WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2020.135239
ARCHIVES OF CIVIL ENGINEERING
Vol. LXVI ISSUE 4 2020

NEW STATIC ANALYSIS METHODS FOR PLATES MADE OF MONOLITHIC AND LAMINATED GLASS

M. GWÓŹDŹ[1], P. WOŹNICZKA[2]

The implementation of a new, high-performance float flat glass manufacturing technology in Europe, in conjunction with the growing interest in new glass functions expressed by the construction industry, has led to significant developments in the theory of glass structures. Long time research conducted in the EU countries has been concluded by the technical document CEN/TC 250 N 1060, drawn up as a part of the work of the European Committee for Standardization on the second edition of Eurocodes (EC). The recommendations pertaining to the design of glass structures have been foreseen in the second edition of the Eurocodes, in particular the development of a separate design standard containing modern procedures for static calculations and stability of glass building structures (cf. works M. Feldmann, R. Kasper, K. Langosch and other).

In this paper new static analysis methods for glass plates made of monolithic and laminated glass, declared in the document CEN/TC 250 N 1060 (2014) and recommended in the national standarization document CNR-DT 210 (National Research Council of Italy, 2013) are presented. These static analysis methods are not commonly known in our national engineering environment, and thus require popularization and regional verification. Numerical and analytical simulations presented in this paper for rectangular plates made of monolithic and laminated glass and having various support conditions are of this character. The results of numerical calculations constitute a basis for the discussion of new static analysis methods for plates.

Keywords: glass, glass strength, nonlinear statics, computer modelling

[1] Prof.,DSc.,PhD.,Eng., University of Bielsko-Biala (ATH), Institute of Building Industry, ul. Willowa 2, 43-309 Bielsko-Biała, Poland, e-mail: magwozdz@ad.ath.bielsko.pl
[2] PhD. Eng., Cracow University of Technology, Faculty of Civil Engineering, Warszawska 24, 31-155 Cracow, Poland, e-mail: pwozniczka@pk.edu.pl

1. INTRODUCTION

Classical static analysis of flat elastic structures is based on the solution of thin plate bending problem according to the linear theory, assuming that the deflection of the midplane $w(x,y)$ is substantially smaller than the plate thickness h. However, elementary lab experiment of the four point monolithic glass plate bending proves that the deflections of the midplane are much larger than the plate thickness. This is accounted for in the nonlinear theory, leading to the following differential equation:

$$\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=h\left(\sigma_x\frac{\partial^2 w}{\partial x^2}+2\tau_{xy}\frac{\partial^2 w}{\partial x\partial y}+\sigma_y\frac{\partial^2 w}{\partial y^2}\right)+q(x,y), \tag{1.1}$$

where:

q(x,y) – area load applied to plate surface,

σ_x ,σ_y – normal stresses due to bending,

τ_{xy} – tangent stresses due to torsion

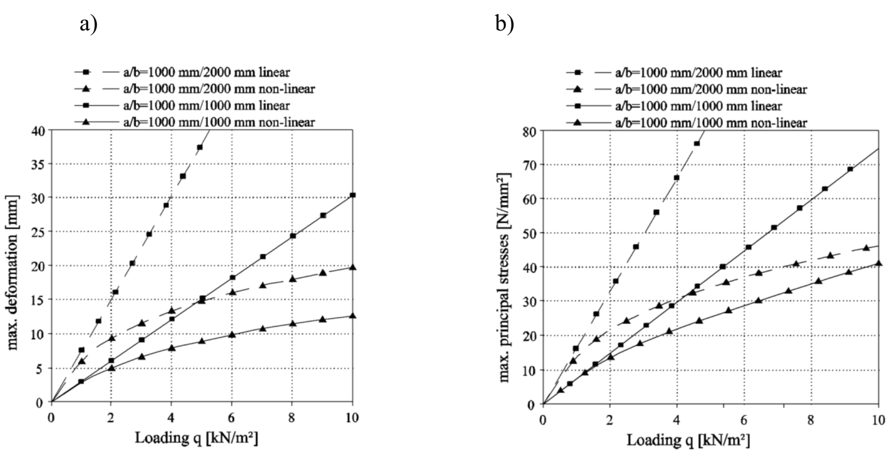


Fig. 1. Bending of sample plates having the dimensions *a*x*b* according to the linear and nonlinear theory: a) deflections, b) extreme stresses. Source [5]

During large deflections ($w > h$) the midplane of the plate is extended, resulting in membrane stresses interfering with pure bending state. Solution of the equation (1.1) for given boundary conditions and known distribution of external loads, for instance in the simple case of $q(x,y) = q = \text{const}$ may be found by two approaches:

a) application of FEM computer modeling, yielding maps of displacements and stresses induced by bending of the plate,

b) application of analytical methods, allowing for presentation of discrete solutions, i.e. usually tabular presentation of results.

Solutions obtained for monolithic glass plates subjected to uniform load and simply supported along edge, according to the linear and nonlinear (formula 1.2) theory, are depicted in Fig. 1 following paper [5] and document [4]. The extreme deflection curves juxtaposed in Fig. 1.a) and extreme normal stresses juxtaposed in Fig. 1.b) indicate, that application of nonlinear theory substantially reduces appropriate static components.

