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VERIFICATION OF ORTHOTROPIC MODEL OF WOOD

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The paper is dedicated to the discussion of elastic coefficients of wood. Parameters for wood presented in the literature are critically evaluated and discussed. The orthotropic mathematical model, with nine different elastic parameters, is one of the most often used models of wood. However, mathematical limitations on these parameters for the correct model are not well known. Based on these limitations, the verification of orthotropic elastic parameters for different species of wood is presented. The analysis shows that the published data are often unclear and sometimes wrong. The attempt to relate experimental results to the mean values specified in the standards is the second aspect considered in this paper. The designer, a user of these standards, should have clear information that the given parameters are specified for specific mathematical model and species of wood. This paper attempts to propose such a classification.

Keywords: *wood, orthotropic model, elastic coefficients, limitations*

1. Introduction

Wood and wooden products are very popular materials used in construction engineering for buildings, bridges, towers and other structures. Wood is a complex, inhomogeneous and anisotropic material, but in engineering practice wood is often idealized as a homogeneous, orthotropic material [2,13,14,26,40,44,45]. Based on the orthotropic model, three principal orthogonal directions of elasticity for wood, i.e., longitudinal (L), tangential (T) and radial (R) are determined. In some papers [11,24,37,38], mostly in numerical simulation, nine independent elastic constants such as three elastic moduli (E_L, E_T, E_R), three transverse elastic moduli (G_{LT}, G_{TR}, G_{RL}) and three Poisson's ratios (ν_{TR}, ν_{TL}, ν_{RL}) are defined. In experimental studies [10,12,17,23,27,29,31,33,39] twelve elastic constants, with six different Poisson's ratios (ν_{TR}, ν_{TL}, ν_{RL}, ν_{RT}, ν_{LT}, ν_{LR}), are determined. According to the

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orthotropic model, these parameters should satisfy the condition of the symmetry thus nine of these constants are independent. This paper demonstrates that the elastic matrix symmetry condition is not always satisfied. In same papers the orthotropic model of wood is expanded to the model that assumes identical properties in the radial and tangential directions. For this case, six elastic constants are defined [8,19]. In paper [28] authors described wood using a model with four elastic constants. Unfortunately, not all parameters of wood are always stated in the papers. For example, in [4,7,32] there are three elastic moduli and six Poisson's ratios but in [18] – only three elastic moduli and three shear moduli. In none of these cases it is possible to verify the model of wood.

This article attempts to verify the orthotropic model of wood on the basis of the research papers above and wood handbooks [15,22] contents. A general scheme of criteria is offered for checking whether the material constants satisfied the condition of the symmetry of compliance matrix. Over a dozen species of wood are analysed. The second purpose of this paper is to classify the wood species analysed according to European code [34,35].

2. Materials and methods

The present paper focuses on the orthotropic model of wood. The considerations are carried out in the Cartesian rectangular coordinate system x_1 x_2, x_3, coincident with the *L, T, R* local system. The constitutive equations for linear elastic materials [1,5,6,9,27] are expressed as:

$$S_{ij} = D_{ijkl}E_{kl}; \;\; i,j,k,l = 1,2,3, \tag{2.1}$$

where:

S_{ij} – component of the symmetric stress tensor, E_{kl} – component of the strain tensor, D_{ijkl} – component of the fourth-order elasticity tensor.

In engineering practice, the convenient two-index notation, referred to as Voigt notation, is used. Then the equation (2.1) takes the form:

$$s_i = d_{ij}e_j \;\text{ or }\; e_i = c_{ij}s_j; \;\; i,j = 1,2,\ldots,6, \tag{2.2}$$

where:

s_i – single column stress matrix, e_j – single column strain matrix, d_{ij} – stiffness matrix and c_{ij} – compliance matrix.

In general, a stiffness matrix and a compliance matrix contain 36 independent components, but taking into account the symmetry of strain energy, 21 distinct components can be set out. This is the most general case of a linear elastic material. The number of independent components of these matrices is further reduced if the material has symmetry planes. There are exactly eight different sets of symmetry planes [5,30,42,43].

