



## Research paper

# Modeling the problem of sequencing projects in the contractor's portfolio of orders

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**Abstract:** It is a usual practice for a contractor to deliver several projects at a time. Typically, the projects involve similar types of works and share the same pool of resources (i.e. construction crews). For this reason, the company's portfolio of orders considered for a particular planning horizon can be modeled as a project with repeatable processes to be performed in heterogeneous units located in a number of construction sites. Its scheduling requires determining the best sequence of the resources' moving from unit to unit while minding the due dates related with particular orders as well as resource continuity constraints. The authors present a model of this scheduling problem in the form of a mixed-integer linear program. The aim is to schedule a portfolio of projects in a way that minimizes the total of the resource idle time-related costs, the indirect costs, and the delay penalties. The model can be solved by means of a general-purpose solver. The model is applied to schedule a portfolio of multifamily housing projects.

**Keywords:** project scheduling, repetitive construction processes, idle time reduction, mathematical modeling, schedule optimization

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## 1. Introduction

The main activity of a construction contractor consists in erecting built facilities or performing works in particular construction sites. From the point of view of the manager, each order the contractor takes is considered a project characterized by a particular time for completion, a due date, a budget, and a price. The contractor's "production plans" comprise a number of projects that are independent in terms of technological relationships between processes. However, the delivery of these projects relies on a shared pool of resources (the plant and the crews) that determine the contractor's capacity. The analyses of in-house resources (skills, productivity, availability), their allocation to particular orders, and the need to reinforce them by subcontracting or by employing new workforce/ investing in new plant, are carried out at each level of enterprise and construction project management. The aim of the manager is to maximize in-house resource utilization rates, but the volatility of the construction market and the uncertainty of winning new jobs make it difficult to keep balance between the capacity and the actual workload [27]. To find the balance, some propose to improve bidding strategies to win more orders [18] or hiring out own resources.

Managing the resource allocation is one of the key issues of planning in construction enterprises. The goal of the construction contractor (construction enterprise) is to seek resource allocation scenarios of the lowest cost [21]. In order to rationally manage the enterprise's own resources, in complex business conditions, it is necessary to implement computer systems that support optimal managerial decisions [26]. At the stage of planning the execution of the company's orders, it is necessary to determine the number of resources to serve particular projects so that the contractual due dates can be met. These due dates are frequently enforced by clients with little consideration for the contractor's other commitments, and failing to meet due dates results in heavy penalties. In market conditions, there exist a gap between the contractor's production capacity and the demand for resources dictated by the portfolio of acquired orders. Nevertheless, it is necessary to strive to reduce the operating costs of the company and increase the degree of utilization of its own resources.

Judging by the literature on the subject, the research on construction enterprise resource optimization is concerned mainly with methods of scheduling isolated projects under pre-defined resource availability constraints. Designing the optimal (i.e. the shortest) schedule for a particular project and its successful implementation does not guarantee the efficiency of the enterprise as a whole [4], but it may determine the possibility of proceeding to the next order in a short time horizon.

The purpose of preparing production plans of construction enterprises should be the harmonization of works at all construction sites. They are the basis for determining the enterprise's demand for materials, machinery and equipment, employment, and funds. Production plans are usually prepared "per period" and adjusted by accounting for new orders. The production program should ensure continuity of work of resources together with timely completion of orders. In many cases, these objectives are conflicting. This is confirmed by the research of Tomczak and Jaśkowski [30] as well as Vanhoucke [31].

Optimization of production plans of construction enterprises involves the three following issues:

1. Sequencing projects and processes – determining the order in which the renewable resources organized into crews should be directed to particular construction sites. For example Fan et al. refer to the ability to reorder tasks as soft logic [5]. In the paper [6] they developed heuristic algorithm to identify the logic sequence of activities, whereas in the article [7] presented a computer system to support processes sequencing which provides an easy input module in addition to scheduling and work-continuity-maintenance modules. Huang et al [16] developed a genetic algorithm-based scheduling method considering soft logic. Podolski [22] presented a model of scheduling and sequencing units based on an NP-hard permutation flow shop problem to minimize project cost with duration constraint. The author used metaheuristic simulated annealing algorithm to solve the problem. Similarly, Rogalska et al. [24] for sequencing activities and time-cost trade-off problem used metaheuristic approach.
2. Simultaneous execution of similar works under availability of several crews able to deliver these works in different units. For example Gounda et al. [9] developed a model to optimize crews routing among various activities considering the allocation of multitasking skilled crews. Huang and Sun [12] proposed a prototype system for the planning and scheduling of processes considering the usage of various crews in an activity groups. Mathematical optimization model of this problem was presented by Jaśkowski i Biruk [14].
3. Defining process execution dates considering rational utilization of resources, reduction of costs and/or project completion times. For example Altuwaim and El-Rayes [2] developed a optimization model for the scheduling of repetitive processes to minimize project duration, crew work interruptions, and interruption costs. Hegazy and Kamarah [10] presented scheduling and cost optimization model considering crew synchronization for prespecified execution deadline. Similar to Zou et al. [32] they allowed processes interruption. Bakry et al [3] proposed model to select among different acceleration strategies for optimized time–cost trade-off.

