondary rheological effects). Their final values can be estimated using the general formula [9–13]:

$$S_\infty = S_0 + C_{\text{creep}} \cdot (S_c - S_0) \tag{3.3}$$

where: S_∞ – final value of the internal force after the completion of rheological processes, S_0 – internal force at the end of the construction process (intermediate state), S_c – internal force calculated as for the final static scheme (continuous), C_{creep} – creep factor, which can be determined by any methods presented in Table 1.

In foreign literature, formula (3.3) was used to estimate internal forces in reinforced concrete nodes (joints) ensuring the continuity of spans made from precast beams, using creep correction multipliers C_{creep} determined by PCA methods (1960s, Portland Cement Association [15]), CTL (National Cooperative Highway Research Program, USA 1989), Method-P (Peterman and Ramirez 1998 [14]).

Table 1. Summary of C_{creep} correction factor formulas according to various methods [12]

Method	**Formula**
PCA	$C_{\text{cr}}^{\text{PCA}} = (1 - e^{-\varphi})$
Method-P	$C_{\text{cr}}^{\text{P}} = \alpha \cdot (1 - e^{-\varphi})$
CTL	$C_{\text{creep}} = \left(1 - e^{-(1-k_t)\cdot\phi_\infty}\right)$
Trost-Bažant	$C_{\text{cr}} = \dfrac{\phi(\infty, t_0)}{1 + \chi\phi(\infty, t_0)}$
According to Eurocode 2	$C_{\text{cr}}^{\text{EC2}} = \dfrac{E_c(t_1)}{E_c(t_0)} \left[\dfrac{\phi(\infty, t_0) - \phi(t_1, t_0)}{1 + \chi\phi(\infty, t_1)}\right]$
	$C_{\text{cr}} = \dfrac{\phi(\infty, t_0) - \phi(t_1, t_0)}{1 + \chi\phi(\infty, t_1)}$

Explanations of symbols:
ϕ – the amount of creep that will occur from the time of load application in the final system,
t_0 – the age of the concrete at the time of permanent load application,
t_1 – the age of the concrete when the support conditions (static scheme) change,
$E_c(t_0)$ – concrete's modulus of elasticity at time t_0,
$E_c(t_1)$ – concrete's modulus of elasticity at time t_1,
$\chi = \chi(tt_0)$ – aging coefficient (for $t = \infty$ it can be assumed 0.8), usually $\chi = 0.6 \div 0.9$,
$\phi(\infty t_0)$ – final value of the creep coefficient,
$\phi(t_1 t_0)$ – creep coefficient from t_0 to t_1, relative to the elastic strain after 28 days,
α – coefficient of the impact of cracking on the continuity node according to [14].

4. Precise methods for creep analysis

More precise methods for creep analysis (e.g., Trost's method, Bažant's method, incremental method according to linear elasticity theory) are based on iterative approaches and require advanced computer implementation, available only in specialized computational systems (e.g., SOFiSTiK, Midas, Lusas) [16–18]. It is necessary to create an accurate numerical model that reflects all intermediate stages of the structure's behavior, resulting from both changes in the static scheme (assembly and final operation) and the sequence of applied loads (load history). Additionally, modeling the prestressing system of the structure and determining the input parameters related to the course of rheological phenomena are essential.

One of the earliest more accurate methods of considering creep was the modified effective modulus method for concrete, also known as Trost's method (1967). It extended the effective modulus method by taking into account the load history (sequence) through a modified concrete modulus based on the relation [7]:

$$\Delta\varepsilon_{co}(t_0) + \Delta\varepsilon_p(t, t_0) = \frac{\Delta\sigma_c(t_0)}{E_{co,ef}} + \frac{\Delta\sigma_c(t) - \Delta\sigma_c(t_0)}{E_{co,m}} \tag{4.1}$$

The modified modulus of elasticity is determined by the formula:

$$E_{co,m} = \frac{E_c(t_0)}{1 + \rho(t, t_0)\varphi(t, t_0)} \tag{4.2}$$

where: $\rho(t, t_0)$ – relaxation coefficient (Trost).