The purpose of this paper is to discuss the linear elastic orthotropic model along with the elastic coefficients. For this model the compliance matrix c_{ij} has the form:

(2.3)
$$c_{ij} = \mathbf{c} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix},$$

where:

E_i – Young's modulus, G_{ij} – Kirchhoff's modulus and ν_{ij} – Poisson's ratio defined as the ratio of passive strain component perpendicular to the load ε_j and the active strain component parallel to the load direction ε_i:

(2.4)
$$\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i}; i,j = 1,2,3 \text{ and } i \neq j.$$

There are twelve material constants in (2.3) but only nine of them are independent because, due to existing symmetry, we can write:

(2.5)
$$\nu_{ij} = \nu_{ji}\frac{E_i}{E_j}.$$

In most papers, Poisson's ratio is calculated according to (2.4) but some researchers [2,12,17,20,25,29,31-33] determine Poisson's ratio from $\nu_{ij} = -\varepsilon_i/\varepsilon_j$. Then the compliance matrix is different from (2.3) and equation (2.5) takes the form:

(2.6)
$$\nu_{ij} = \nu_{ji}\frac{E_j}{E_i}.$$

2.1. LIMITATIONS OF TECHNICAL CONSTANTS

Due to positive-definiteness of the strain energy, the compliance matrix (2.3) must be positive-definite [1,3,5,6,9,25,30,41-43,46]. The matrix is positive-definite if all of the leading principal minors are positive (Sylvester's criterion). This leads to subsequent, known from the literature [25,27,36,46] conditions:

$$E_1 > 0, E_2 > 0,\ E_3 > 0, G_{23} > 0, G_{13} > 0, G_{12} > 0,$$

$$1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13} > 0, \tag{2.7}$$

$$\nu_{12}\nu_{21} < 1,\ \nu_{23}\nu_{32} < 1, \nu_{13}\nu_{31} < 1.$$

Another original way to define limitations of elastic constants is to determine eigenvalues λ of the compliance matrix:

$$det(\mathbf{c} - \lambda\mathbf{I}) = 0. \tag{2.8}$$

The compliance matrix is positive-definite if all eigenvalues satisfy the condition:

$$\lambda_i > 0;\ (i = 1,2, \dots ,6). \tag{2.9}$$

Obtaining positive eigenvalues is equivalent to the fulfilment of conditions (2.7). In the case of orthotropic material, it is impossible to provide a solution to equation (2.8), but it is possible to study the eigenvalues for the data adopted. For an isotropic material described by Young's modulus E and Poisson's ratio ν the solutions of (2.8) are the following:

$$\lambda_1 = \frac{1-2\nu}{E}, \lambda_{2,3} = \frac{1+\nu}{E}, \lambda_{4,5,6} = \frac{2(1+\nu)}{E}. \tag{2.10}$$

The condition (2.9) leads to known from the literature [3,27,46] dependences:

$$E > 0 \text{ and } \nu \in (-1.0; 0.5). \tag{2.11}$$

The constraint on Poisson's ratio $(2.11)_2$ is true only for isotropic materials. For orthotropic materials this range is much wider. Some researchers [3,20,25,36,42,43,46] focused only on determining the values of Poisson's ratios.

The limitations described above as well as the symmetry condition are the basis for the discussion and verification of the correctness of wood elastic coefficients reported in the literature.

2.2. Technical constants of wood in standards

A wooden structure is designed according to standards [34,35]. Wood is allocated into fifteen strength classes: nine for poplar and coniferous species and six for deciduous species. Strength properties such as bending, tension along and across the fibre, and compression along and across the fibre are defined for each class. Also, elastic properties such as the longitudinal elastic modulus $E_1 = E_{0,mean}$, the tangential elastic modulus $E_2 = E_{90,mean}$, and the mean shear modulus $G = G_{mean}$, are determined. The standards do not include information about Poisson's ratios and the value of mean shear modulus is not clear as it cannot be interpreted as the mean in terms of the three planes. It should be emphasized that these parameters do not apply in the case of the isotropic description due to the absence of interrelation among the elastic modulus, the shear modulus and Poisson's ratio. The standard does not specify that mean values are specified for orthotropic description. Further considerations assume that this is so. The relationship between the parameters laid down in the standard and the results reported in the literature for orthotropic models remains an open question.

3. Results and discussion

3.1. Orthotropic constants for wood

This study examines orthotropic conditions for different species of wood and verifies the wood constants given in the reference material. When all elastic constants are given [7,10,11,12,15,16,21,22,23,29,31,33,39] it is sufficient see whether condition (2.9) is met (Table 1, Table 2) because if eigenvalues are positive, the compliance matrix is positive-definite. For incomplete data [4,15,22] the conditions (2.7) are reviewed (Table 3).