As can be seen from the examples above, the same questions are encountered both in production planning and in scheduling repetitive projects. Therefore, the problem of creating a construction enterprise's production plan can be approached as scheduling a repetitive project that involves erection of a fixed number of built facilities within a predefined planning horizon.

The highest levels of harmonization would be achieved if the production plan involves projects very similar in terms of building size, type, and construction methods (Repetitive Projects). In this case, labor intensity of works of the same type in particular sites is the same, and it is possible to set crew compositions in such a way that execution times of processes in all the units are the same (typical repetitive activities); this makes possible to minimize the execution time of the whole task and achieve resource continuity.

A number of scheduling techniques have been developed for repetitive projects whose units differ in size (repetitive nonlinear projects), and where process durations in units differ

(atypical repetitive activities) [12]. It is usually assumed that the sequence of processes to be executed in each unit is the same. However, in the construction practice the sequence of works in units needs not be constant [13], especially if the units are structurally unrelated (e.g. separate buildings).

With repetitive non-linear projects, the order of execution of buildings/units affects the execution time of the entire production plan and the downtime of renewable resources [5]. The problem of determining the optimal sequence of orders for the production schedules in the machining industry is known as the Permutation Flow Shop scheduling, used for example to minimize makespan [15] or only to ensure that the due date is not exceeded [25]. Its adaptation to the construction environment resulted in a number of algorithms to find the best permutation, such as those based on the Branch and Bound (B&B) algorithmic framework [11], heuristic, and matheuristic algorithms [28]. The no-wait flow shop scheduling problem with the makespan objective is a special case of the Asymmetric Traveling Salesman Problem (ATSP) [19]. It is thus possible to use many algorithms dedicated to this problem, and use general-purpose optimization tools [20].

Tomczak [29] presented a mathematical model of selecting resources for construction project that involved erection of multiple buildings, and determining the optimal order buildings. The model based on a multicriteria objective function to account for the need to minimize the project duration, to minimize the downtime of crews, and to improve continuity of work in units (buildings).

The problem of multi-mode execution (selection of crews of different composition) combined with simultaneous determination of the optimal order of execution of buildings (units) was presented by Sroka et al. [28]. The model takes into account the monthly cash flows, contractual delay penalties, the cost of crew downtime, and borrowing losses. As introducing the possibility to choose from a number of process modes to the classic Permutation Flow Shop Problem significantly increases the solution space, the authors employed a combination of two simulated annealing algorithms, and SA algorithm supported by Genetic Search Algorithm, to find the optimal solution.

The concept of soft logic relations (non-fixed work sequences) between the same type of construction processes conducted in separate units improves flexibility of the plans. Soft logic allows changing the sequence of the sub-activities of an activity by adjusting the logic relations between units [33]. The activities can be conducted in sequence, at the same time (with the number of simultaneously executed processes limited by resource availability), or in a partial overlap. Allowing for such possibilities helps reduce the duration of the entire project [6], minimize the total cost of a repetitive project [5], and improve work continuity [7].

In many cases, the contractor disposes of a number of similarly specialized crews and can subcontract work. The resource allocation problem can then be modeled as a repetitive project scheduling problem with execution modes. The modes represent crews of different composition, and equipment, which results in different duration and/or cost of the same processes. With these assumptions, the optimization problem may consist in minimizing the project makespan, tardiness [8], minimizing the bid price, or maximizing the net present value [1]. With the possibility to employ multi-skilled crews, it is possible to further improve the continuity of works and shorten project durations [17].

If the works in individual buildings or units can be delivered independently, and if the contractor has a larger number of crews of the same type with appropriate equipment, the works can be scheduled simultaneously in several units. Naturally, this concurrent delivery of units must follow the Finish-Start relations resulting from construction methods [9]. Huang and Sun [12] proposed a workgroup-based approach for scheduling this type of projects. The project should be divided not into repetitive units, but into workgroups consisting of processes that can be executed by crews of the same type (belonging to a resource group). The durations and costs of the processes allotted to a workgroup depend on the selection of the crew (as crews of the same group differ in productivity. Activities in a workgroup can be performed in an arbitrary order as specified by the planner. Relations between groups result from technological dependencies between the construction processes. This allows simultaneous execution of processes from a group subject to availability of resources from the corresponding resource group. The authors [13] developed a genetic algorithm-based optimization model to support the allocation of resources (work crews) to maintain the continuity of their work. A mathematical model of this problem was presented by Jaśkowski and Biruk [14].