The modified effective modulus method, initially presented as an approximate method, proved to be sufficiently accurate when the deformation changes linearly with the creep coefficient or when the stress changes linearly with the relaxation coefficient. Trost's method has been implemented, among others, in the SOFiSTiK software. In this method, the quantities $\rho(t, t_0)$ and $E_{co,m}$ do not take into account aging creep.

This effect can be captured by the *age-adjusted effective modulus method (*AAEM), in which $E_{co,m}$ is replaced with $E''(t, t_0)$, and $\rho(t, t_0)$ is replaced by the ageing coefficient $\chi(t, t_0)$, as shown by Bažant in 1972 [19]. Taking into account the aging effect, the errors of the AAEM method for loads applied at a young age of concrete, which is typical for balanced cantilever method or incremental launching method, are significantly smaller than in the Trost's method. This approach is more precise concerning various loading paths, including not only stress reduction over time (relaxation) but also its increase (long-term buckling). In this method, the formula for the effective modulus of concrete takes the form:

$$E''(t, t_0) = \frac{E(t_0) - R(t, t_0)}{\phi(t, t_0)} \tag{4.3}$$

where: $R(tt_0)$ – relaxation function.

The aging coefficient can be calculated from the relaxation function and vice versa:

$$\chi(t, t_0) = \frac{E(t_0)}{E(t_0) - R(t, t_0)} - \frac{1}{\phi(t, t_0)} \tag{4.4}$$

In subsequent years, Bažant presented more complex models of shrinkage and creep [20]. Mathematical descriptions of these phenomena, based on advanced theoretical foundations, are the B3 (Bažant and Baweja 1995, 2000) and B4 (Bažant et al. 2014) models. Model B3 assumes calibration based on short-term (1–3 months) measurements of creep and shrinkage, in order to better reflect the composition and strength of the designed concrete, which is particularly important for high-strength concretes. The parameter values of this model can be easily obtained using linear regression based on short-term measurement data. Model B4 (Bažant, Hubler, Qiang Yu [21]) also assumes model calibration using short-term tests but introduced a correction to the computational equation to achieve a more accurate long-term creep curve slope. A set of short-term measurements was developed in parallel, serving as a database for calibration and verification of improved creep prediction models.

In case of the Bažant's models, the total creep strain due to the load at age t, caused by a uniform axial load that acts from the load application age t_0, is determined from the formula:

$$\varepsilon_c(t) = \sigma_c(t) \cdot J(t, t_0) \tag{4.5}$$

where the creep function is in the form:

$$J(t, t_0) = q_1 + C_0(t, t_0) + C_d(t, t_0, t_c) \tag{4.6}$$

where: q_1 – instantaneous deformation caused by unit stress, $C_0(t, t_0)$ – compliance function concerning basic creep, $C_d(t, t_0, t_c)$ – additional compliance function accounting for drying creep, t, t_0, t_c – concrete age at the considered time, the age when drying begins, and the age of load application.

According to the presented model, basic creep consists of three components: *an aging viscoelastic term, a nonaging viscoelastic term, and an aging flow term.* The additional creep compliance function accounts for the drying process of concrete before the first load application. Many other formulas for predicting concrete creep are described in the literature, standards, and guidelines. They are usually derived not from theoretical considerations but empirical methods. These include the American Concrete Institute [24] model and the European Model Code [22, 23]. The guidelines of ACI Committee 209 developed by Branson and Christiason in 1971, after several minor modifications, were presented in 1992 and then reapproved in 2008 [24] (without changes). The creep function for this model is as follows:

$$J(t, t_0) = \frac{1 + \phi(t, t_0)}{E_{cmto}} \tag{4.7}$$

where: E_{cmto} – modulus of elasticity at the time of load application t_0, $\phi(tt_0)$ – creep coefficient at *age* t, which is the ratio of creep strain to elastic strain at the beginning of loading at age t_0, described by the formula:

$$\phi(t, t_0) = \frac{(t - t_0)^{\psi}}{d + (t - t_0)^{\psi}} \phi_u \tag{4.8}$$

where: d, ψ – coefficients dependent on the shape and size of the element, ϕ_u – final creep coefficient.