This study analyses six species of softwoods and eleven species of hardwoods. The softwoods include Spruce (*Picea*), Sitka Spurce (*Picea sitchensis*), Engelman Spruce (*Picea engelmannii*), Norway Spruce (*Picea abies* [*L*]. *Karst*), Pine (*Pinus sylvestris L*.) and Western Larch (*Larix occidentalis*). The hardwoods include Maple (*Acer platanoides L.*), Sugar Maple (*Acer saccharum*), Yellow Birch (*Betula alleghaniensis*), Oak (*Quercus*), Common Oak (*Quercus robur L.*), White Oak (*Quercus alba*), Red Oak (*Quercus rubra*), Beech (*Fagus*), European Beech (*Fagus sylvatica L.),* Black Walnut (*Juglans nigra*) and White Ash (*Fraxinus americana*).

The eigenvalues of the compliance matrix for the wood species are positive (Table 1, Table 2) and limitations of technical constants (2.7) are satisfied (Table 3). However some wood specimens cannot be referred to as orthotropic as in some cases the compliance matrix is non-symmetric. Black Walnut [15,22], Oak [4] and Beech [4] were found to show orthotropic behavior. If the accuracy of calculation terms of the compliance matrices are reduced, Spruce [4] and Maple [11] can also be assumed to be orthotropic. The elastic constants for Pine in [21] were calculated using the orthotropic material model, assuming the symmetry of the compliance matrix. Orthotropic of the Pine is confirmed by experiments discussed in [13,14]. However, in the case of pine analyzed in [4], in which ultrasonic measurements of wood were carried out, the compliance matrix is non-symmetric (Table 3). Additionally, Poisson's ratio $\nu_{12} = 1.46$ is more than 3.7 times as high as and Poisson's ratio ν_{32}=0.016 being almost 29 times less than that given in [21] (Table 1).

Table 1. Off-diagonal terms and eigenvalues of the compliance matrix (2.3) for species of softwood

	data found in the literature				**calculations in this paper**			
	E_1	G_{12}	ν_{12}	ν_{21}	$-c_{12}$	$-c_{21}$	λ_1	λ_4
	E_2	G_{13}	ν_{13}	ν_{31}	$-c_{13}$	$-c_{31}$	λ_2	λ_5
	E_3	G_{23}	ν_{23}	ν_{32}	$-c_{23}$	$-c_{32}$	λ_3	λ_6
	[MPa]		[-]		[10^{-12}Pa^{-1}]		[10^{-6}Pa^{-1}]	
Pine [21]	6919	262	0.388	0.015	56	56	0.00014	0.00382
	271	354	0.375	0.024	54	54	0.00170	0.00423
	450	34	0.278	0.462	1027	1027	0.00282	0.02941
Sitka Spruce [15,22]	11880	725	0.467	0.025	49	39	0.00008	0.00138
	511	760	0.372	0.040	43	31	0.00087	0.00216
	927	36	0.245	0.435	469	480	0.00013	0.02778
Sitka Spruce [16]	10820	660	0.470	0.020	43	43	0.00009	0.00151
	470	690	0.370	0.040	48	34	0.00096	0.00236
	840	30	0.240	0.440	524	511	0.00145	0.03333
Engelman Spruce [15,22]	9790	1175	0.462	0.058	100	47	0.00009	0.00085
	578	1214	0.422	0.083	66	43	0.00064	0.00190
	1253	98	0.255	0.530	423	441	0.00082	0.01020
Engelman Spruce [16]	9800	1180	0.460	0.060	103	47	0.00009	0.00085
	580	1220	0.420	0.080	64	43	0.00064	0.00189
	1250	90	0.260	0.530	424	448	0.00082	0.01111
Norway Spruce [17]	12800	587	0.450	0.014	35	35	0.00008	0.00170
	397	617	0.360	0.018	29	28	0.00127	0.00295
	625	53	0.210	0.480	768	529	0.00162	0.01887