Zou et al. [33] analyzed the problem of scheduling repetitive projects with a multi-crew execution option. Employing multiple crews to execute sub-activities allows for simultaneous execution of work of the same type in several units. The authors developed a multi-objective mixed-integer linear programming model with three different objectives: minimizing the project completion time, total interruption time and total project cost. The model takes into account total mobilization and demobilization cost and time as well as crew movement cost. The logic relation between units is modeled by a logic matrix that sets priorities for executing sub-process. Their values are set arbitrarily, ignoring the possibility of improving the value of the objective function by changing the order of execution of processes and independent units.

In literature there are few works devoted to optimization of production scheduling in the scale of the entire construction company. Therefore, in this paper an attempt was made to develop a model supporting scheduling of a portfolio of orders, including simultaneous execution of many construction structures.

Specific criteria affecting the efficiency of the company's activity were taken into account. In contrast to the realization of individual facilities, in the scale of the whole enterprise it is not necessarily important to strive for minimizing the individual project duration, and even paradoxically it can lead to interruption in the work of crews and increase the cost of their employment. Particularly important from the economic point of view is the reduction of operating costs, including the labor cost and resources usage cost.

In the article, the production plan of construction company is modeled as a set of processes performed on facilities ordered for execution by the company. It takes into account the heterogeneous nature of structures, which may differ in size and labor intensity of works.

It was assumed that the company has crews that can be allocated to carry out the same processes simultaneously but on different sites. It should be noted that in the methods of scheduling individual repetitive projects, as a rule, the limitation of the lack of parallelism

of the execution of processes of the same type is adopted and sometimes it is allowed to change the order of units execution is allowed, while the order is the same for all crews.

The execution of a certain portfolio of orders of the enterprise over a predefined planning horizon involves multiple crews employed by the enterprise. Therefore, the model presented in this paper draws from the approach proposed by Zou et al. [33], seeking to determine the optimal sequence of order execution and taking into account specific conditions and execution requirements.

## 2. Assumptions for the project portfolio scheduling model

To complete the portfolio of projects contracted for an assumed planning horizon, the contractor is to deliver  $n$  types of construction processes repeated in  $m$  units (the units represent orders, so buildings). Process  $i$  delivered in unit  $j$  is further referred to as a sub-process  $(i, j)$ . Processes are assumed to be executed in sequence. This sequence is fixed and the same in all units. Therefore, in each unit  $j = 1, 2, \dots, m$ , a successive sub-process  $(i + 1, j)$  may start only as its predecessor  $(i, j)$  has been completed.

Each process  $i = 1, 2, \dots, n$  is assigned a set of specialized crews  $R_i$  able to deliver it. It is assumed that the sets of crews are disjoint (i.e. a crew cannot be assigned to more than one process). The crews represent renewable resources: workers skilled in particular trades equipped with necessary tools and machines. The crews of set  $R_i$  may differ in productivity, which translated into the duration of sub-processes entrusted to them: the duration of sub-process  $(i, j)$  if delivered by crew  $r \in R_i$  is  $t_{i,j,r}$ . Allocation of crews to sub-processes is modeled with binary variables  $x_{i,j,r} \in \{0, 1\}$  that assumes the value of 1 if sub-process  $(i, j)$  is intended for crew  $r$ , and 0 otherwise. It is possible that the same process runs in several units at the same time if its sub-processes are entrusted to separate crews.

Only one sub-process can be executed at a time in a unit. The crews assigned to their execution move from unit to unit in a certain order that is not defined in advance (it is going to be established in the course of analysis).

However, it is important to prevent the same crew from being allocated to sub-processes scheduled to run simultaneously in a number of units. Let us consider two sub-processes,  $(i, u)$  and  $(i, v)$ . If entrusted to the same crew  $r \in R_i$ , then  $x_{i,u,r}$  and  $x_{i,v,r}$  equal 1. They must not be scheduled for simultaneous execution. Therefore, another binary variable is applied,  $y_{i,u,v} \in \{0, 1\}$  ( $y_{i,u,u} = 1$ ), that equals 1 if sub-process  $(i, u)$  is to be completed prior to process  $(i, v)$ , and equals 0 otherwise.

To construct the schedule, the planner needs to determine sub-process dates. In the model, the dates are expressed by the consecutive number of the working day counted from the beginning of the planning horizon. The start dates are modeled in the form of variables  $s_{i,j}, i = 1, 2, \dots, n, j = 1, 2, \dots, m$ . Once the crews are allocated to sub-processes and thus the sub-process durations are known  $(d_{i,j})$ , the finish dates  $f_{i,j}, i = 1, 2, \dots, n, j = 1, 2, \dots, m$  can be easily calculated as the sums of start dates and durations.

