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Research paper

Extended Force Density Method for cable nets under self-weight. Part II – Examples of application

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Abstract: This paper is a continuation of part I – Theory and verification and presents some examples of application of the Extended Force Density Method. This method allows for form-finding of cable nets under self-weight and is based on the catenary cable element which assures high accuracy of the results and enables solving wide range of problems. Some essentials of the method are highlighted in this article. A computer program UC-Form was developed in order to perform the calculations and graphically present the results. Main advantages and features of the program are presented in this paper. Subsequently the program is used to perform calculations for a few practical examples with taut and slack cables. Input data is provided in order to enable reproducing calculations by other researchers. The outcomes are shown in the paper and prove that EFDM is an efficient tool for analysis of behaviour of cable nets under self-weight in different configurations.

Keywords: cable nets, form-finding, extended force density method, self-weight

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1. Introduction

Analysing form and forces of cable nets requires different approach than in the case of "rigid" structures (consisting of beams, plates, shells). Imposing the final shape and forces is not possible, mainly due to geometrically nonlinear behaviour (see [4]). It means that geometry of such structures strongly depends on the applied loads including self-weight. As Otto indicated in [5], in the case of correctly prestressed cable net the influence of self-weight is not as significant as in structures containing slack elements. However, it can be shown that even slight changes in geometry can make a great difference in tensile force values. Therefore, it is strongly recommended to analyse cable structures under self-weight in each phase of design starting from form-finding and ending on patterning. Moreover, taking self-weight into account is crucial when dealing with slack cables (see [2]). Such elements can appear in a cable net during the erection or after removal of some elements as a result of failure or planned action.

There are also some types of cable roof structures which are based on slack cables. In order to deal with configuration and force distribution in such structures the Extended Force Density Method and its computer implementation called UC-Form were introduced. Main features of the method and a computer program are presented in subsequent paragraphs.

2. The basics of Extended Force Density Method

Extended Force Density Method (EFDM) uses the main assumptions of Schek's concept of the Force Density Method (FDM) (see [6]). These can be summarized as follows:

- a cable net is a system consisting of linear, weightless elements connected by nodes which can be anchored (fixed) or free;
- point loads are applied in chosen free nodes;
- free nodes coordinates are calculated with the aid of equilibrium equations for nodes in 3 directions;
- in order to linearise equilibrium equations due to unknown coordinates the force density is defined as a ratio between element force and element length;
- defining a specific set of force density values results in a particular structure configuration;
- values of calculated free nodes coordinates are the basis for obtaining element lengths and tensile forces.

EFDM enables analysis of cable nets not only under nodal point loads but also under self-weight. For that purpose elastic, catenary element equations were implemented which ensure high accuracy of results for taut and slack cable elements which cannot be achieved in the case of a parabolic cable element (see [1, 2]). Due to the fact that force densities in a cable net under self-weight are unknown a new system of equations is introduced in the EFDM. The iterative procedure in which these equations and nodal equilibrium equations are solved leads to obtaining geometry and forces in the cable net. The nodal equilibrium



equations for vertical direction from the FDM are modified by adding reaction forces under self-weight to other nodal forces. More details of the EFDM are presented in the first part of this article [7].

3. Main features of UC-Form program

Application examples of the Extended Force Density Method presented in subsequent chapters were calculated with the use of self-developed computer program called UC-Form. It consists of 25 Scilab files which execute calculations and present results in a graphical form. Input data can be defined directly in the program or with the aid of an auxiliary MS Excel file which is more convenient in the case of large and complicated structures. It also enables two different methods of defining structure geometry which correspond to the form of output data which can be obtained from other computational or graphical software (e.g. Abaqus, Ls-Dyna, Formian). On the basis of some essential geometrical input given by the user program can automatically create orthogonal cable nets covering specific contours which are rectangle, elipse and double parabolic arch (see Fig. 1).



Fig. 1. Examples of orthogonal nets created automatically

After entering input data user can choose one of the three possible paths of performing calculations:

- 1) form-finding without self-weight;
- form-finding without self-weight and with additional constraints (e.g. imposing force values in elements or nodes distance);
- 3) form-finding with self-weight.

After performing calculations and displaying obtained geometry of structure user can indicate the elements to be deleted in order to analyse an incomplete structure (see Subsection 4.3). The final stage performed by the program is creating and displaying an interactive window with the current view of the structure which can be shifted, rotated and zoomed. Selected information about obtained structure configuration can be switched on or off in a control panel (e.g. elements and nodes numbers, loads, tensile forces). Current view can be exported to a graphical file (format .png or .eps). An example with a simple cable net and its element lengths is shown in Fig 2.