Table 2. Off-diagonal terms and eigenvalues of the compliance matrix (2.3) for species of hardwood

	data found in the literature				calculations in this paper			
	E_1	G_{12}	ν_{12}	ν_{21}	$-c_{12}$	$-c_{21}$	λ_1	λ_4
	E_2	G_{13}	ν_{13}	ν_{31}	$-c_{13}$	$-c_{31}$	λ_2	λ_5
	E_3	G_{23}	ν_{23}	ν_{32}	$-c_{23}$	$-c_{32}$	λ_3	λ_6
	[MPa]		[-]		$[10^{-12}\ Pa^{-1}]$		$[10^{-6}\ Pa^{-1}]$	
Maple [11]	13810	753	0.500	0.025	37	36	0.00006	0.00133
	678	1013	0.460	0.044	34	33	0.00041	0.00184
	1311	255	0.420	0.820	625	619	0.00099	0.00392
Yellow Birch [15,22]	13900	945	0.451	0.024	35	32	0.00007	0.00106
	695	1029	0.426	0.043	40	31	0.00051	0.00186
	1084	236	0.426	0.697	643	613	0.00097	0.00424
Yellow Birch [16]	13850	940	0.450	0.020	29	32	0.00007	0.00106
	690	1020	0.430	0.040	37	31	0.00050	0.00187
	1080	240	0.430	0.700	648	623	0.00098	0.00417
Common Oak [39]	12827	766	0.466	0.053	84	36	0.00007	0.00142
	633	703	0.478	0.045	33	37	0.00050	0.00183
	1344	337	0.371	0.621	462	586	0.00130	0.00297
European Beech 11.3% [29,31,32]	10560	930	0.580	0.040	55	55	0.00009	0.00107
	730	1240	0.430	0.040	26	41	0.00048	0.00156
	1510	380	0.310	0.610	404	425	0.00081	0.00263
European Beech 12.5% [12]	13900	855	0.240	0.090	149	17	0.00007	0.00117
	606	1280	0.270	0.070	37	19	0.00041	0.00177
	1900	486	0.270	0.640	337	446	0.00078	0.00206
European Beech 12.7% [31,33]	9560	930	2.260	0.110	224	236	0.00006	0.00107
	490	1240	0.080	0.020	9	8	0.00034	0.00219
	2200	380	0.230	1.020	464	469	0.00081	0.00263
Air-dry Beech 10-15% [10]	16000	720	0.020	0.360	419	1	0.00006	0.00139
	860	1010	0.038	0.430	299	2	0.00041	0.00145
	1440	190	0.720	0.380	264	837	0.00100	0.00526
Air-dry Beech*	16000	720	0.360	0.020	23	23	0.00006	0.00139
	860	1010	0.430	0.038	26	27	0.00041	0.00145
	1440	190	0.380	0.720	500	442	0.00100	0.00526
Black Walnut [15,22]	11600	719	0.632	0.035	54	54	0.00008	0.00139
	650	986	0.495	0.052	42	43	0.00050	0.00186
	1230	244	0.378	0.718	584	582	0.00101	0.00410

Table 3. Off-diagonal terms of the compliance matrix (2.3) and conditions (2.7) for species of woods

	data found in the literature				**calculations in this paper**			
	E_1	G_{12}	ν_{12}	ν_{21}	$-c_{12}$	$-c_{21}$	$\nu_{12}\nu_{21} < 1$	
	E_2	G_{13}	ν_{13}	ν_{31}	$-c_{13}$	$-c_{31}$	$\nu_{13}\nu_{31} < 1$	Eq. (2.7)$_2$
	E_3	G_{23}	ν_{23}	ν_{32}	$-c_{23}$	$-c_{32}$	$\nu_{23}\nu_{32} < 1$	
	[MPa]		[-]		[10^{-12}Pa^{-1}]		[-]	
Pine [4]	6634	-	1.460	0.025	221	220	0.037	
	113	-	0.337	0.016	50	51	0.005	0.949
	320	-	0.535	0.016	50	4735	0.009	
Spruce [4]	5038	-	2.350	0.043	467	466	0.101	
	92	-	0.255	0.022	51	51	0.006	0.200
	433	-	0.377	1.738	4014	4098	0.655	
Western Larch [15,22]	12900	890	0.276	-	-	21	-	
	839	813	0.355	-	-	28	-	-
	1019	90	0.352	0.389	382	420	0.137	
Sugar Maple [15,22]	10250	646	0.476	0.037	56	46	0.018	
	666	1138	0.424	0.065	48	41	0.028	0.660
	1353	-	0.349	0.774	572	524	0.270	
White Oak [15,22]	10600	-	0.428	0.036	47	40	0.015	
	763	912	0.369	0.074	43	35	0.027	0.755
	1728	-	0.300	0.618	358	393	0.185	
Red Oak [15,22]	13000	1053	0.448	0.033	31	34	0.015	
	1066	1157	0.350	0.064	32	27	0.022	0.786
	2002	-	0.292	0.560	280	274	0.164	
Oak [4]	4745	-	0.750	0.177	158	158	0.133	
	1122	-	0.823	0.242	173	173	0.199	0.510
	1399	-	0.239	0.298	213	213	0.071	
Beech [4]	9160	-	0.900	0.102	98	98	0.092	
	1037	-	1.244	0.251	136	136	0.312	0.492
	1851	-	0.146	0.261	141	141	0.038	
White Ash [15,22]	10700	824	0.440	0.051	60	41	0.022	
	856	1166	0.371	0.059	44	35	0.022	0.684
	1338	-	0.360	0.684	511	421	0.246	