2. STATIC ANALYSIS METHODS

2.1. MONOLITHIC GLASS PLATES

Computer modelling of plates is currently preferred in each case, and especially in the case of thorough static analyses of large aluminium – glass structures. The analytical (semi-empirical) methods do not present an alternative for computer modelling, but may be applied during engineering design of facades and other simple glass – aluminium structures. The procedure of simplified calculations, quite often applied in Poland, cf. [9], is based on estimation of the maximum deflections and stresses in the plate (extreme or effective) according to the formulas contained in the code DIN-EN 13474 [2]. This procedure has inherent limitations, and especially due to the tabular form of presentation of static calculations results in some cases may lead to incorrect solutions. An alternative analytical static calculation method for monolithic glass plates is contained in the document CNR-DT 210/2013 [3], prepared by the National Research Council of Italy. In this document the formulae for extreme deflections and stresses have the following form:

$$w_{\max} = k_{\mathrm{w}} \frac{(a \cdot b)^2}{h^3} \cdot \frac{q_{\mathrm{k}}}{E} \quad \text{for } k_{\mathrm{w}} = k_{\mathrm{w}}(p_k^*) \tag{2.1}$$

$$p_k^* = \left(\frac{a \cdot b}{4h^2}\right)^2 \frac{q_{\mathrm{k}}}{E}, \tag{2.2}$$

(2.3) $$\sigma_{\max} = k_\sigma \frac{a \cdot b}{h^2} q_d \quad \text{for} \quad k_\sigma = k_\sigma(p_d^*),$$

(2.4) $$p_d^* = \left(\frac{a \cdot b}{4h^2}\right)^2 \frac{q_\mathrm{d}}{E},$$

The extrapolation formulae for k_w and k_σ coefficients have been listed in the document [3] for plates (simple support along all four edges having the lengths of *a* and *b*, respectively) as the following analytical functions:

(2.5) $$k_w = \frac{\sqrt{\left(\sqrt{\frac{1}{z_1^4} + 4(p_k^*)^2} - \frac{1}{z_1^2}\right)}}{16\sqrt{2} \cdot p_k^*},$$

where

$$z_1 = 192(1-\nu^2)(a \cdot b)^2 \left\{0{,}00406 + 0{,}00896\left[1 - \exp\left(-1{,}123\left(\frac{1}{a \cdot b} - 1\right)^{1{,}097}\right)\right]\right\}.$$

(2.6) $$k_\sigma = \frac{1}{4 \cdot \sqrt{\frac{1}{z_2^2} + \frac{(p_d^*)^2}{z_3^2 + (z_4 p_d^*)^2}}},$$

where

$$z_3 = 4{,}5\left(\frac{1}{a \cdot b} - 1\right)^2 + 4{,}5, \quad z_4 = 0{,}585 - 0{,}05\left(\frac{1}{a \cdot b} - 1\right),$$

$$z_2 = 24a \cdot b\left\{0{,}0447 + 0{,}0803\left[1 - \exp\left(-1{,}17\left(\frac{1}{a \cdot b} - 1\right)^{1{,}073}\right)\right]\right\}.$$

The static scheme of rectangular plate with simple support along all four edges is appropriate for traditional glazing of external partitions made of double glazing with monolithic glass panes. To verify the analytical procedure static calculations have been conducted for rectangular plates having the thickness of $h = 5$ mm and the width to height ratio of $a/b = 2000/2000 = 1{,}00$ and $a/b = 1250/1900 = 0{,}66$. The extreme climate loads have been assumed for the submontane zone ($H = 1023$ m above sea level) during 3-5 sec. long wind gusts and isochoric pressure in the air chamber conforming to the location of the double pane manufacturer at the height H_p = 210 m above sea level.

Table 1. Juxtaposition of the results obtained for a 2000x2000 mm plate by analytical methods and FEM modelling

Static component	Analytical method according to		FEM modelling	
	DIN-EN 13474	CNR-DT 210/2013	Program 1.	Program 2.
(1)	(2)	(3)	(4)	(5)
Linear theory of thin plates – simple support				
w_{max} [mm]	-	-	85,7	85,8
σ_{max} [MN/m^2]	-	-	63,7	63,78
Linear theory of thin plates – sliding support				
w_{max} [mm]	-	-	85,7	85,8
σ_{max} [MN/m^2]	-	-	63,7	63,78
Nonlinear theory of thin plates – simple support				
w_{max} [mm]	-	-	10,5	10,6
σ_{max} [MN/m^2]	-	-	14,0	13,9
Nonlinear theory of thin plates – sliding support				
w_{max} [mm]	24	23	22,9	**22,9**
σ_{max} [MN/m^2]	< 39,6	31,4	29,5 (40,7)*	**30,3 (35,9)***
$(\sigma_{max})^*$ - local stresses at corners of the plate				

The assumptions listed above result in characteristic load q_k = 1,0 kPa and computational load q_d = 1,5 kPa acting on the plate. The results of static calculations performed according to formulae (2.1)÷(2.6) have been verified by the computer simulations using FEM. Two independent computer programs (RFEM [20] and AxisVM [19], hereinafter referred to as Computer program 1. and Computer program 2., respectively) applying the theory of nonlinear thin plates have been used. Both these programs are suitable for engineering purposes. Static analysis was conducted in both linear and nonlinear cases of FEM calculations. Shell elements have been applied. To keep the model simple linear elastic material type has been used. The dimensions of considered slabs were insignificant, thus it was possible to use a quite fine FEM mesh (0.05x0.05 m) and avoid excessive computational cost. The final results were not affected by further mesh refinements.