In the case of standard concrete curing conditions and no specific creep data for local aggregates, the suggested average value of the final creep coefficient is $\phi_u = 2,35$. To determine the creep coefficient $\phi(t, t_0)$, one must define the age of concrete at loading, the relative humidity of the environment, the volume-to-surface ratio of the element (or the average dimension of the element), and the type of cement.

The problem of concrete theology has also been addressed by working groups within the international association FIB (International Federation for Structural Concrete). Publications by FIB, such as Model Code 1990 [22] and Model Code 2010 [23], form the basis for designing concrete structures (including the first version of Eurocodes [10, 28]). The constitutive relation of concrete considering creep, presented in Model Code 2010 [23], takes the form [5]:

$$\varepsilon_c(t) = \sigma_c(t_0)\left[\frac{1}{E_{ci}(t_0)} + \frac{\varphi_{28}(t,t_0)}{E_{ci}}\right] + \int_{t_0}^{t}\left[\frac{1}{E_{ci}(\tau)} + \frac{\varphi_{28}(t,\tau)}{E_{ci}}\right] d\sigma_c(\tau) \tag{4.9}$$

where: $\sigma_c(t_0)$ – stress at time t_0, $J(t,t_0) = \dfrac{1}{E_{ci}(t_0)} + C(t,t_0)$ – creep function, $E_{ci}(t_0)$ – time-varying modulus of elasticity at load application time t_0, $C(t,t_0) = \dfrac{\varphi(t,t_0)}{E_{ci}}$ – creep measure: $\varphi(t,t_0)$ – creep coefficient, E_{ci} – modulus of elasticity of concrete at the age of 28 days.

After modifying the formula (4.9) into a recursive form and considering shrinkage strains, one obtains the constitutive equation of concrete in the general incremental method presented in PN-EN1992 [10] (the dependence of concrete strain over time on stress – Fig. 1):

$$\varepsilon_c(t) = \frac{\sigma_0}{E_c(t_0)} + \phi(t,t_0)\frac{\sigma_0}{E_c(28)} + \sum_{i=1}^{n}\left(\frac{1}{E_c(t_i)} + \frac{\phi(t,t_i)}{E_c(28)}\right)\cdot\Delta\sigma(t_i) + \varepsilon_{cs}(t,t_s) \tag{4.10}$$

where: σ_0 – initial stress in concrete, $E_c(t_0), E_c(28), E_c(t_i)$ – modulus of elasticity at timen t_0; after 28 days; in time interval t_i, $\phi(t,t_0), \phi(t,t_i)$ – creep coefficient from time t_0 to t and from time t to t_i, $\Delta\sigma(t_i)$ – stress increments in the cross-section in individual time intervals, $\varepsilon_{cs}(t,t_s)$ – deformation of contraction during t to t_s.

The first component of Equation (4.10) describes the immediate deformations caused by the stress applied at time t_0. The second term captures creep caused by the long-term effect of stress application at time t. The third expression represents the sum of immediate deformations and creep deformations resulting from the change in stresses in the analyzed time intervals (tt_i), i.e., the influence of the load history. The fourth component of the formula stands for the strain caused by shrinkage.

According to the general method, the progression of deformations over time is considered incrementally with respect to successive time intervals, taking into account the stress value in concrete from the previous interval in the following intervals (Fig. 1).

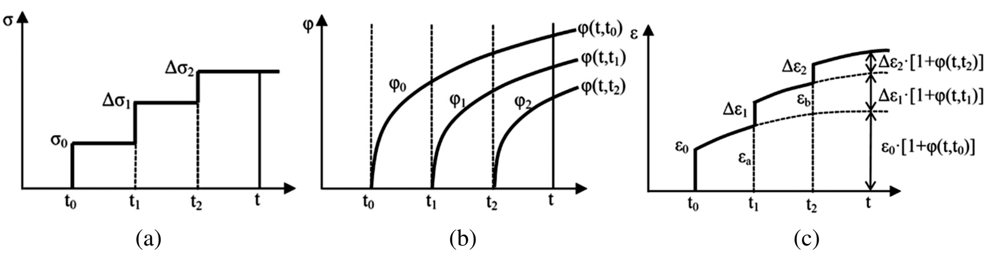


Fig. 1. Illustration of the development of concrete deformations caused by stress increments over time and the change in creep coefficient: (a) increase in stresses over time, (b) change in the creep coefficient over time, (c) development of deformations over time [8, 25, 26]