The completion date of unit  $j$  (i.e. the contractor's order: a building or a whole contracted set of works in a particular site) is  $T_j$  and represents the date of completing the last sub-process in this unit. The model assumes that if this date proves later than the contractual due date  $T_j^D$ , the contractor is expected to pay delay penalties in the amount of  $P_j$  directly proportional to the scale of delay. The daily penalty rate may differ unit to unit and is modeled by  $P_j^d$ .

With repeatable project, scheduling longer project completion times helps improve resource continuity: the sub-process floats become greater and there is more flexibility in adjusting process start dates to the availability of resources. Resource downtime generates unproductive costs as well as results in the so called forgetting effect detrimental to productivity. The financial losses of downtime are: reduced wages for the workers (with performance-related pay), reduced contractor's throughput (with time-related pay), and cost moving resources to other construction sites. In the model, the total financial losses due to downtime are  $FL$ , and they are calculated according to the crew-specific daily rates  $FL_r^d$  and downtime duration. For each crew  $r \in R_i$ , the downtime is the difference between the crew's time of employment over the planning horizon,  $\Delta_{i,r}$ , and its productive time being the sum of durations of all sub-processes assigned to the crew.

While lavishly scheduled projects offer more flexibility for resource management, they become costly in terms of time-dependent indirect costs: the construction site equipment must be hired for a longer period, the salaries to the staff are paid longer etc. Financial costs related to the contractor's means tied up in the product in process add to the problem. In the model, the total indirect costs specific to unit (project)  $j$  are  $IC_j$ . They are calculated according to the project-specific daily rate  $IC_j^d$  and the actual project duration.

### 3. The scheduling model

The project portfolio scheduling process (resource assignment and setting process dates) is aimed at reducing the total of the time-dependent indirect costs, the resource downtime cost, and the delay penalties. The model parameters are as follows:

$n$  – the number of process types,

$m$  – the number of units,

$R_i$  – the set of crews available to complete process  $i$ ,

$t_{i,j,r}$  – the duration of sub-process  $(i, j)$  if delivered by crew  $r \in R_i$ ,

$T_j^D$  – the contractual due date of unit  $j$ ,

$P_j^d$  – the daily delay penalty rate for unit  $j$ ,

$FL_r^d$  – the daily rate of losses due to resource  $r$  downtime,

$IC_j^d$  – the daily rate of indirect costs for unit  $j$ .

The model is applied to define the optimal values of the following decision variables:

$x_{i,j,r}$  – a binary variable that assumes the value of 1 if sub-process  $(i, j)$  is entrusted to crew  $r$ , and equals 0 otherwise,

$y_{i,u,v}$  – a binary variable that assumes the value of 1 if sub-process  $(i, u)$  precedes sub-process  $(i, v)$  and equals 0 otherwise,

$s_{i,j}$  – the start date of sub-process  $(i, j)$ ,

$d_{i,j}$  – the duration of process  $(i, j)$ ,

$f_{i,j}$  – the finish date of sub-process  $(i, j)$ ,

$T_j$  – the completion date of unit  $j$ ,

$S_j$  – the start date of unit  $j$ ,

$P_j$  – the total amount of delay penalty for unit  $j$ ,

$FL$  – the total amount of losses due to resource downtime,

$\Delta_{i,r}$  – the total time of employment of crew  $r \in R_i$  for the execution of all orders within the planning horizon,

$IC_j$  – the total time-dependent indirect cost for unit  $j$ .

The problem was modeled as a mixed-integer (binary) linear program. The linear form of the objective and the constraint functions helps find optimal solutions in a relatively short time [23].

The basic criterion for the selection of optimal solutions is to minimize the financial losses associated with the costs of resource downtime, delay penalties and the indirect costs. The objective function is thus

$$(3.1) \quad \min C: C = FL + \sum_{j=1}^m P_j + \sum_{j=1}^m IC_j$$

The downtime losses are calculated per crew, as a product of the crew's total downtime and the daily downtime rate. The total cost of downtime over the whole planning horizon is

$$(3.2) \quad FL = \sum_{i=1}^n \sum_{j=1}^m \sum_{r \in R_i} (\Delta_{i,r} - t_{i,j,r} \cdot x_{i,j,r}) \cdot FL_r^d$$

The amount of contractual penalties is calculated as the product of the difference between the project completion time and the directive time and the daily penalty rate (Eq. (3.3)). It was assumed that early completion is neither penalized nor rewarded (Eq. (3.4)):

$$(3.3) \quad P_j \geq (T_j - T_j^D) \cdot P_j^d, \quad j = 1, 2, \dots, m$$

$$(3.4) \quad P_j \geq 0, \quad j = 1, 2, \dots, m$$

Considering the form of the objective function (minimization), the delay penalty is going to be set to the lowest possible level despite the fact that 3.3) is in fact an inequality.