Fig. 2. An example of a graphic window showing final configuration

4. Practical application of the EFDM

4.1. Form-finding of a cable roof structure

The main aim of form-finding methods is to obtain the initial configuration of a cable structure which would be the starting point for the static and dynamic analysis under live and dead loads. Adding self-weight in this phase of analysis leads to better estimation of geometry and forces for further design phases. Correct initial configuration of a cable structure should ensure:

- lateral stiffness;
- architectonically and functionally desired shape and size;
- all cables working in tension;
- possibly regular force distribution (at least within separated groups of cables).

The example presented in this subchapter is inspired by one of Frei Otto's projects called here a sawtooth roof (see Fig. 3). Due to lack of precise geometric data in Otto's



Fig. 3. Sawtooth roof by Frei Otto (see [5])

book, size and shape obtained here are only similar to the original. Compressed masts and their guy cables are not included in this model. All cables are divided into three groups:

- main upper concave elements;
- tensioning lower convex elements;
- transverse connecting main and tensioning cables.

One type of spiral strand rope with undermentioned data is assumed for all the elements:

- diameter 13 mm;
- self-weight $8.34 \cdot 10^{-3}$ kN/m;
- longitudinal stiffness $EA = 19 \cdot 10^3$ kN;
- design value of tension resistance 104 kN.

Table 1 summarizes fixed nodes numbers and coordinates of the cable net.

Fixed	Coordinates [m]			Fixed	Coor	dinates	[m]	Fixed	Coordinates [m]		
node no.	x	у	x	node no.	x	у	z	node no.	x	у	z
1	0.0	3.0	0.0	46	1.5	12.0	6.0	70	22.5	0.0	6.0
9	24.0	3.0	0.0	54	22.5	12.0	6.0	71	1.5	24.0	6.0
10	0.0	9.0	0.0	55	1.5	18.0	6.0	72	5.0	24.0	6.0
18	24.0	9.0	0.0	63	22.5	18.0	6.0	73	8.5	24.0	6.0
19	0.0	15.0	0.0	64	1.5	0.0	6.0	74	12.0	24.0	6.0
27	24.0	15.0	0.0	65	5.0	0.0	6.0	75	15.5	24.0	6.0
28	0.0	21.0	0.0	66	8.5	0.0	6.0	76	19.0	24.0	6.0
36	24.0	15.0	0.0	67	12.0	0.0	6.0	77	22.5	24.0	6.0
37	1.5	6.0	6.0	68	15.5	0.0	6.0				
45	22.5	6.0	6.0	69	19.0	0.0	6.0				

Table 1. Fixed nodes coordinates for a sawtooth roof

Form-finding is an iterative procedure. In order to analyse geometry and forces in each step, ordinates of a central point (no. 50 in Fig. 4) and tensile force ranges in three groups of cables are presented in Table 2. Ordinates are measured from the level of lower fixed nodes.

In the first step self-weight is omitted and force densities in all elements are assumed 1.0 kN/m. As we can see in Fig. 4, in this case cable net is almost flat and its lateral stiffness is low. This configuration shows that higher prestress in main and tensioning cables is necessary. Therefore, in the second step force density value in these groups of elements is assumed 15.0 kN/m. Fig. 5 shows that structure resulting from this analysis has similar configuration to the original one and it is regarded here as a target in next steps.

In the third step self-weight is included and a first set of unstretched cable lengths is assumed as indicated below:

- main cables 3.10 m;
- tensioning side cables 1.35 m, tensioning interior cables 2.70 m;
- transverse side cables 5.00 m, transverse interior cables 4.50 m.





Fig. 4. Sawtooth roof without self-weight, force densities equal to 1.0 kN/m (step 1)



Fig. 5. Sawtooth roof without self-weight, force densities equal to 15.0 and 1.0 kN/m (step 2)

In this configuration whole structure hangs freely on the main cables and its tensile forces along with lateral stiffness are low (see Fig. 6).

In the next step unstretched lengths are taken from step 2. As we can see in Fig. 7 transverse cables are slightly curved and tensile forces in all cables have very low values. It means that pretension of the cable net is still insufficient.

Next 4 steps are attempts to find geometry similar to the one from the second step and providing high lateral stiffness. In step 5 unstretched lengths for all cable segments are 0.01 m shorter than in previous one. Then initial lengths taken from step 2 are shortened by 0.05 m in the main cables (step 6) or in tensioning cables (step 7 and 8). As maximum tensile force in step 7 exceeds design value of tension resistance a spiral strand rope with 16 mm diameter is assumed for main and tensioning cables (step 8).