This method captures the dependence of concrete creep in each cross-section on the course of stresses over time (load history). Specialized computer programs enable creep analysis in the numerical model of the structure separately for each of the segmented parts of the model, in which the strain changes according to the formula (4.10), while maintaining the conditions of equilibrium and inseparability.

In creep models used in PN-EN [10], ACI [24] (except MC 2010 [23] and models B3, B4 [20]), creep curves approach the final horizontal asymptotic limit or finite upper creep limit, which does not actually exist because long-term creep is logarithmic, as demonstrated in [21, 27]. According to the B3 and B4 models, the long-term asymptote of the creep curve is logarithmic. After a few years, both the creep curve and the deflection become linearly increasing on a logarithmic time scale (Fig. 2).

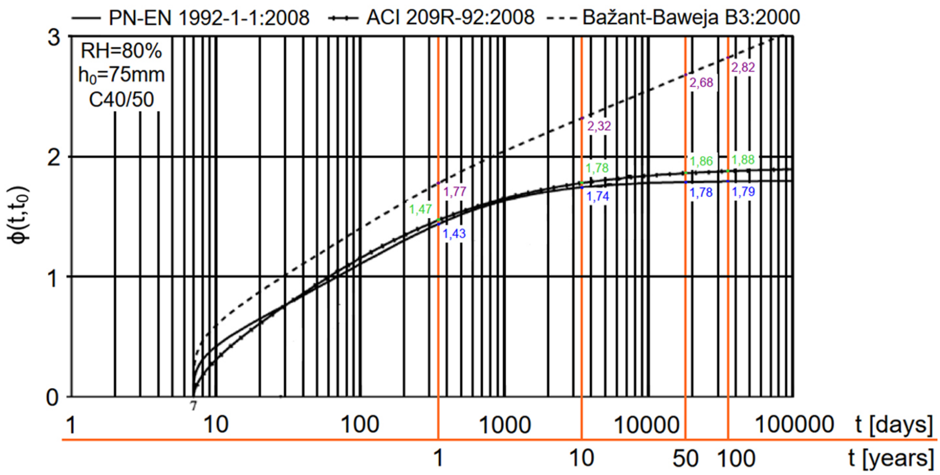


Fig. 2. Comparison of creep coefficients of concrete $\varphi(t,t_0)$ [27] according to PN-EN [10], ACI [24] and B3 [20] for concrete class C40/50, RH=80%

5. Creep in design standards PN-S-10042 and PN-EN 1992

In the current standards PN-EN 1992-1-1:2008 [28] and PN-EN 1992-2:2010 [10], the creep course can be estimated more accurately than in the withdrawn Polish standard PN-S-10042:1991 [29]. The PN-S-10042:1991 standard presented creep coefficient values assuming the concrete age at the time of loading to be 7, 28, and 90 days, respectively, for typical technological conditions and normal curing conditions. The coefficient values for intermediate curing conditions had to be determined by interpolation.

In Eurocodes [10, 28], creep is accounted for according to the fib MC 2010 model, where values for certain parameters may differ slightly. Formulas are provided to track changes in the creep coefficient over time, as per equation (5.1). The basic creep coefficient φ_0 is calculated from the relation (5.2). The influence of humidity is taken into account by the φ_{RH} coefficient according to the formulas given in [28]. The influence of the concrete age at the time of loading on the creep course $\beta(t_0)$ is described by the relation (5.3).

$$\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0) \tag{5.1}$$

$$\phi_0 = \phi_{RH}\beta(f_{cm})\beta(t_0) \tag{5.2}$$

$$\beta(t_0) = \frac{1}{0,1 + t_0^{0,20}} \tag{5.3}$$

The function of the creep course over time $\beta_c(t, t_0)$ has the form:

$$\beta_c(t, t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0}\right]^{0,3} \tag{5.4}$$