The indirect costs stay in direct proportion to the actual duration of each project:

$$(3.5) \quad IC_j = (T_j - S_j) \cdot IC_j^d, \quad j = 1, 2, \dots, m$$

Each sub-process  $(i, j)$  must be assigned precisely one crew selected from the set of available crews qualified to deliver process  $i$ :

$$(3.6) \quad \sum_{r \in R_i} x_{i,j,r} = 1, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m$$



The duration of sub-process  $(i, j)$  delivered by the selected crew  $r \in R_i$  is:

$$(3.7) \quad d_{i,j} = \sum_{r \in R_i} t_{i,j,r} \cdot x_{i,j,r}, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m$$

The sub-processes start dates must be non-negative:

$$(3.8) \quad s_{i,j} \geq 0, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m$$

The sub-processes completion dates are calculated as the total of the start date and the duration (this means that the sub-processes are to run continuously):

$$(3.9) \quad f_{i,j} = s_{i,j} + d_{i,j}, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m$$

The completion of a unit means all its sub-processes have been completed, so on completion of the last sub-process:

$$(3.10) \quad T_j = f_{n,j}, \quad j = 1, 2, \dots, m$$

Similarly, the unit's start date is the start date of its first sub-process:

$$(3.11) \quad S_j = s_{1,j}, \quad j = 1, 2, \dots, m$$

The sub-processes in a unit must be conducted in a predefined order, one after another:

$$(3.12) \quad s_{i+1,j} \geq f_{i,j}, \quad i = 1, 2, \dots, n-1, \quad j = 1, 2, \dots, m$$

The dates for given process in individual units depend on which and how many crews have been assigned to carry it out. If one crew has been assigned to carry out several sub-processes, it has to carry them out in a certain order, and start the next sub-process not earlier than after the previous one has been finished. The relationship between the dates of process  $i$  in any two units is thus as follows:

$$(3.13) \quad f_{i,u} \leq s_{i,v} + M \cdot (1 - y_{i,u,v}) + M \cdot (2 - x_{i,u,r} - x_{i,v,r}), \\ i = 1, 2, \dots, n, \quad u, v = 1, 2, \dots, m, \quad u \neq v, \quad \forall r \in R_i$$

considering that

$$(3.14) \quad y_{i,v,u} = 1 - y_{i,u,v}, \quad i = 1, 2, \dots, n, \quad u, v = 1, 2, \dots, m, \quad u < v$$

where  $M$  is a sufficiently large number.

If a pair of sub-processes  $(i, u)$  and  $(i, v)$  is not entrusted to the same crew  $r$  ( $2 - x_{i,u,r} - x_{i,v,r} > 0$ ), then condition (3.13) is fulfilled, and the sub-processes can be conducted concurrently. If these sub-processes are to be delivered by the same crew ( $x_{i,u,r}, x_{i,v,r} = 1$ ), their dates depend on variables  $y_{i,u,v}$  and  $y_{i,v,u}$  that model their succession. If  $y_{i,u,v} = 1$ , then sub-process  $(i, v)$  may start after  $(i, u)$  has been completed, as comes from condition (3.13). In this case,  $y_{i,v,u} = 0$  according to condition (3.14), and (3.13) is

automatically met. In contrast, if  $y_{i,u,v} = 0$  then (3.13) is automatically met,  $y_{i,v,u} = 1$  following condition (3.14) and, as comes from (3.13), sub-process  $(i, u)$  is to succeed sub-process  $(i, v)$ .

The period of employment of crew  $r \in R_i$  for the execution of all orders is calculated as the maximum difference between the completion date and the start date of the sub-processes entrusted to it:

$$(3.15) \quad \Delta_{i,r} \geq f_{i,v} - s_{i,u} - M \cdot (1 - y_{i,u,v}) - M \cdot (2 - x_{i,u,r} - x_{i,v,r}), \\ i = 1, 2, \dots, n, \quad u, v = 1, 2, \dots, m, \quad \forall r \in R_i$$

$$(3.16) \quad \Delta_{i,r} \geq 0, \quad \forall r \in R_i, \quad i = 1, 2, \dots, n$$

$$(3.17) \quad x_{i,j,r} \in \{0, 1\}, \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m, \quad \forall R_i$$

$$(3.18) \quad y_{i,u,v} \in \{0, 1\}, \quad y_{i,u,u} = 1, \quad i = 1, 2, \dots, n, \quad u, v = 1, 2, \dots, m$$

The general-purpose solvers, such as LINGO, AIMMS, AMPL, CPLEX, Gurobi, can be applied to solve the model.