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Fig. 6. Sawtooth roof with self-weight and initial values of unstretched lengths (step 3)



Fig. 7. Sawtooth roof with self-weight and unstretched lengths taken from step 2 (step 4)

As we can see in Table 2 higher pretension in main cables (step 6) provides more space under the cable net (node 50 ordinate equals to 3,131 m) but also produces high values of tensile forces. It means that larger wire rope sections should be used in such case. Similar situation occurs in the case of higher prestress in tensioning cables, but profit in clearance height of structure is not sufficient compared to step 5. Step 5 configuration is also most beneficial because it provides high margin of tension resistance for adding live loads.

Although presented analysis is not a complete designing process, it shows the influence of self-weight and unstretched cable lengths on geometry and forces in a cable net. It also presents how form-finding without self-weight can be used as a starting point for next design steps including this type of load.



Step no.	Node 50 ordinate [m]	Tensile force value ranges [kN]	Step no.	Node 50 ordinate [m]	Tensile force value ranges [kN]
1	2.827	$N_{\text{main}} = 2.92 \div 4.18$ $N_{\text{tens}} = 2.29 \div 3.05$ $N_{\text{trans}} = 3.00 \div 3.68$	5	2.810	$N_{\text{main}} = 66.13 \div 76.36$ $N_{\text{tens}} = 58.52 \div 65.29$ $N_{\text{trans}} = 4.83 \div 7.62$
2	2.848	$N_{\text{main}} = 43.81 \div 49.07$ $N_{\text{tens}} = 39.70 \div 40.79$ $N_{\text{trans}} = 3.14 \div 5.09$	6	3.131	$N_{\text{main}} = 295.84 \div 328.79$ $N_{\text{tens}} = 189.28 \div 207.60$ $N_{\text{trans}} = 19.98 \div 37.99$
3	0.180	$N_{\text{main}} = 1.00 \div 1.15$ $N_{\text{tens}} = 0.06 \div 0.12$ $N_{\text{trans}} = 0.03 \div 0.13$	7	2.872	$N_{\text{main}} = 95.52 \div 108.22$ $N_{\text{tens}} = 84.63 \div 93.37$ $N_{\text{trans}} = 7.83 \div 13.18$
4	2.773	$N_{\text{main}} = 1.93 \div 2.26$ $N_{\text{tens}} = 0.60 \div 0.80$ $N_{\text{trans}} = 0.11 \div 0.19$	8	2.872	$N_{\text{main}} = 135.15 \div 153.08$ $N_{\text{tens}} = 119.64 \div 132.04$ $N_{\text{trans}} = 11.08 \div 18.53$

Table 2. Node 50 ordinates and tensile force ranges in eight different configurations

4.2. Erection phases of a cable roof structure

Self-weight of a cable structure plays crucial role in a process of erection when other loads along with pretension are absent. Shape and forces can dramatically change in particular phases of erection and also can strongly differ from their values in the final state (with all dead and live loads). Therefore Extended Force Density Method can be an effective and simple tool for finding form and forces in the cable net and designing its erection process.

In the next example a cable net from Subsection 4.1 is used to present 7 phases of its erection process. It is assumed that whole structure is assembled on the ground level. Each segment (except side segments of main cables) has unstretched length according to previous form-finding analysis. End points of tensioning and transverse cables should be anchored in lower level. End fittings of main cables should be adapted for hydraulic wire rope tensioner. With its use side segments of main cables are shortened from 6,0 m to 3,26 and 3,17 m (different lengths for side and interior cables) (see Fig. 8 and Table 3).

Table 3. Unstretched lengths of main cables side segments in different erection phases

Phase	1	2	3	4	5	6	7
Unstretched length L_0 [m]	6.00	4.50	3.50	3.40	3.30	3.26	3.26 / 3.17

In Table 4 first three phases of erection process are presented along with unstretched lengths of prestressed cable segments L_0 , ordinates of node no. 50 and tensile force ranges in each phase are given. Due to slight changes in geometry in next four phases views of structure are omitted.

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Table 4. Three phases of erection of the sawtooth roof

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Fig. 8. Prestressing main cables in the sawtooth roof

Figure 9 presents how maximum values of tensile forces in each group of cables change while a cable net is being prestressed. In this case there is no risk of achieving higher values of forces than in the final state, so erection process is safe.