In PN-EN [28], as in PN [29], the composition of the concrete mix can be taken into account. The influence of the type of cement (class S, N or R) on the creep course is taken into account by correcting the age of the concrete at the time of load t_0 according to the formula:

$$t_0 = t_{0,T}\left(\frac{9}{2 + t_{0,T}^{1,2}} + 1\right)^{\alpha} \geq 0,5 \tag{5.5}$$

In formulas (5.1)–(5.5) the following notations are used: $\phi(t, t_0)$ – creep coefficient over time, ϕ_0 – basic creep coefficient, $\beta(t_0)$ – coefficient for influence of concrete age at the time of load, β_H– coefficient for influence of moisture and geometry of the element, $\beta_c(t, t_0)$ – function of the creep course over time, ϕ_{RH}– coefficient for influence of relative humidity, $\beta(f_{cm})$ – coefficient for the influence of concrete strength, α – exponent depending on the type of cement, t – age of concrete at the considered moment (in days), t_0 – age of concrete at the time of loading, $t_{0,T}$ – age of concrete at the time of loading, taking into account the influence of temperature on the maturation of concrete.

In Annex KK of the PN-EN 1992-2 standard [10], simplified methods are provided to account for the influence of creep on the redistribution of internal forces in structures built in stages using creep correction coefficients C_{creep}, referencing methods developed in the second half of the 20th century (PCA, CLT, Method-P). Multipliers accounting for the proportions of internal forces induced by creep can be estimated using formulas (5.6)–(5.8) [10, 16, 25].

$$C_{\text{creep}}^{\text{EN}-1} = \frac{E_c(t_1)}{E_c(t_0)}\left[\frac{\phi(\infty, t_0) - \phi(t_1, t_0)}{1 + \chi\phi(\infty, t_1)}\right] \tag{5.6}$$

$$C_{\text{creep}}^{\text{EN}-2} = \frac{\phi_{t=\infty}}{1 + \chi\phi_{t=\infty}} \tag{5.7}$$

$$C_{\text{creep}}^{\text{EN}-3} = \frac{\phi(\infty, t_0) - \phi(t_1, t_0)}{1 + \chi\phi(\infty, t_1)} \tag{5.8}$$

In Eurocodes [10, 28], the relationships are derived based on the theory of the effective modulus of elasticity supplemented by the influence of the aging coefficient $\chi(tt_0)$. In this approach, it is not possible to consider the influence of load history on creep. Calculations pertain to creep over an infinite time, thus discretization of time is not required. According to PN-EN [10, 28], the linear-elastic model of creep is valid for characteristic stresses $\sigma_c \leq 0.45 f_{ck}(t)$ in concrete under a nearly constant combination of actions (f_{ck}– characteristic compressive strength of concrete). Time-dependent ehaviour of concrete is recommended to be determined using the creep coefficient or creep function, or by the relaxation function. For higher compressive stresses, nonlinear creep effects must be considered. In practice, this requires iterative methods and specialized software using the finite difference method and FEM. For creep strain calculations, due to the application of loads at different times, the principle of superposition can be applied.

In the "bridge" Eurocode [10] Annex KK) it is recommended to take into account the rheological effects and the associated redistribution of internal forces in the ultimate limit states (ULS) and serviceability limit states (SLS) according to several methods of varying precision: general incremental method, methods derived under the assumption of linear viscoelasticity, the aging coefficient method, and the simplified aging coefficient method. It is noted that the normative approximate methods for calculating shrinkage and creep may differ from experimental values by ±30%.

6. Examples of creep-induced redistribution of internal forces

6.1. Viaduct made of prestressed precast beams

The structural analysis of bridge composite structures made of precast beams (prestressed concrete-concrete type) is complex due to the influence of rheological phenomena, cracking of reinforced concrete monolithic parts and different static schemes at various load stages.