The summary of the results obtained for a plate having the dimensions of 2000x2000 mm is presented in the table 1, where the columns (2) and (3) contain the results determined by

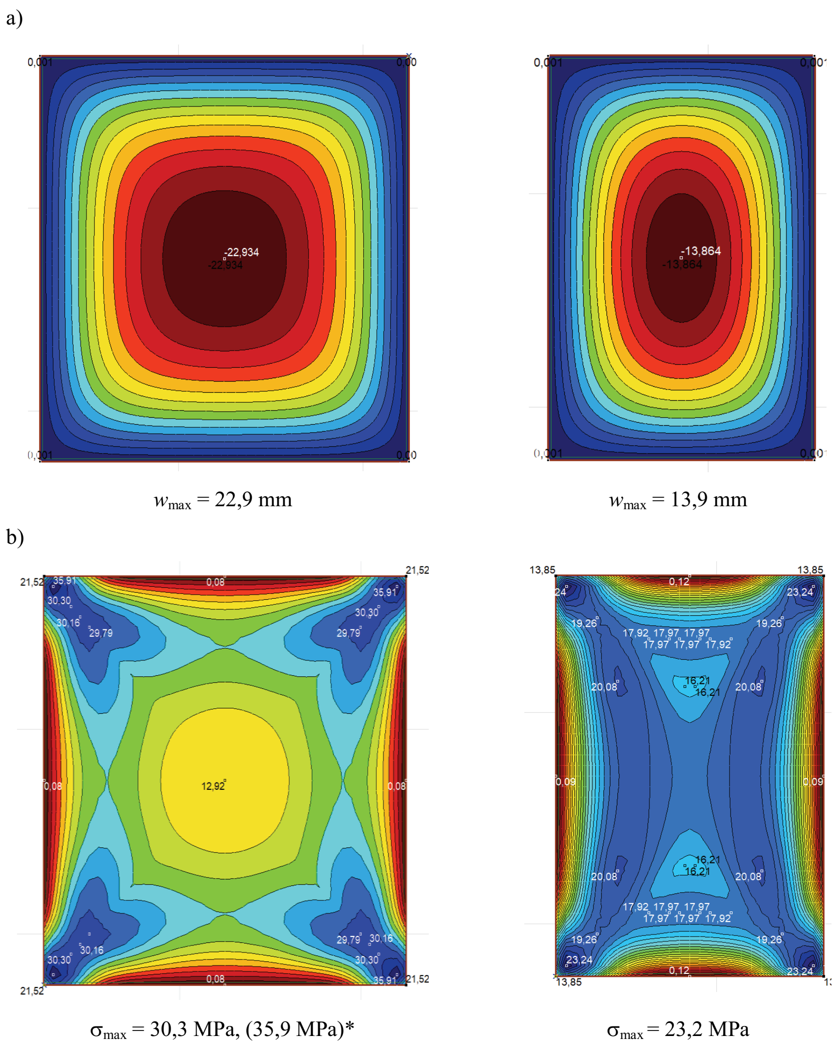


Fig. 2. Results of FEM modelling according to the nonlinear theory of thin plates: a) maps of deflections, b) maps of tensile principal stresses

analytical method while columns (4) and (5) contain the results calculated using FEM, which are convergent, according to both computer programs (cf. Table 1.). Fig. 2 depicts representative results of the modelling, in particular Fig. 2a) depicts maps of deflections for plates 2000/2000 and 1250/1900 according to the nonlinear theory, while Fig 2b) depicts maps of principal stresses according to nonlinear theory for both plates. Values of deflections and stresses at mid span and extreme stresses (local phenomena) are shown. One should note, that mid span analytical (computed according to formulae given in [3]) and numerical results are quite similar. This observation is true for both calculated deflections and stresses. At the same time mid span stresses calculated according to [2] are slightly overestimated. On the other hand, analytical methods are unable to take into account local phenomena, which could be crucial in case of certain special boundary conditions.