## 4. Example

Operation of the model is presented on the example: a notional contractor specialized in structural works acquired six contracts (i.e. units) for multifamily housing blocks to be delivered within one year. All these projects involve a similar scope of works: earthworks, monolithic substructure with external insulations, superstructure works including monolithic frames and flat roof, complete insulation of the envelope (roof covering and ETICS on external walls). The due dates for the completion of the orders and the data necessary for the calculation of indirect costs and contractual penalties are summarized in Table 1.

Table 1. Model parameters (input for the example)

Unit $j$	Unit due date $T_j^D$ [days]	Daily delay penalty rate $P_j^d$ , EUR/day	Daily rate of indirect costs $IC_j^d$ , EUR/day
1	150	9000	2000
2	200	10500	2200
3	160	10000	2100
4	200	12000	2400
5	210	11000	2200
6	240	14000	2500

The due dates correspond to the number of the working day when the projects need to be completed calculated from the beginning of the year. The scope of works in each project was divided into 4 processes. Table 2 presents the potential crews able to deliver the processes and duration of sub-processes if delivered by them.

Table 2. Sub-process durations  $t_{i,j,r}$  (in working days) according to unit and crew

Unit $j$	Earthworks and foundations		RC frame			Flat roof		Façade works	
	crew A	crew B	crew C	crew D	crew E	crew F	crew G	crew H	crew I
	$t_{1,j,1}$	$t_{1,j,2}$	$t_{2,j,1}$	$t_{2,j,2}$	$t_{2,j,3}$	$t_{3,j,1}$	$t_{3,j,2}$	$t_{4,j,1}$	$t_{4,j,2}$
1	15	12	60	58	59	10	12	10	11
2	22	20	72	70	71	16	18	20	22
3	20	19	70	68	68	15	16	15	16
4	28	26	75	72	73	18	19	25	27
5	25	24	80	78	79	20	22	20	22
6	35	32	90	86	88	22	24	30	34

For each crew, the daily rate of losses due to crew downtime was assumed 2500 EUR/day.

The model was solved using LINGO 15.0. Fig. 1 presents the schedule corresponding to the optimal solution.

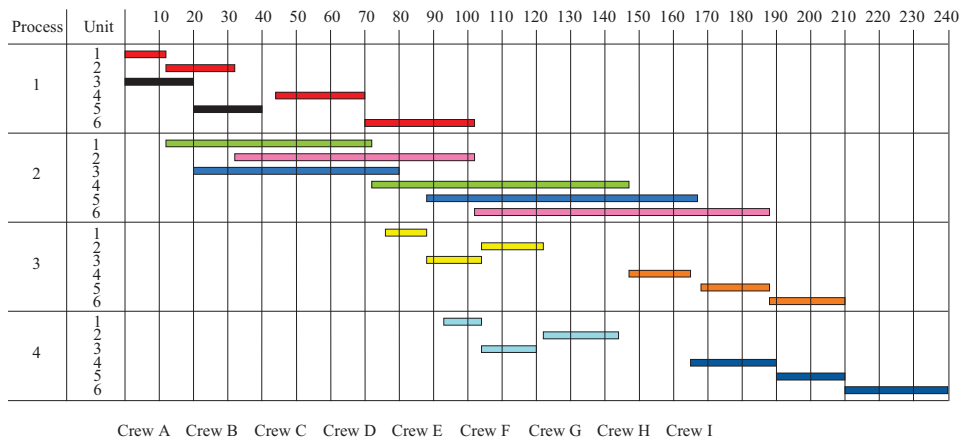


Fig. 1. Optimal schedule for 6 objects (example)

The minimum of the objective function, so the total of the costs of crew downtime, indirect costs, and delay penalties, is 1 986 300 EUR. With the optimal solution, no unit is completed later than on the due date, so there are no penalties to be paid. Units 1, 2, 3, and 4 will be delivered ahead of time (in particular, unit 1 is completed by day 104, unit 2 by day 144, unit 3 by day 120, and unit 4 by day 190). Early completion is considered welcome by the contractor if the payment can be claimed on actual completion (in such case, the contractor's capital tied up in the work in progress can be released). Regardless

of the payment conditions arranged with the clients, early completion means the resources can be engaged in new projects and generate income.

The downtime costs for the brigades amounted to EUR 42 500 (17 days of downtime, the downtime rates are equal for all crews). Most of the downtime is that of Crew B (12 days), whereas Crew F is scheduled only three unproductive days, and Crew I – two days. The remaining crews work continuously.