Creep caused by the effect of long-term loads applied before continuity (self-weight, beam prestressing) and after continuity (weight of equipment, removal of assembly supports) results in changes in the values of support moments [9,11,14,15,30–34]. Additionally, temporary supports are removed after assembly is completed. Computationally, this corresponds to the application of concentrated forces equal to the reactions of the assembly supports to the hyperstatic scheme. At this time, the formwork of monolithic joints is also removed. This corresponds to the application of equivalent loads to the continuous structure due to the weight of these elements, causing elastic redistribution of moments that affects the overall creep of the concrete structure.

The redistribution of internal forces caused by concrete creep in a bridge structure with prefabricated beams is illustrated using the example of the WD-8 structure designed as part of the construction of the S19 expressway. The structure's load-bearing scheme is a two-span continuous beam with spans of 21.2 + 21.2 m, framed over the intermediate support. The cross-sectional width is 11.80 m, and the angle of intersection with the obstacle is 66.2° (Fig. 3).

Prefabricated beams are made (reinforcement, concreting, and prestressing) earlier than the rest of the structure. During construction, they operate in a simply supported scheme. Due to the occurrence of creep from self-weight, the internal force system in the final scheme tends towards the state that would occur if it had been built as continuous from the start. Creep from self-weight causes the slow increase of support moments (Figs. 4a–4e). This trend can be strongly disrupted by creep from the prestressing force (Figs. 4b–4f), which occurs in beams already operating in the continuous system [33].

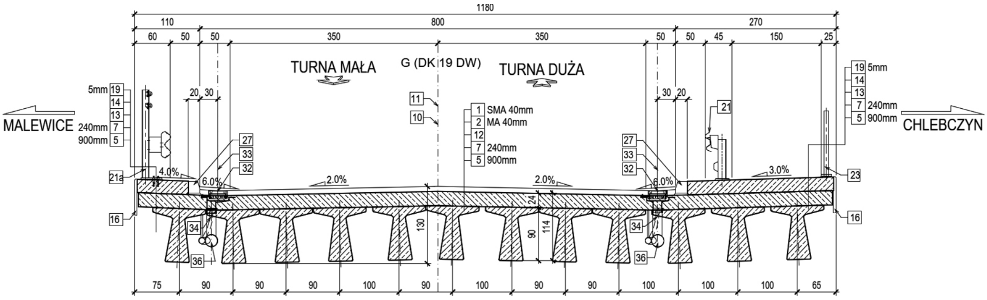


Fig. 3. Cross-section of the WD-8 viaduct

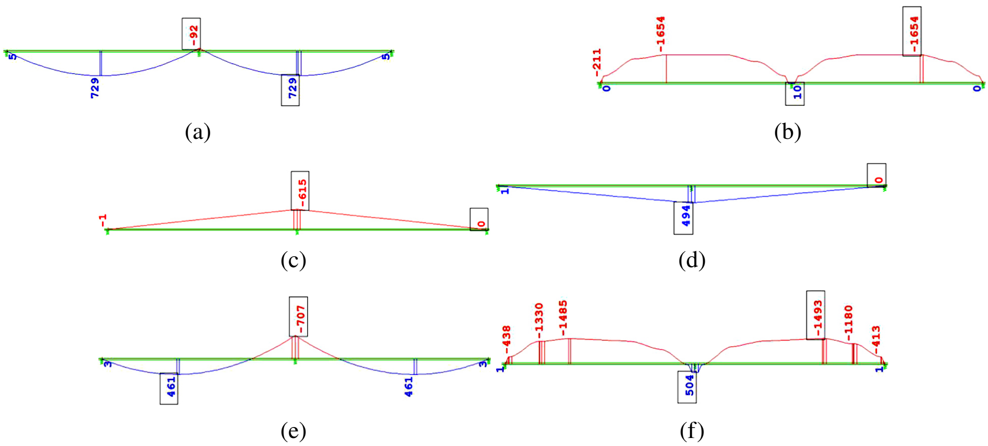


Fig. 4. Diagrams of the bending moment M_y [kN·m] caused by: (a) self-weight in the continuous system immediately after monolithization of the joints, (b) concrete creep from self-weight over 100 years of use, (c) self-weight after redistribution caused by concrete creep (d) prestressing of prefabricated elements in the continuous system immediately after monolithization of the joints, (e) concrete creep due to prestressing force over 100 years of use, (f) prestressing of prefabricated elements after redistribution caused by concrete creep

The distribution of internal forces in these structures, as a result of creep, tends towards the state as if the scheme had been statically indeterminate from the beginning. The degree to which the force distribution approaches this state depends on the time of beam installation relative to their construction, the storage conditions of the beams, the flexibility of the nodes, and the degree of cracking of the monolithic parts of the structure.