2.2. LAMINATED GLASS PLATES

Laminated safety glass is in general manufactured with PVB film cf. [1, 12, 14], and simplified modelling during phase I allows for unrestrained sliding of panes, i.e. does not account for fusing by the film. However, the newest research, for instance [6, 10, 11, 13, 15], shows that the laminating layer, in spite of susceptibility to rheological phenomena, exhibits noticeable fusing effect. The simplified calculations of laminated glass plates apply the procedures described above (formulae (2.1)÷(2.4)) for monolithic glass) but replace the real laminated glass plate thickness $h = \Sigma t_i$ by effective thickness $h = t_{eff}$. The effective thickness concept is based on the consistent curvature criterion for an element subjected to bending: a laminated element having n layers and exhibiting stiffness EJ_{full} and quasi-monolithic element exhibiting stiffness EJ_{eff}, thus the following relationship holds:

(2.7)
$$\frac{1}{\rho} = \frac{M}{EJ_{full}} = \frac{M}{EJ_{eff}}.$$

In the equation (2.7) the bending stiffness EJ_{full} of a pane consisting of two glass plates as depicted in Fig. 3 is given as:

(2.8)
$$EJ_{full} = E(J_1 + J_2) + E\frac{A_1 A_2}{A_1 + A_2} \cdot z^2,$$

where

(2.9) $$z = z_1 + z_2 = \frac{1}{2}\left(t_1 + t_2\right) + t_{\text{int}}, \quad z_1 = z\frac{t_2}{t_1 + t_2}, \quad z_2 = z\frac{t_1}{t_1 + t_2},$$

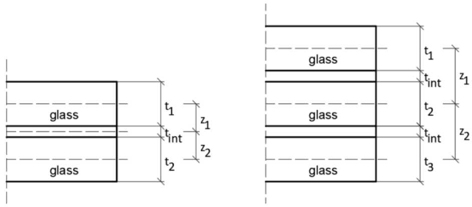


Fig. 3. Designations of dimensions for two and three layer laminated components

where:

$A_1\, J_1$, $A_2\, J_2$ - cross sectional area and moment of inertia with respect to the axis of symmetry of glass panes 1 and 2.

The effective bending stiffness may be determined by the application of Wölfel-Bennison model [18], which leads to the following formula for the effective moment of inertia:

(2.10) $$J_{\text{eff}} = \Gamma J_{\text{full}} + (1-\Gamma)\sum_{i=1}^{n} J_i\,.$$

For the glass pane consisting of two layers according to Fig. 3, the formula (2.10) takes the following form:

(2.11) $$J_{\text{eff}} = J_1 + J_2 + \Gamma\frac{A_1 + A_2}{A_1 + A_2}z^2,$$

(2.12) $$\Gamma = \frac{1}{1 + 9{,}6\dfrac{t_{\text{int}} E I_{\text{s}}}{G_{\text{F}} L^2 z^2}}; \quad I_{\text{s}} = t_1 z_1^2 + t_2 z_2^2,$$

where:

G_{F} - shear modulus of the bonding layer,

L - beam (plate) span.

The effective thickness of the layered plate, due to the extreme deflections w is equal to:

(2.13) $$t_{\text{eff, w}} = \sqrt[3]{t_1^3 + t_2^3 + 12\Gamma I_s},$$

in addition, the effective thickness of each single pane in the layered glass plate, due to the normal stresses σ induced by element bending, is equal to:

(2.14)
$$t_{1,\text{eff},\sigma} = \sqrt{\frac{t_{eff,w}^3}{t_1 + 2\Gamma z_1}}, \quad t_{2,\text{eff},\sigma} = \sqrt{\frac{t_{eff,w}^3}{t_2 + 2\Gamma z_2}}.$$

The function Γ is a measure of coupling between panes, and its value remains within the interval $0 \leq \Gamma \leq 1$. In particular complete coupling is indicated by $\Gamma = 1$, while complete lack of coupling, i.e. the situation when glass panes are free to move with respect to each other, is given by $\Gamma = 0$. The shear modulus G_F of the bonding layer present in the formula (2.12), as well as the Young modulus E_F are affected by the rheological phenomena, i.e. their values decrease over loading time t of this layer. Extract from tests of rheological properties of the Saflex PVB film [17] having the thickness of 0.76 mm by one manufacturer is presented in the Table 2.

Table 2. Values of Saflex PVB film shear modulus G_F [MPa] according to [17]

Loading time t	Temperature								
	20°C	25°C	30°C	35°C	40°C	45°C	50°C	55°C	60°C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3 seconds	341	237	108	27	3,7	1,3	0,8	0,7	0,6
30 minutes	140	28	1,9	0,8	0,6	0,5	0,4	0,4	0,3
24 hours	22	1,7	0,7	0,5	0,4	0,3	0,2	0,1	0,1
1 month	1,8	0,7	0,5	0,4	0,2	0,1	0,1	-	-
1 year	0,8	0,6	0,4	0,2	0,1	-	-	-	-
10 years	0,6	0,5	0,3	0,1	-	-	-	-	-
50 years	0,6	0,4	0,2	-	-	-	-	-	-
Young modulus of Saflex PVB: $E_F = 2G_F(1+\nu)$ where $\nu \approx 0{,}476$									