Fig. 9. Maximum tensile force values in different erection phases

Analysis of force changes from phase 1 to 5 reveals that in the case of tensioning cables maximum force declines at first (see Fig. 10). It is associated with faster increase of force in main cables. Then, starting from phase 4, forces in each group rise up to final values.

This example shows that analysis of erection phases of a cable structure is necessary to examine changes in geometry and forces in order to design safe erection procedure. Obtained geometrical data can be helpful in geodesic control on the site. Calculated force values can be compared to the measured ones to avoid overloading of cables or connections.





Fig. 10. Maximum tensile force values in phases from 1 to 5

4.3. Cable net behaviour after removal of selected elements

In this chapter an example of a cable net after accidental removal of selected elements is analysed. This is the case of an accidental load in which a dynamic response should be included. As the Extended Force Density Method is purely static in this example an approach of Eurocode is utilised. It assumes that dynamic effect can be substituted by a static effect multiplied by a dynamic coefficient (see formula (2.4) from [3]). All force values after elements removal are calculated according to this approach.

An open hexagonal cable net with fixed vertices situated on different ordinates is shown in Fig. 11. Coordinates of fixed nodes are given in Table 5. As analysed structure has antisymmetric shape, in each stage two elements with maximum tensile force are removed. Changes in force values and shape of the cable net are analysed subsequently.



Fig. 11. Hexagonal cable net under self-weight



Fixed node no.	Coordinates [m]			Fixed node no	Coordinates [m]			
	x	у	x	Tixed node no.	x	у	z	
1	0.0	5.0	0.0	23	20.0	5.0	0.0	
18	5.0	0.0	-3.0	41	5.0	10.0	-5.0	
19	15.0	0.0	-5.0	61	15.0	10.0	-3.0	

Table 5. Fixed nodes numbers and coordinates of hexagonal cable net

Figure 12 shows the configuration of structure after removal of first two elements. As we can see, the effect is local, only few nearest elements became slack but the whole structure is still prestressed.



Fig. 12. Hexagonal cable net after removal of 2 elements

Figure 13 shows removed elements and also elements in which tensile forces increased. They are divided into two groups according to the level of this increase.



Fig. 13. Force increases in the hexagonal cable net after removal of 2 elements



After removal of next two elements external parts of the cable net became completely slack as we can see in Fig. 14.



Fig. 14. Hexagonal cable net after removal of 4 elements

Tensile forces increased in only 6 elements shown in Fig. 15. The increase levels are quite high but absolute values of forces in these elements are lower than 1 kN. It means that whole structure lost its initial prestress and transverse stiffness.



Fig. 15. Force increases in the hexagonal cable net after removal of 4 elements

Removing next two elements results in completely slack configuration of the cable net shown in Fig. 16 from two different angles. Although in many elements tensile force values are higher than in previous stage (see Fig. 17) maximum tensile force is 1,58 kN so the structure has very low stiffness and load carrying capacity.

Figure 18 shows ordinates of central profile of the cable net (from node 1 to 23) in each stage. It shows global changes in geometry of the structure. After removal of first two





Fig. 16. Hexagonal cable net after removal of 6 elements



Fig. 17. Force increases in the hexagonal cable net after removal of 6 elements



Fig. 18. Central profile of hexagonal cable net in different stages

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elements the configuration is very similar to the initial one. In next stages great parts of the cable net become nearly flat or even concave. It can lead to destruction of cladding and also cumulation of rainwater or snow (called ponding) which can cause local load increase and higher values of tensile force.

Analysis presented in this chapter shows sequence of progressive collapse of the structure. It can help to identify the weakest elements or areas of the cable net. On this basis larger sections or additional elements can be designed in order to increase safety.

5. Conclusions from presented examples

As it was shown in the previous chapter Extended Force Density Method enables analysis of wide range of problems associated with cable structures. Presented here examples prove that this kind of structures have some specific features compared to "rigid" ones and they require different attitude. It is mainly due to their geometrical nonlinearity and catenary shape under self-weight.

First example proves that form-finding including self-weight can result in more realistic initial geometry and force distribution which can help to assume correct wire rope sections. Defining proper values of unstretched cable lengths have great influence on behaviour of a cable structure. It was shown that these lengths can be obtained on the basis of analysis without self-weight.

After finding the initial form of a cable structure and performing complete design process there is a great need to analyse the erection process. In different stages of this process a cable net is not fully prestressed. Slack configurations along with tensile force distribution can be easily found with the aid of EFDM. Finding overloaded cables or connections can be helpful to plan safe erection process.