In [14] is describing orthotropic elastic behavior of Larch but the analysis in this paper (Table 3) is impossible since only some of the elastic parameters are known. The elastic constants for Spruce reported in [4] almost satisfy the orthotropic conditions (Table 3) but data given in [15-17,22] fail to do so (Table 1). The elastic constants for five species of spruce shown are comparable in Table 1 but different in Table 3. The value of longitudinal Young's modulus $E_1 = 5038$ MPa is ½ the value of the mean in Table 1 ($E_{1,mean} = 11018$ MPa); the tangential Young's moduli $E_2 = 92$ MPa – 1/5 the mean value ($E_{2,mean} = 507$ MPa); the radial Young's moduli $E_3 = 433$ MPa – less than ½ the mean ($E_{3,mean} = 979$ MPa). In case of Poisson's ratio, the largest differences are for ν_{12} and for ν_{32}. The mean values in Table 1 are the following: $\nu_{12,mean} = 0.462$, $\nu_{32,mean} = 0.483$ and the corresponding values in Table 3 are: $\nu_{12} = 2.35$ (five times as big) and $\nu_{32} = 1.738$ (over three times as big).

Similar results are obtained for Beech wood. The elastic constants for beech given in [4] satisfy the orthotropic conditions (Table 3), unlike the data given in [12,29,31-33] (Table 2). Five papers, written by same research team [12,29,31-33] show the results for different moisture contents 11.3%, 12.5% and 12.7%. The results vary because elastic parameters are moisture content-dependent. However, it is hard to understand why at the 12.7% moisture content Poisson's ratios for beech ν_{12} and ν_{32} are respectively over nine and almost twice as high as for 12.5% moisture content; whereas ν_{13} for 12.5% is three times as high as for 12.7% moisture content.

Table 2 shows elastic parameters for the dry beech 10-15% given in [10]. The data for Poisson's ratios differ by the order of magnitude given in [12,29,31,32] and there are huge differences in the compliance matrix. In [10] the relationship between elastic constants expressed by Eq. (2.5) but parameters seem to satisfy condition (2.6) (Table 2: results for air-dry beech*).

3.2. REFERENCE TO VALUES IN STANDARDS

Table 4. Strength classes of wood [34]

	Poplar and coniferous species									**Deciduous species**					
	C14	C16	C18	C22	C24	C27	C30	C35	C40	D30	D35	D40	D50	D60	D70
E_1 [GPa]	7	8	9	10	11	12	12	13	14	10	10	11	14	17	20
E_2 [GPa]	0.23	0.27	0.30	0.33	0.37	0.40	0.40	0.43	0.47	0.64	0.69	0.75	0.93	1.13	1.33
G [GPa]	0.44	0.50	0.56	0.63	0.69	0.75	0.75	0.81	0.88	0.60	0.65	0.70	0.88	1.06	1.25

The values of longitudinal and tangential moduli of elasticity are the basis for determining the class of wood. The mean elastic parameters for different species of wood, according to standard [34], is showed in Table 4. Classification, however, is neither simple nor clear. As shown in Table 5 species of wood were assigned to particular classes based on mean values of elastic moduli E_1 and E_2. The classes obtained with E_1 vary from those obtained with E_2. Only for White Oak the class is the same

– C14. For three species of wood is impossible determining the class of wood – the moduli of elasticity are smaller (Spruce[2]) or larger (Spotted Gum, Tallowwood) then those given in the classification. Also, it not clear how the shear modulus specified in the standard should be understood if for orthotropic materials as many as three moduli are given.