Data listed in the Table 2 indicates, that the bonding layer made of Saflex PVB film subjected to external loads exhibits creep due to the reduction in shear modulus G_F, and the structures made of laminated glass located in the air conditioned environment (T = 20°C) just after one year of service under load are affected by the reduction of the G_F modulus to less than 1 MPa. In the case of external exposure, during hot summer (T = 30°C) this process is accelerated and the reduction to below 1 MPa occurs just after 30 minutes. The concept of effective laminated glass layer thickness may be developed from a model alternative to the formula (2.10), elaborated by L. Galuppi and G. Royer-Carfagni [8], (cf. the document CNR-DT 210/2013 [3]), which for a bar structure may be described by the formula:

$$\frac{1}{J_{\text{eff}}} = \frac{\eta_1}{J_{\text{full}}} + \frac{1-\eta_1}{\sum_{i=1}^{n} J_{\text{i}}}, \tag{2.15}$$

or by a formula analogous to (2.15) for plate [7]:

$$\frac{1}{D_{\text{eff}}} = \frac{\eta_2}{D_{\text{full}}} + \frac{1-\eta_2}{\sum_{i=1}^{n} D_{\text{i}}}, \tag{2.16}$$

where the coefficients η_1 and η_2 for a plate consisting of two panes are as follows:

$$\eta_1 = \frac{1}{1+\dfrac{t_{\text{int}} E(J_1+J_2)}{G_{\text{F}} J_{\text{full}}} \dfrac{t_1 \cdot t_2}{t_1+t_2} \Psi}, \quad \eta_2 = \frac{1}{1+\dfrac{t_{\text{int}} E(D_1+D_2)}{G_{\text{F}}(1-\nu^2) D_{\text{full}}} \dfrac{t_1 \cdot t_2}{t_1+t_2} \Psi}. \tag{2.17}$$

where the coefficients η_1 and η_2 for a plate consisting of two panes are as follows:

$$\sum_{i=1}^{n} D_i = \frac{E t_i^3}{12(1-\nu^2)}, \quad D_{full} = \frac{E}{12(1-\nu^2)} \cdot \left[(t_1^3 + t_2^3) + \frac{t_1 \cdot t_2}{t_1+t_2} \cdot z^2 \right], \tag{2.18}$$

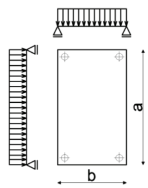
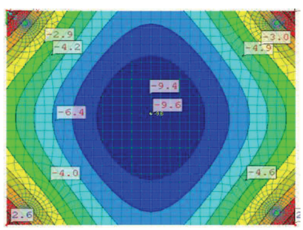
Ψ - coefficient depending on the static scheme (support conditions) and loading conditions. For instance for the simply supported beam evenly loaded with a load of intensity q_{d} along the whole span length L the coefficient $\Psi = 168/17\text{L}^2$; tabulated values for rectangular plates having various support conditions are listed in [3]. The strength criterion of each plate has the form:

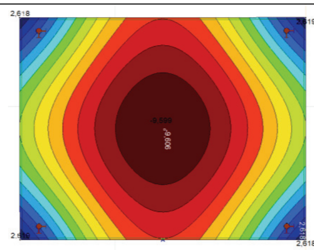
$$\sigma_{\text{i,max}} = \frac{6 \cdot |M_{\text{max}}|}{b \cdot t_{\text{i,eff},\sigma}^2} \le R_d, \tag{2.19}$$

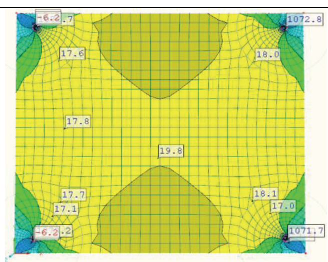
where

(2.20)
$$t_{i,\mathrm{eff},\sigma}=\sqrt{\dfrac{1}{\dfrac{2\eta|z_i|}{\sum_{i=1}^{n}t_i^3+12\sum_{i=1}^{n}(t_i\cdot z_i^2)}+\dfrac{t_i}{t_{\mathrm{eff,w}}^3}}},\qquad t_{\mathrm{eff,w}}=\sqrt[3]{\dfrac{1}{\dfrac{\eta}{\sum_{i=1}^{n}t_i^3+12\sum_{i=1}^{n}(t_i\cdot z_i^2)}+\dfrac{1-\eta}{\sum_{i=1}^{n}t_i^3}}}.$$

Table 3. Results of FEM modelling of 1250x1500 mm laminated plate – point support

<table>
<tr><td>Computer program 1.</td><td>Computer program 2.</td></tr>
<tr><td colspan="2">Non-sliding supports – non-linear plate theory

</td></tr>
<tr><td>

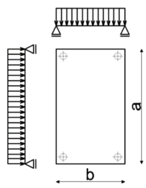
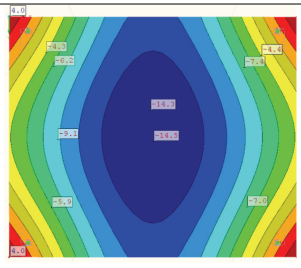
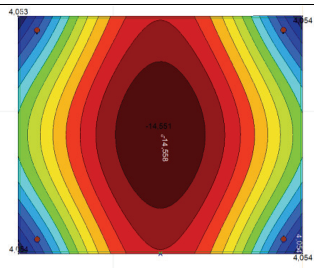
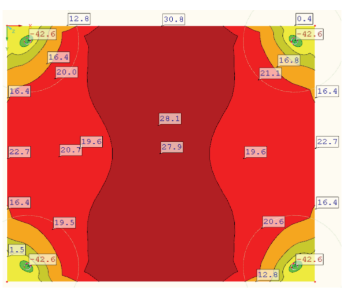
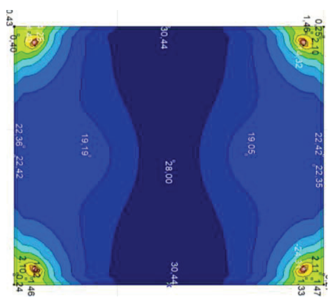
w_{max} = 9,6 mm</td><td>

