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THE METHOD OF DETERMINING HORIZONTAL CURVATURE IN GEOMETRICAL LAYOUTS OF RAILWAY TRACK WITH THE USE OF MOVING CHORD

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In the paper, the issue of determining the horizontal curvature of the railway track axis was discussed to define unknown geometric characteristics of the measured route (location of straight and arched sections, circular arc radii, transition curve lengths, etc.). This problem has not been solved yet, and so far geometrical characteristics have been identified by approximate methods (e.g. horizontal arrow chart). Operating the angles of tangent to the geometrical layout (resulting from the very definition of curvature) seems very difficult in a real railway track reproduced on the basis of measurements. Therefore, a new concept has emerged to determine the curvature of the track not with the use of tangent but corresponding chords. In this way, the idea of curvature determination using the method of changing the slope angles of the moving chord was developed. Verification of the proposed method, carried out on a clearly defined basic geometric system of tracks, showed a sufficient compliance of the obtained curvature charts with the charts of the corresponding geometric solution. In order to use this method, one must know the coordinates of the points of a given section of the route in the Cartesian system.

Keywords: Track Geometry, Horizontal Curvature, Use of Moving Chord, Verification of a New Method

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1. INTRODUCTION

The geometrical shape of the railway track plays a key role in the safety of rail transport. The designation and assessment of the shape of the track axis in the horizontal plane, determining the possible train speed, is of particular importance here. The purpose of this operation is to define the basic geometric parameters of the route:

- location and length of straight sections,
- location of circular arcs together with their radius and length,
- location of transition curves with specification of their type and length.

Basing on this data, it becomes possible to simulate the train passing in a given geometrical layout (and thus determining the speed), as well as to obtain data for designing the track axis adjustment, which seems to be the fundamental issue.

The present measuring methods have a very long tradition and although they are subject to various innovations, they are characterized by high labour intensity and the associated necessity to incur significant financial outlays. The rules for performing measurements are similar in different railway managements [1-6]. In accordance with the provisions in force in Poland [7], straight sections are measured using the ordinate and abscissa method along the measuring lines, with an appropriate geodetic matrix. The measurements on the arc include the measurement of horizontal arrows related to the chord determined by the theodolite target axis (the so-called direct method) or the track axis adjustment signs (indirect method); measurement accuracy is up to 1 mm.

A more modern measuring system is the Total Station placed on a trolley; it allows the measurement of track inclination (using an inclinometer) and its width. The position is determined by indenting back to four geodetic network points (with reference to the 1st class state surveying grid). Coordinates are measured at intervals of 10 m on straight sections and every 5 m on sections located in the arc, in relation to the point on the inside of the head of the right rail track.

A major improvement of the existing situation should be ensured by the mobile satellite measurement method developed in Poland 10 years ago [8-10]. It involves continuous registration of track axis coordinates using GNSS receivers installed on a moving flat car. It also means using collected measurement data in appropriate calculation algorithms (so-called analytical design method [11-12]). The advantages of this method have been recognized by the national infrastructure manager – PKP Polskie Linie Kolejowe S. A. It was proved in the launch of the BRIK research project [13], which aims to achieve a practical solution.

The number of track axis coordinates obtained in mobile satellite measurements depends on the speed of the measurement flat car and the frequency of receivers used. For example, for a speed of 20 km/h and 20 Hz antenna frequency, the distance between the measuring points is 28 cm; measuring a distance of 1 km takes 3 minutes. These are, therefore, best values compared to other methods.

As a result of measurements a set of numerical data is available, which, after proper processing, creates a set of coordinates in the appropriate Cartesian system. In Poland the national space reference system 2000 (the so-called PL-2000 system) is in use. Fig. 1 shows an example section of rail route in a horizontal plane [14]. The horizontal axis in chart *Y* denotes the east, and vertical axis *X* – the north.

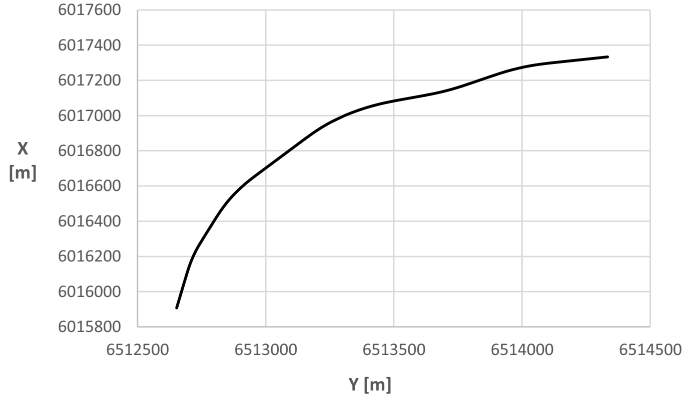

Fig. 1. Illustrative segment of the railway route in the national space reference system 2000 [14] (where the axes represent two different scales)

Visualization of the route shown in Fig. 1 is undoubtedly of great value but from the point of view of the basic aim of the undertaken actions, i.e. determining geometrical parameters of the route, the matter is not so simple. The transposition of measurement points into the local coordinate system, used in the analytical design method [11], in which the adjacent main directions of the route are arranged symmetrically, is not very helpful.