Table 5. Values of longitudinal E_1, tangential E_2 moduli of elasticity for wood and class of wood

		E_1 [MPa]	$E_{1,mean}$ [MPa]	Class	E_2 [MPa]	$E_{2,mean}$ [MPa]	Class
Softwoods	**Spruce**[1]	8900-11400	10150	**C24**	650-832	741	**>C40**
	Spruce[2]	-	5038	**<C14**	-	92	**<C14**
	Pine[1]	8200-13700	10950	**C24**	312-521	416	**C27,C30**
	Larch[1]	12900	12900	**C35**	839	839	**>C40**
	Yew[3]	10500	10500	**C22-C24**	627	627	**>C40**
Hardwoods	**Maple**[1]	7900-12600	10250	**D30-D35**	514-819	666	**D30-D35**
	Birch[1]	11000-15000	13000	**D40-D50**	550-750	650	**D30**
	White Oak[1]	7100-14100	10600	**D40**	511-1015	763	**D40**
	Red Oak[1]	10300-15700	13000	**D40-D50**	742-1130	1066	**D60**
	Beech[4]	9560-13900	11775	**D40**	490-606	548	**<D30**
	Spotted Gum[5]	24467-26512	26147	**>D70**	1457-1540	1499	**>D70**
	Tallowwood[5]	20720-21548	21078	**>D70**	1317-1535	1426	**>D70**
	Poplar[1]	10900	10900	**C24**	469	469	**C40**
	Walnut[1]	11600	11600	**D40**	650	650	**D30**
	Ash[1]	9400-12000	10700	**D40**	752-960	856	**D40-D50**

[1] [22], [2] [4], [3] [17], [4] [12,33], [5] [7]

3. Conclusions

Review of the literature concerning experimental studies of elastic coefficients of wood naturally raises the question of the correctness of the results reported and of the design standard-compliant allocation of a given wood species to appropriate class.

Selected literature data were analyzed with constraints on elastic constants of a linear elasticity orthotropic model as correctness criterion. The results reported are often ambiguous or incorrect with regard to the model adopted. As different methods are used, the comparison of the results is difficult and may lead to the conclusion that the material being examined is not orthotropic.

The findings of this study indicate that experimental research should be complemented by in-depth theoretical investigations, with special reference to the constraints the results have to meet if they are to be assigned to a certain mathematical model.

Attempts to allocate wood species under this analysis to design classes do not bring conclusive results. The author of this paper believes that three mean values (two elasticity moduli and one shear modulus) are insufficient for a careful reader to be able to verify the results of the experimental work.

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Tab. 5. Wartości podłużnych E_1 i stycznych E_2 modułów sprężystości dla drewna oraz klasyfikacja drewna

WERYFIKACJA ORTOTROPOWEGO MODELU DREWNA

Słowa kluczowe: drewno, model ortotropowy, współczynniki sprężystości, ograniczenia

STRESZCZENIE:

Do opisu drewna i materiałów drewnopochodnych najczęściej przyjmowany jest liniowo sprężysty model ortotropowy. W literaturze znajdziemy wiele informacji na ten temat. Nie zawsze jednak informacje te są spójne i zgodne z zasadami teoretycznymi. Dane dostępne w literaturze są często błędne lub niejasne – dlatego sprawdzanie warunków ograniczających na stałe techniczne nabiera szczególnej wartości i pozwala określić ich przydatność.

W niniejszej pracy podjęto próbę uporządkowania rozważań o ortotropowym modelu drewna i podjęto próbę weryfikacji, czy podawane w literaturze stałe materiałowe spełniają warunki ograniczeń dla stałych technicznych materiału ortotropowego. Oryginalnym sposobem zdefiniowania ograniczeń na stałe techniczne (wynikających z dodatniej określoności macierzy podatności), zaproponowanym w pracy, jest wyznaczenie wartości własnych macierzy podatności. Jeżeli wszystkie wartości własne są dodatnio określone, to macierz podatności jest dodatnio określona. W przypadku materiału ortotropowego nie udaje się podać rozwiązania zagadnienia własnego macierzy podatności w postaci ogólnej, ale można badać wartości własne dla konkretnych danych. Oprócz sprawdzenia ograniczeń na stałe techniczne istotne jest również sprawdzenie, czy otrzymane z badań eksperymentalnych stałe spełniają warunek symetrii macierzy podatności.