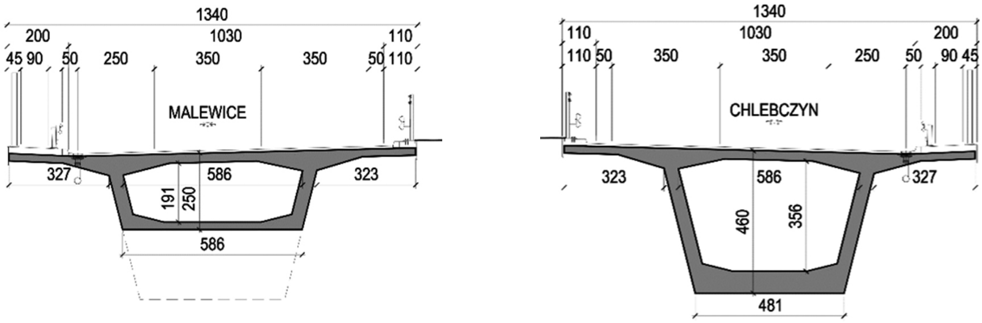
In general, it can be stated that in a hyperstatic structure made of prestressed prefabricated beams, constructed in stages:

– redistribution from creep caused by self-weight increases, compared to the assembly state, the negative moments at supports and decreases the positive moments in spans,

– concrete creep from beam prestressing (superposition of forces from prestressing before continuity and forces induced by creep in the continuous system) leads to a reduction of span moments from prestressing and the emergence of positive moments at the supports.

6.2. Bridge constructed be balanced cantilever method

The redistribution of internal forces from self-weight caused by creep in a bridge constructed by the balanced cantilever method is illustrated using the example of the MS-5 structure designed as part of the construction of the S19 expressway. The structure is a three-span continuous beam with spans of 54.0 + 90.0 + 54.0 m and the cross-section shown in Fig. 5.



(a) (b)

Fig. 5. Cross-sections of the MS-5 bridge: (a) span section, (b) support section

Figure 6 shows the redistribution of bending moments from self-weight caused by creep after the closure of the load-bearing structure. It should be noted that immediately after the closure, the bending moment corresponds to the cantilever schemes. However, due to

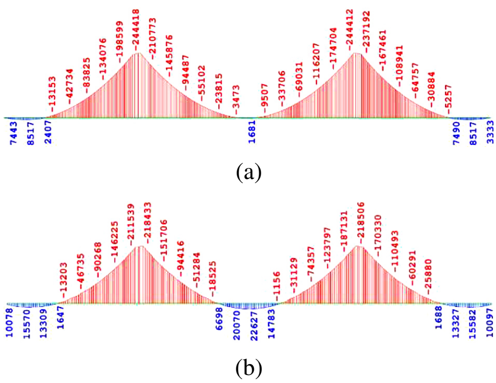


Fig. 6. Total bending moment M_y [kNm] from the self-weight of the bridge: (a) immediately after closure, (b) after redistribution caused by concrete creep over 100 years of use [35]

creep, the internal force system from the time of span closure tends towards the continuous scheme from the start. This is associated with a reduction in support moments (negative) while simultaneously emerging or increasing span moments (positive).

The small positive moment in the main span (M_y = 1681 kN·m) results from the use of a rigid steel structure connecting the ends of the cantilevers during the casting of the keystone. Its value increases several times to M_y = 22627 kN·m. Meanwhile, the negative support moment is reduced by 11%. The positive moments of the end spans increase by about 83% compared to the construction phase. The effects of internal force redistribution caused by creep must be considered when verifying the normal stresses of the load-bearing structure in the SLS and during the design of critical sections for ultimate load capacity in the final operation phase.