Unit 3 and 6 are processed without interruption, whereas the continuity of works in other units is not maintained. However, the interruptions in the flow of works are generally small, with the longest gap of 43 days between the foundations and the frame of Unit 5. Short interruptions may be considered advantageous as they may act as buffers to protect timely execution of processes in case of possible disturbances. Long interruptions, as in the case of Unit 5, significantly increase indirect costs. They can be eliminated by postponing the start date of the first process in this unit, but in this particular case, the continuity of work of Crew A would suffer; this solution would also be more costly. If such a long stoppage with Unit 5 was unacceptable from the point of the client, one could consider finding some other employment for Crew A, so directing them to another construction site during this period to carry out work commissioned by other contractors.

The solution prompts allocation of resources to tasks and defines the order of the resources' moving from unit to unit. Crew A execute Process 1 first in Unit 3 and moves to Unit 5, Crew B – moves successively between Units 1, 2 and 4. Process 2 is executed by Crew C first in Unit 1 and then in Unit 4, while Crew D serves Unit 2 and then Unit 6, while Crew E starts with Unit 3 and proceeds to Unit 5. These crews are scheduled to work simultaneously in different units. Process 3 is initially carried out by Crew G in Units 1, 3 and 2 and then, after a break of 25 days, by Crew F in Units 4, 5 and 6. Similarly, Process 4 is started by Crew I in Units 1, 3 and 2 and, after a break of 21 days, by Crew H, successively in units 4, 5 and 6.

## 5. Conclusions

Preparing production schedules from the perspective of a construction company is a complex issue. The literature typically discusses its selected aspects. Construction orders are usually acquired by competitive tendering. The outcome of the procedure is uncertain as affected by factors independent from the bidder. Therefore, the portfolio of orders can be determined in a deterministic way only in a short planning horizon. In the course of scheduling, the planner should strive to meet contractual deadlines and use resources as efficiently as possible – to reduce fixed costs and maximize the company's profits.

Optimizing resource utilization per project does not guarantee the optimal schedule for the enterprises' portfolio of project. Therefore, it is important to develop scheduling methods that help optimize the allocation of in-house labor and reduce the operating costs of the enterprise in the assumed time horizon, for which it is possible to determine (with an assumed level of certainty) the portfolio of orders.

The approach proposed in the papers facilitates reducing downtime, which is a source of financial losses. A mathematical model was developed to improve resource continuity as well as continuity of work related with the orders, the latter to curtail time-dependent indirect costs, as long as it does not cause a significant delay over contractual due dates and payment of delay penalties. The proposed selection of decision variables and mathematical formulation of constraints bring the problem down to the mixed-integer (binary) program possible to be solved using general-purpose solvers. Obviously, this feature holds for models of problems of low computational complexity. However, this offers the opportunity to develop a base of test instances and to verify the quality of dedicated algorithms created in the future.

Any methods of resource allocation, defining the production plan, or the production potential, account for the industry-specific conditions. In the case of construction, these are the need to transfer resources between the construction sites, or the possibility of subcontracting. In their further research, the authors intend to expand the model by allowing the in-house crews to be used as subcontractors to third party projects, and employing subcontractors to reinforce crews busy with projects within the company's portfolio to prevent exceeding due dates.

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## Modelowanie problemu kolejności realizacji zleceń przedsiębiorstwa budowlanego

**Słowa kluczowe:** planowanie przedsięwzięć, procesy powtarzalne, redukcja czasu przestoju brygad roboczych, modelowanie matematyczne, optymalizacja harmonogramów

**Streszczenie:**

Opracowanie harmonogramu realizacji planu produkcyjnego przedsiębiorstwa budowlanego jest zagadnieniem złożonym i dotychczas podejmowanym w literaturze w ograniczonym zakresie – odnoszącym się do wybranych zagadnień szczegółowych. Planując realizację zleceń należy dążyć do zachowania terminów umownych i efektywnego wykorzystania zasobów w celu redukcji kosztów stałych zatrudnienia i maksymalizacji zysków przedsiębiorstwa. Celem sporządzania planów produkcyjnych przedsiębiorstw powinna być harmonizacja robót prowadzonych na wszystkich placach budów. Istotne jest zatem rozwijanie metod harmonogramowania pozwalających na optymalizację alokacji własnych zasobów pracy i redukcję kosztów funkcjonowania przedsiębiorstwa w przyjętym horyzoncie czasu, dla którego można określić (z przyjętym poziomem pewności) portfel zleceń. Zarządzanie rozdziałem zasobów jest jednym z głównych problemów planowania produkcji w przedsiębiorstwie budowlanym. Celem wykonawcy budowlanego jest poszukiwanie takiego scenariusza działań (alokacji zasobów), aby całkowite koszty poniesione przez niego były jak najniższe. Na etapie planowania realizacji zleceń przedsiębiorstwa należy ustalić niezbędną liczbę zasobów kierowanych na poszczególne obiekty, tak aby zlecenia były realizowane w dyrektywnie określonych terminach. Wykonawca zwykle jest zobligowany do zapłaty kar umownych za przekroczenie terminów dyrektywnych realizacji zleceń. W warunkach rynkowych istnieje zawsze luka pomiędzy zdolnością produkcyjną a liczbą realizowanych zleceń. Jest jednak konieczne dążenie do redukcji kosztów funkcjonowania przedsiębiorstwa i zwiększenia stopnia wykorzystania własnych zasobów.