W artykule podjęto próbę weryfikacji ortotropowego modelu drewna na podstawie danych literaturowych. Przeanalizowano sześć gatunków drewna iglastego i jedenaście gatunków drewna liściastego (Tabela 1, Tabela 2, Tabela 3). Analiza piśmiennictwa dotyczącego eksperymentalnych badań sprężystych współczynników drewna podnosi kwestię poprawności otrzymywanych wyników. Ponieważ w badaniach stosowane są różne metody, porównanie jest trudne i może prowadzić do wniosku, że badany materiał nie jest ortotropowy. Wyniki analiz otrzymane w tej pracy wskazują, że badania eksperymentalne powinny być uzupełnione dogłębnymi badaniami teoretycznymi, ze szczególnym uwzględnieniem ograniczeń, jakie stałe materiałowe muszą spełnić, jeśli mają zostać przypisane do określonego modelu matematycznego.

Innym dość istotnym problemem jest wykorzystanie stałych materiałowych podanych w literaturze do przypisania, zgodnego z normami projektowania, danego gatunku drewna do odpowiedniej klasy. Wartości podawane w normach dotyczą uśrednionych wartości potrzebnych do projektowania – odniesienie tych wartości do opisu ortotropowego, raportowanego w literaturze, nie jest jednoznaczne i wymaga krytycznego spojrzenia. Projektując konstrukcje drewniane istotne są klasy wytrzymałości i wynikające z nich wartości wytrzymałości na zginanie, rozciąganie i ściskanie. Dla litego drewna konstrukcyjnego określone jest 15 klas wytrzymałości: dla topoli i gatunków iglastych – 9 klas, a dla gatunków liściastych – 6 klas. Dla każdej klasy podano wytrzymałości charakterystyczne na zginanie oraz na rozciąganie i ściskanie wzdłuż i w poprzek włókien. Z właściwości sprężystych podane są wartości średniego podłużnego modułu sprężystości, średniego poprzecznego modułu sprężystości oraz średniego modułu odkształcenia postaciowego (Tabela 4). W normie brak jest informacji o współczynnikach Poissona, a ponadto, nie jest jasne, jak należy rozumieć wartość średniego modułu

odkształcenia postaciowego, jeśli dla materiałów ortotropowych podano trzy moduły. Przy czym, należy podkreślić, że parametry te nie mają zastosowania w przypadku opisu izotropowego ze względu na brak współzależności między modułem sprężystości, modułem odkształcenia postaciowego i współczynnikiem Poissona. Z normy wynika, że na podstawie danych dotyczących średnich wartości podłużnego i poprzecznego modułu sprężystości należy określić klasę dla analizowanego gatunku drewna. Klasyfikacja ta jednak nie jest jednak prosta i jednoznaczna. W Tabeli 5 podano wartości modułów sprężystości i na tej podstawie próbowano dokonać klasyfikacji drewna. Próby przydzielenia gatunków drewna w ramach tej analizy do klas projektowych nie przyniosły rozstrzygających wyników. Tylko w przypadku białego dębu (White Oak) można jednoznacznie określić klasę drewna – C14. Natomiast w przypadku trzech gatunków drewna niemożliwe jest określenie klasy drewna – moduły sprężystości są mniejsze (Spruce[2]) lub większe (Spotted Gum, Tallowwood) niż te podane w klasyfikacji normowej. Dodatkowo należy zwrócić uwagę na to, że dla klas C27 i C30 podane są takie same wartości stałych sprężystych (Tabela 4), a klasy te różnią się właściwościami wytrzymałościowymi. A zatem pojawia się pytanie, czy na podstawie danych materiałowych jest w ogóle możliwa klasyfikacja drewna? Autor tego artykułu uważa, że trzy średnie wartości stałych materiałowych (dwa moduły sprężystości i jeden moduł odkształcenia postaciowego) są niewystarczające, aby uważny czytelnik (projektant) mógł zweryfikować wyniki pracy eksperymentalnej.