7. Summary and conclusions

Methods for estimating the effects of concrete creep in bridge structures allow for varying degrees of accuracy in the assessment of force redistribution. Accurate computational representation of this phenomenon is challenging and feasible in advanced FEM systems dedicated to bridge structures.

The advanced creep models currently in use primarily differ in the time-dependent progression of the creep function. In the computational approach presented in PN-EN [28] and ACI [24] (with the exception of MC 2010 [23] and models B3, B4), the creep curves approach a final horizontal asymptotic limit or a finite upper creep limit. In reality, such a limit does not exist. According to contemporary research, long-term creep exhibits a logarithmic nature.

In both presented computational examples, the distribution of internal forces due to creep tends towards the state as if the system were continuous from the beginning. In the case of structures made of prefabricated beams in a hyperstatic system, the impact of creep can be estimated using simplified methods. For the design of more critical bridge structures constructed using the balanced cantilever method, it is advisable to use more accurate methods, implemented in modern software aimed at bridge construction.

Finally, we note the potential to expand the modeling of viscoelastic properties of engineering structures using rheological structures described by fractional-order derivatives (fractal derivatives). Such models are applied, for example, in the design of flexible and semi-rigid road pavement structures [36, 37]. For instance, the Kelvin–Voigt model, where the viscous damper is replaced by a fractal element, is described by the following fractional differential equation of order α:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d^\alpha \varepsilon(t)}{dt^\alpha}, \alpha \in (0, 1) \tag{7.1}$$

The definition of the fractal derivative is explained in the fundamental monograph [38]. The solution of equation (7.1) with respect to strain (creep test) is as follows:

$$\varepsilon(t) = \frac{\sigma_0}{E} \left(\frac{t}{\lambda}\right)^\alpha E_{\alpha,\alpha+1} \left[-\left(\frac{t}{\lambda}\right)^\alpha\right] \tag{7.2}$$

where $E_{\alpha,\beta}(z) := \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$ denotes the two-parameter Mittag–Leffler function. It can be shown that as $\alpha \to 1$, equation (7.2) reduces to (2.2), meaning the fractal model simplifies to the classical Kelvin–Voigt structure. The use of non-classical, fractal rheological structures allows for a better fit of the mathematical model to experimental research results. Further work by the authors of this article will explore the application of fractal derivative concepts in bridge

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Metody oceny pełzania betonu w mostowych konstrukcjach sprężonych

Słowa kluczowe: reologia, lepko-sprężystość, pełzanie betonu, redystrybucja sił wewnętrznych, pochodne ułamkowego rzędu

Streszczenie:

W referacie omówiono zjawisko pełzania betonu, jego modele mechaniczne oraz uproszczone i bardziej wyrafinowane sposoby szacowania efektów pełzania stosowane w projektowaniu konstrukcji mostowych. W części dotyczącej metod uproszczonych opisano metodę – efektywnego modułu sprężystości oraz metody współczynnika korekcyjnego C_{creep} stosowane w przypadku przęseł z belek prefabrykowanych. Spośród dokładnych metod przedstawiono m.in. metodę zmodyfikowanego efektywnego modułu sprężystości (Trost 1967), metodę efektywnego modułu sprężystości betonu dostosowanego do wieku jego obciążenia (Bažant 1972) oraz metodę przyrostową według liniowej teorii sprężystości. Scharakteryzowano metody obliczeniowego ujęcia pełzania według aktualnych przepisów PN-EN, wycofanych norm polskich oraz w zaleceniach i literaturze zagranicznej. Zaprezentowano wpływ pełzania na redystrybucję sił wewnętrznych przy etapowym wznoszeniu ustroju na przykładzie wiaduktu z belek prefabrykowanych i mostu budowanego metodą betonowania nawisowego. Zwrócono uwagę na możliwości rozszerzenia opisu zagadnień pełzania w betonowych konstrukcjach mostowych przy użyciu aparatu pojęciowego pochodnych ułamkowego rzędu.

Received: 2024-05-27, Revised: 2024-08-13