In the last example EFDM was used to trace one of possible progressive collapse paths resulting from sequential removal of elements with highest values of tensile force. Dynamic effect was taken into account by the use of Eurocode approach. This kind of analysis is useful to identify overloaded zones or elements of a structure which can be strengthen.

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Rozszerzona Metoda Gęstości Sił do analizy siatek cięgnowych pod ciężarem własnym. Część II – przykłady zastosowania

Słowa kluczowe: siatki cięgnowe, kształtowanie, rozszerzona metoda gęstości sił, ciężar własny

Streszczenie:

Druga część artykułu dotyczącego Rozszerzonej Metody Gęstości Sił (RMGS) zawiera opisy praktycznych przykładów jej zastosowania oraz wnioski z nich wynikające.

W artykule skrótowo przypomniano założenia i zasady stworzonej do analizy siatek cięgnowych pod ciężarem własnym Rozszerzonej Metody Gęstości Sił. Jest to metoda oparta na iteracyjnej procedurze, w której naprzemiennie rozwiązywany jest układ równań równowagi węzłów siatki cięgnowej oraz ukł ad równań opisujących zachowanie cięgna pod ciężarem własnym. Z drugiego układu równań otrzymuje się aktualne wartości gęstości sił, dzięki którym przy użyciu pierwszego układu równań oblicza się współrzędne węzłów wolnych siatki. Zaprezentowana metoda opiera się na dokładnym opisie kształtu cięgna pod ciężarem własnym, czyli na linii łańcuchowej.

Do obliczeń RMGS stworzono autorski program UC-Form przy użyciu pakietu obliczeń matematycznych Scilab. W artykule przedstawiono możliwości programu i skrócony algorytm działania uwzględniający kilka możliwych ścieżek wprowadzania danych i wykonywania obliczeń.

W następnej części artykułu zaprezentowano wraz z danymi wejściowymi trzy przykłady wykorzystania RMGS i programu UC-Form do rozwiązania praktycznych problemów specyficznych dla konstrukcji cięgnowych. Pierwszym z nich jest wstępne kształtowanie siatki cięgnowej, które jest początkowym etapem obliczeń poprzedzającym wykonanie tradycyjnych analiz statycznych i dynamicznych oraz wymiarowania konstrukcji. W przykładzie pokazano, w jaki sposób można wykorzystać analizę bez ciężaru własnego do prawidłowego kształtowania siatki z uwzględnieniem ciężaru własnego. Pokazano również wpływ ciężaru własnego na geometrię i siły w siatce.

W kolejnym przykładzie przedstawiono kilka faz wznoszenia konstrukcji cięgnowej. Ze względu na to, że na etapie montażu siatki cięgnowe nie są sprężone, ich kształt może być opisany tylko przy użyciu modeli uwzględniających ciężar własny. RMGS jest zatem dobrym narzędziem analizy kształtu i sił występujących w konstrukcji na różnych etapach montażu. Dzięki tego typu analizie można w bezpieczny sposób zaprojektować proces wznoszenia i wyeliminować sytuacje, w których mogłoby nastąpić przekroczenie nośności wybranych elementów lub łączników. Informacje uzyskane na podstawie takich analiz mogą również być wykorzystywane przy geodezyjnej kontroli wznoszenia konstrukcji.

Ostatni przykład ilustruje w uproszczony sposób zjawisko postępującej katastrofy powstającej w wyniku usuwania w kolejnych etapach obliczeń elementów, w których wystąpiła maksymalna siła. Również w tym przykładzie kluczowa jest możliwość uwzględnienia ciężaru własnego, ponieważ już w pierwszych fazach zniszczenia konstrukcji pojawiają się elementy luźne. Ma to bezpośredni wpływ na możliwość powstawania zniszczeń w elementach pokrycia lub wyposażenia lokalnie lub globalnie, a także na przejmowanie obciążeń przez inne elementy, co może doprowadzić do ich



przeciążenia. W zaproponowanym przykładzie wykonywano jedynie analizy statyczne, ale efekt dynamiczny został uwzględniony w sposób przybliżony poprzez zwiększający współczynnik dynamiczny zaproponowany w Eurokodzie. Dzięki tego typu analizie można wyodrębnić elementy lub fragmenty konstrukcji, które mogą być kluczowe w trakcie awarii, a także zaproponować możliwe metody wzmocnienia.

Przedstawione przykłady obliczeń przy użyciu RMGS mają praktyczne znaczenie przy projektowaniu konstrukcji cięgnowych. Uwzględnienie ciężaru własnego pozwala na analizowanie konstrukcji w pełni napiętych, ale również częściowo lub całkowicie luźnych.

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