w_{max} = 9,6 mm</td></tr>
<tr><td>

σ_{max} = 19,8 (σ=1073) MPa</td><td>
σ_{max} = 21,9 (σ = 128) MPa</td></tr>
</table>

In the formulae (2.20) one should substitute $\eta = \eta_1$ for beams or $\eta = \eta_2$ for plates, respectively. The analytical formulae (2.13) and (2.14), derived from the Wölfel-Bennison model are widely applied in the contemporary European normalization. The alternative formulae (2.20), derived from the Galuppi-Royer-Carfagni model constitute a new proposal, which should be subjected to testing.

Table 4. Results of FEM modelling of 1250x1500 mm laminated plate – point support

Computer program 1.	Computer program 2.
Sliding supports – non-linear plate theory 	
 w_{max} = 14,5 mm	 w_{max} = 14,6 mm
 σ_{max} = 30,8 MPa	 σ_{max} = 30,4 MPa

Appropriate static calculations for rectangular plates consisting of two layers have been performed to verify the analytical procedure (2.17)÷(2.20). Considered plates have their own structural prototypes. The first laminated plate represents a repetitive segment of a roof spanned over the entrance to a building. The static scheme is a plate having the dimensions of 1250x1500 mm with a pinned support over four point supports at the distance of 1100x1300 mm. The structure of two layer glass plate is 8-0.76-8 mm with 8 mm thick glass panes and 0.76 mm thick bonding layer.

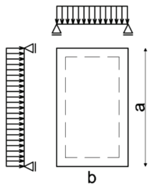
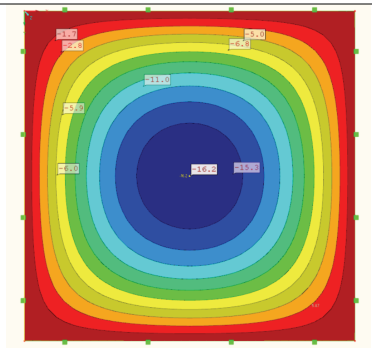
The constant and environmental loads are equal to: characteristic $q_k = 2.5$ kN/m^2 and computational $q_d = 3.7$ kN/m^2. The nominal length of the dominant snow load action on the bonding layer has been assumed as 3 months, resulting in the reduction in shear modulus of this layer to $G_F = 2.0 - 0.2\log(7.776\text{x}10^6) = 0.622$ MPa.

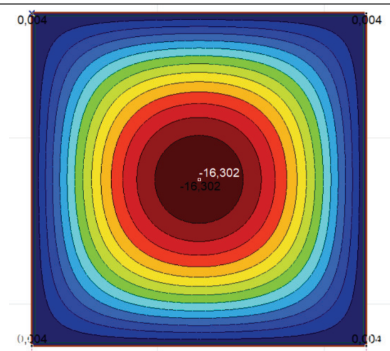
The plate stiffnesses $\Sigma D_i = 6.222$ kNm and $D_{full} = 8.087$ kNm determined using formulae (2.18) with the value of $\Psi = 7.9015\text{x}10^{-6}$ mm^{-2} (assumed according to CNR-DT 210/2013 [3]) – result in the value of parameter $\eta_2 = 0.3193$. The effective thicknesses determined using analytical formulae (2.20) are equal to: for deflections – $t_{eff,w} = 1.11$ cm and for stresses – $t_{eff,\sigma} = 1.24$ cm. Computer modelling using FEM is necessary to obtain the final results (extreme stresses and deflections) for a given static scheme of a plate with pointwise support. Such calculations have been performed using two independent computer programs, and the obtained results are juxtaposed in the Tables 3 and 4. Computational assumptions were similar to those applied in the case of monolithic glass plate, however finer mesh near the point supports was used (Tab. 3). In particular maps of deflections and tensile principal stresses obtained for a pointwise supported plate with restricted in plane displacements are depicted in the Table 3, while Table 4 contains analogous maps determined for a pointwise supported plate with unrestricted in plane displacements.