So, to determine the unknown geometrical characteristics of the measured route (location of straight and arched sections, radii of circular arches, lengths of transition curves and others), one should use the graph of measured horizontal arrows. The measurement of arrows (both horizontal and vertical) has for many years been the basis for diagnostic methods relating to the assessment of the geometrical condition of railway tracks [15-17]. Another approximate method is the so-called "curvature chart" built in an approximate way – on circles designated by three points located along the track. To improve the existing situation a solution consisting in determining the curvature of the geometric layout seems to be the first thing that comes to one's mind in this situation.

2. DEFINING THE TRACK CURVATURE

The curvature of the track is determined along its length l in a linear coordinate system. The measure of the track curvature is the ratio of the angle by which the direction of the longitudinal axis of the vehicle changes after passing a certain arc, to the length of that arc (Fig. 2). The curvature of curve K at point M is the limit to which the ratio of the acute angle $\Delta\theta$ contained between the tangents to curve K at points M and M_1 to the arc length Δl tends, when M_1 follows curve K to point M.

$$k = \lim_{\Delta l \to \infty} \left| \frac{\Delta\Theta}{\Delta l} \right| \tag{2.1}$$

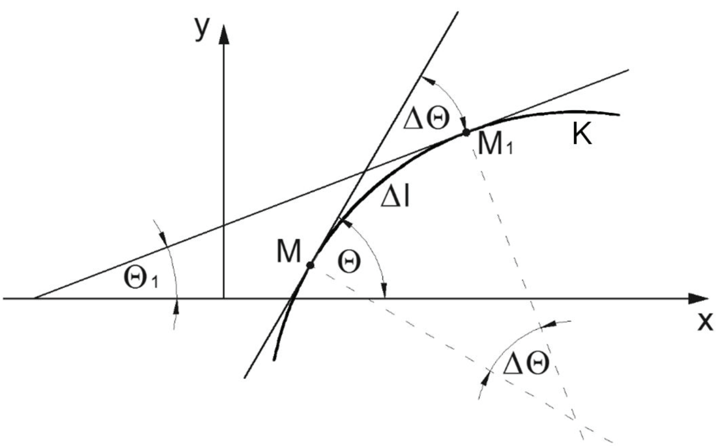

Fig. 2. Schematic diagram explaining the concept of curvature

If the analytical notation of function $\Theta(l)$ is known, then the following formula applies

$$k(l) = \frac{d}{dl}\Theta(l) \tag{2.2}$$

Based on formula (2.1), it is very easy to determine the curvature for a circular arc (i.e. a circle) with radius R. The angle $\Delta\theta$ here is equal to the angle between the radiuses reaching the contact points, which leads to obtaining a constant curvature $k = 1/R$, expressed in (rad/m).

The issue of determining curvature on sections with variable curvature (i.e. on transition curves) is much more complex. However, there is no reason to treat any curve as a series of circular arcs with a changing radius (which is widely practiced). The curvature distribution should be formed here along the length of the curve for variable l. It also applies to the railway track deformed by frequent use (as in Fig. 1).

Designing a geometric system, however, requires operating in a Cartesian coordinate system. As a result of this process, the track axis coordinates are determined, allowing it to be delineated on site. Determining the curvature of a given geometrical layout is therefore difficult because direct use of formulas from a linear system is impossible.

3. THE IDEA OF THE PROPOSED METHOD OF CHANGING THE INCLINATION ANGLES OF THE MOVING CHORD

The need to use geometric layout tangent inclination angles results from the very definition of curvature. This is obviously not a problem, provided you possess an analytical record of a given curve. However, due to the routine operation and deformation of the railway track in real conditions, determining the location of straight tangents is very difficult. The situation is completely different in relation to stretched measuring chords, the position of which is always clearly determined. Therefore, the concept appeared to use corresponding chords rather than tangents when determining the curvature of the track. It was based on the assumption that for the considered small sections of the track they are parallel to each other, while the contact points project perpendicularly onto the centre of a given chord. Fig. 3 presents a schematic diagram for determining the curvature according to the proposed method of changing the angles of the measuring chord.

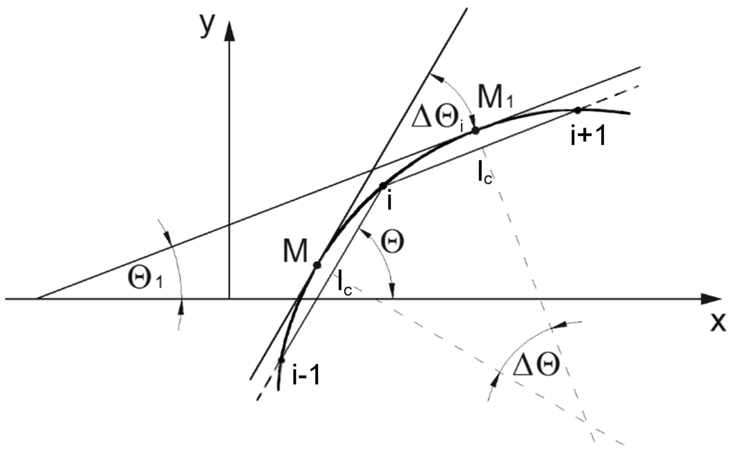

Fig. 3. Schematic diagram of the method for changing the inclination angles of a measuring chord