Przedsiębiorstwa budowlane realizują zazwyczaj równocześnie kilka przedsięwzięć obejmujących roboty o podobnym zakresie rzeczowym. Są one powierzane do wykonania dysponowanym brygadam roboczym, które w pewnym ustalonym porządku przechodzą z jednego na inny plac budowy. Pod względem ilości wykonywanych robót i stosowanych szczegółowych rozwiązań konstrukcyjnych oraz technologicznych procesy realizowane na różnych obiektach różnią się, co wynika z ustaleń zawartych w indywidualnych opracowaniach projektowych. Portfel zleceń przedsiębiorstwa w przyjętym horyzoncie planowania można zatem modelować jako wieloobiektowe przedsięwzięcie

obejmujące zbiór procesów powtarzanych do wykonania na niejednorodnych działkach roboczych. Harmonogramowanie realizacji zleceń wymaga ustalenia kolejności zajmowania działek przez brygady robocze i ustalenia terminów realizacji procesów. Wpływa to na terminowość i czas realizacji zleceń a także na ciągłość pracy brygad roboczych.

W artykule opracowano model matematyczny umożliwiający przydział brygad roboczych do realizacji poszczególnych procesów, spośród będących w dyspozycji przedsiębiorstwa w przyjętym horyzoncie planowania, a także na ustalenie harmonogramu ich pracy – terminów realizacji przydzielonych im procesów na wznoszonych obiektach. Model ma na celu zapewnienie redukcji łącznych kosztów pośrednich i przestoju w pracy brygad oraz kar umownych. Straty spowodowane przestojami w pracy każdej brygady są obliczane jako iloczyn czasu przestoju po wykonaniu procesu na działce roboczej oraz jednostkowych (dziennych) kosztów przestoju. Wysokość kar umownych jest obliczana jako iloczyn różnicy między czasem realizacji przedsięwzięcia a czasem dyrektywnym oraz jednostkowej kary. W przypadku ukończenia realizacji w czasie krótszym od dyrektywnego wykonawca nie zostanie obciążony karą finansowymi, przyjęto również, że nie uzyska za to bonusu.

Zaproponowany sposób doboru zmiennych decyzyjnych oraz zapisu analitycznego ograniczeń problemu o charakterze permutacyjnym pozwolił na sformułowanie modelu w postaci modelu mieszanego całkowitoliczbowego, do którego rozwiązania można stosować dostępne na rynku solvery. Oczywiście dotyczy to modeli problemów o niewielkiej złożoności obliczeniowej, lecz stwarza możliwość opracowania bazy przykładów testowych i weryfikacji jakości tworzonych w przyszłości algorytmów dedykowanych.

Zaproponowane podejście do modelowania i rozwiązywania problemu szeregowania zleceń przedsiębiorstwa przedstawiono na przykładzie realizacji stanu surowego zamkniętego sześciu budynków wielorodzinnych wznoszonych w technologii monolitycznej (fundamenty, ściany i stropy żelbetowe monolityczne; stropodach z żelbetowych płyt prefabrykowanych z warstwami izolacyjnymi; ściany ocieplone z wykorzystaniem ETICS (External Thermal Insulation Composite System)). Realizacja każdego obiektu wymaga wykonania następujących procesów powierzanych do wykonania odrębnym brygadam branżowym: roboty ziemne i fundamentowe (stan zero), konstrukcja monolityczna żelbetowa (stan surowy), dach, elewacja. Realizacja tych obiektów stanowi portfel zleceń analizowanego przykładowego przedsiębiorstwa w okresie jednego roku.

Zaproponowane w artykule podejście umożliwia redukcję przestoju, które stanowią źródło strat finansowych i dodatkowych kosztów. Opracowano model matematyczny pozwalający zapewnić ciągłość pracy brygad roboczych oraz ciągłą realizację zleceń (redukcję kosztów pośrednich budowy), o ile nie powoduje to znacznego przekroczenia terminów dyrektywnych i zapłaty kar za niedotrzymanie warunków kontraktowych. Autorzy zamierzają, jako kierunek dalszych badań, rozwijać zastosowane podejście. Proponuje się uwzględnić w opracowanym modelu możliwość zatrudnienia niewykorzystanych brygad na innych budowach (jako podwykonawcy) oraz zatrudnienia podwykonawców z zewnątrz w przypadku niedotrzymywania terminów dyrektywnych.