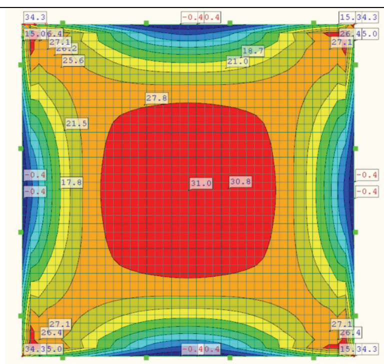
A comparison of extreme deflections and stresses in the Tables 3 and 4 indicates, that FEM modelling yields smaller deflections and stresses in the case of supports with restricted in plane displacements. One should note, however, that application of point supports in form of the so called rotule does not ensure the restriction of displacement due to application of elastic pads. Another remark resulting from the comparison of modelling results concerns the full convergence of the results obtained by two different, generally available, computer programs. Standard FEM computer programs include nonlinear theory of thin plates, appropriate for precise modelling of plates made of monolithic and laminated glass. Finally, it is necessary to indicate that extreme values of stresses in the support zone (Tab. 3) are inaccurate. This is due to the simplified boundary conditions and the application of shell elements. In order to obtain more precise results, 3D finite analysis based on solid elements should be performed. Several other possibilities are given in [3].

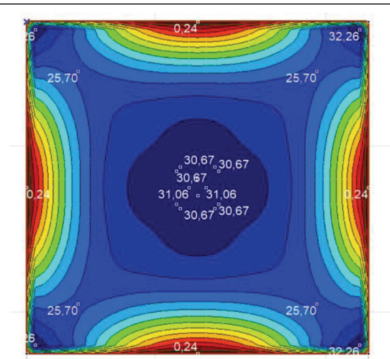
The second laminated plate is a repetitive segment of a walkway inside a building. A 2000x2000 mm plate supported along four edges with or without restraining the in plane displacements. A nominal 10 years long time of live loads acting on the bonding layer (in the temperature of 25 °C) has been assumed.

Table 5. Results of FEM modelling of 2000x2000 mm laminated plate – support along 4 edges

<table>
<tr><td>Computer program 1.</td><td>Computer program 2.</td></tr>
<tr><td colspan="2">Silding supports – non-linear plate theory

</td></tr>
<tr><td>

w_{max} = 16,2 mm</td><td>

w_{max} = 16,3 mm</td></tr>
<tr><td>

σ_{max} = 31,0 MPa (34.3 MPa)*</td><td>

σ_{max} = 30,7 MPa (32,6 MPa)*</td></tr>
<tr><td colspan="2">$(\sigma_{max})^{*}$ - local stresses at corners of the plate</td></tr>
</table>

This reduces the shear modulus of the bonding layer according to the Table 2 to G_F = 0.5 MPa. The plate stiffnesses ΣD_i = 12.15 kNm and D_{full} = 15.67 kNm, calculated using formulae (2.18) for the value $\Psi = 4.9705 \text{x} 10^{-6}$ mm^{-2} (assumed according to CNR-DT 210/2013 [3]) – result in the value of parameter $\eta_2 = 0.3189$. The effective thicknesses determined using analytical formulae (2.20) are equal to: for deflections – $t_{eff,w} = 1.385$ cm and for stresses – $t_{eff,\sigma} = 1.554$ cm.

The results of FEM modelling for linear edge supports allowing for in plane displacements are presented in the Table 5. A comparison of extreme displacements and stresses confirms the convergence of results obtained by two independent FEM computer programs. In particular the maximum deflections have been determined as $w = 16.2$ mm and 16.3 mm, while the maximum tensile stresses $\sigma_{max} = 31.0$ MN/m^2 and $\sigma_{max} = 30.7$ MN/m^2.

3. SUMMARY

The static calculations of glass plates made of monolithic or laminated glass should be based on FEM computer modelling with application of nonlinear thin plate theory. In the simple cases of monolithic glass structures analytical methods may be applied, and in particular the new procedure recommended in the document CNR-DT210/2013. The comparative analysis conducted in this paper indicates, that the new procedure yields results convergent with the results of computer modelling. The alternative semi-empirical method according to the DIN-EN 13474 code may not be applied in every case, as the results are based on discrete (tabulated) solutions with restricted scope of normalized loads. The procedure according to formulae (2.1)÷(2.4) does not pose such complications, as the plate deflection and principal stress function parameters are formulated in an analytical manner and hold in the full load variability range.

Positive results of static analysis of laminated glass panels have been obtained using a new calculation method according to the Galuppi-Royer-Carfagni model. This procedure is universal, in the sense that static calculations of plates with different boundary conditions and uniform or concentrated loads are possible. In cases of pointwise support, however, the method requires computer support (analytical method can be used for edge support), and the correct results are obtained in the form of σ_{max} tensile principal stress maps. It should be underlined here, that for glass plates the maps of equivalent stresses according to the H-M-H hypothesis are incorrect, since glass exhibits large difference between the tensile and compressive strength and thus the principal stresses are authoritative.

REFERENCES

1. J. Belis, D. Mocibob, A. Luible A, M. Vanderbroek, "On the size and shape of initial out-of-plane curvatures in structural glass components" in Constriction and Building Materials, vol 25, no 5, pp. 2700-2712. 2011.
2. DIN-EN 13474. Glas im Bauwesen: Teil 1: Allgemeine Grundlagen für Entwurf, Berechnung und Bemes-sung, 1999. Teil 2: Bemessung von Glasscheiben; Bemessung für gleichmäßig ver-teilte Belastungen, 2000.
3. Document CNR-DT 210/2013, "Guide for the design. Construction and control of buildings with structural glass elements". National Research Council of Italy. 2013.
4. M. Feldmann, R. Kasper,... "Guidance for European Structural Design of Glass Components". JRC and Policy Reports, Document CEN/TC 250 N 1060. 2014.
5. M. Feldmann, R. Kasper, K. Langosch, "Glass für tragende Bauteile".Werner Verlag 2012.
6. M. Feldmann, K. Langosch, "Zum Biegeverchalten von VSG-Laminaten unter Quer- oder Längsbelastung". Stahlbau Spezial 2011: Glasbau – Glass in Building. Heft März. Ernst & Sohn Verlag 2011.
7. L. Galuppi, L. G. Royer-Carfagni, "The effective thickness of laminated glass plates", Journal of Mechanics of Materials and Structures, **7**: 375-400, 2012.
8. L. Galuppi, G. Royer-Carfagni, "Effective thickness of laminated glass beams: New expression via a variational approach", Engineering Structures, 38:53-67, 2012.
9. M. Gwóźdź, "Konstrukcje szklane i aluminiowo-szklane". Wydawnictwo Politechniki Krakowskiej, Kraków 2020.
10. M. Gwóźdź, "Formulae for buckling load bearing capacity of glass structure elements". Archives of Civil Engineering. Vol. LXVI, Issue 2,2020.
11. T. Holberndt, "Entwicklung eines Bemessungskonzepts für den Nachweis von stablilitätsgefährdeten Glasträgern unter Biegebeanspruchung". Dissertation, TU Berlin, Germany 2006.
12. K. Langosch, "Das tragverhalten von Glasstützen mit Mono- und Verbundquerschnitten". Dissertation RWTH Aachen University, Lehrstuhl für Stahlbau und Leichtmetalbau
13. K. Langosch, M. Feldmann, "Wandartige Monoglasstützen unter axialen Drucklasten Und Biegung". In Glasbau 2013, Hrsg. Weller B., Tasche S., Ernst & Sohn.
14. A. Luible, "Stabilität von Tragelementen aus Glas". Dissertation Thèse No 3014, EPFL Lausanne, Switzerland, 2004.
15. J. Liess, "Bemessung druckbelasteter Bauteile aus Glas". Dissertation, Universität Kasse, Germany 2001.
16. pr-EN 16612, "Glass in building. – Determination of the load resistance of glass panes by calculation and testing". NA 005-09-25 AA N870, CEN/TC 129/WG8-N312.
17. Saflex® DG structural interlayer (www.mepla.net/media/medien/product...). Architectural Technical Applications Center. 2015
18. E. Wölfel, "Elastic Composite: An Approximation Solution and its Application Possibilities". Stahlbau, 6: 173-180. 1987.
19. AxisVM X5 User's Manual, Inter-CAD Kft., Hungary 2020.
20. Program RFEM 5. Program Description, Dlubal Software GmbH, Germany 2016

LIST OF FIGURES AND TABLES:

NOWE METODY ANALIZY STATYCZNEJ PŁYT ZE SZKŁA MONOLITYCZNEGO I LAMINOWANEGO

Słowa kluczowe: szkło, wytrzymałość, szkła, płyty, statyka nieliniowa, modelowanie komputerowe

STRESZCZENIE. Wdrożenie w Europie nowej, wysokowydajnej technologii produkcji szkła płaskiego float, w powiązaniu z rosnącymi wymaganiami budownictwa, dotyczącymi nowych funkcji szkła, doprowadziło do znaczącego rozwoju teorii konstrukcji szklanych. Wieloletnie badania naukowe prowadzone w krajach Unii Europejskiej zostały zwieńczone dokumentem technicznym CEN/TC 250 N 1060, zredagowanym w ramach prac Europejskiego Komitetu Normalizacyjnego nad drugą edycją Eurokodów (EC). W drugiej edycji Eurokodów przewidziano rekomendacje w/z projektowania konstrukcji szklanych, a w szczególności opracowanie odrębnej normy projektowania, zawierającej nowoczesne procedury w zakresie obliczeń statycznych i stateczności konstrukcji budowlanych szklanych (por. prace M. Feldmann, R. Kasper, K. Langosch i inne). W artykule podano nowe metody analizy statycznej płyt ze szkła monolitycznego i laminowanego, zadeklarowane w dokumencie CEN/TC 250 N 1060 (2014) i rekomendowane w dokumencie normalizacyjnym krajowym CNR-DT 210 (Włoski Komitet Normalizacyjny, Włochy, 2013). Przywołane metody analizy statycznej płyt nie są w krajowym środowisku inżynierskim powszechnie znane dlatego wymagają popularyzacji i weryfikacji regionalnej. Taki charakter mają przeprowadzone w pracy symulacje analityczne i numeryczne dla płyt prostokątnych ze szkła monolitycznego i laminowanego, o różnych warunkach podparcia. Rezultaty obliczeń numerycznych stanowią podstawę przeprowadzonej dyskusji nowych metod analizy statycznej płyt.

Received: 08.07.2020, Revised: 22.09.2020