The curvature k_i is determined by the following formula:

$$k_i = \frac{\Delta\Theta_i}{l_c} \tag{3.1}$$

where l_c is the length of the measuring chord, angle $\Delta\Theta_i$ results from the difference in the inclination angle of the chords that intersect at point i, i.e.

$$\Delta\Theta_i = \Theta_{i\div(i+1)} - \Theta_{(i-1)\div i} \tag{3.2}$$

The application of this procedure requires the knowledge of a given curve coordinates in the Cartesian system (written analytically or discretely), because the angle values $\Theta_{(i-1)\div i}$ and $\Theta_{i\div(i+1)}$ result from the coefficients of the inclination of the straight lines describing both chords.

4. Verification of the proposed method

The verification of the method of changing the inclination angles of the measuring chord was carried out on an elementary geometrical track layout, consisting of a circular arc and two symmetrical transition curves (of the same type and length). The universal mathematical notation of such a layout is presented in [11]. As part of the verification, many cases were considered for different train speeds, with different types of transition curves and route diversion angles used. Two types of transition curves were used:

- the clothoid commonly used on railroad, described by parametric equations

$$x(l) = l - \frac{1}{40R^2l_k^2}l^5 + \frac{1}{3456R^4l_k^4}l^9 - \frac{1}{599040R^6l_k^6}l^{13} \tag{4.1}$$

$$y(l) = \frac{1}{6Rl_k}l^3 - \frac{1}{336R^3l_k^3}l^7 + \frac{1}{42240R^5l_k^5}l^{11} \tag{4.2}$$

- the new transition curve proposed in [18], described by parametric equations

$$\begin{aligned} x(l) = l &- \frac{1}{40R^2l_k^2}l^5 - \frac{1}{36R^2l_k^3}l^6 + \frac{5}{504R^2l_k^4}l^7 + \frac{1}{96R^2l_k^5}l^8 + \left(\frac{1}{3456R^4l_k^4} - \frac{1}{288R^2l_k^6}\right)l^9 + \\ &+ \frac{1}{1440R^4l_k^5}l^{10} + \frac{1}{6336R^4l_k^6}l^{11} - \frac{19}{31104R^4l_k^7}l^{12} \end{aligned} \tag{4.3}$$

$$\begin{aligned} y(l) = &\frac{1}{6Rl_k}l^3 + \frac{1}{12Rl_k^2}l^4 - \frac{1}{20Rl_k^3}l^5 - \frac{1}{336R^3l_k^3}l^7 - \frac{1}{192R^3l_k^4}l^8 + \frac{1}{2592R^3l_k^5}l^9 + \\ &+ \frac{69}{19440R^3l_k^6}l^{10} + \left(\frac{1}{42240R^5l_k^5} - \frac{1}{6336R^3l_k^7}\right)l^{11} + \left(\frac{1}{13824R^5l_k^6} - \frac{1}{1152R^3l_k^8}\right)l^{12} \end{aligned} \tag{4.4}$$

In the analytical design method [11], the area of a circular arc is described by an explicit function $y(x)$. In the case of transition curves, the applied procedure allows to operate with the parametric equations $x(l)$ and $y(l)$ (as in this paper), as well as with the equations $y(x)$. The latter mainly relate to roads [19-23].

The course of proceedings consists of two main stages. First, the coordinates of successive points of the curve, separated from each other by l_c (i.e. the length of the chord, in a straight line) are determined. As part of the verification, l_c = 5 m was adopted and, due to the symmetry of the geometric layout, the procedure started from the point located in the centre of the layout (i.e. on the circular arc), first dealing with the right side of the system (Fig. 4). Next, like in the mirror image, the required data for the left side of the system was completed.

In the second stage, the curvature of the track axis is determined. The main focus is on determining angles $\Theta_{(i-1)\div i}$ and $\Theta_{i\div(i+1)}$. To do this, first determine the inclination coefficients of both chords in contact, using the following formulas:

(4.5)
$$s_{(i-1)\div i} = \frac{y_i - y_{i-1}}{x_i - x_{i-1}}$$

(4.6)
$$s_{i\div(i+1)} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

Because $\Theta_{(i-1)\div i} = \arctan s_{(i-1)\div i}$, and $\Theta_{i\div(i+1)} = \arctan s_{i\div(i+1)}$, formula (3.2) may be applied and the curvature may be established using formula (3.1).

Fig. 4 shows a horizontal ordinate graph $y(x)$ obtained by the analytical design method [11] for the route diversion angle $\alpha = \pi/6$ rad, the radius of the circular arc $R = 800$ m and the transition curve in the form of a clothoid $l_k = 105$ m long. Fig. 5 shows the corresponding graph of the ordinates of curvature $k(x)$ determined by the method of changing the angles of inclination of the moving chord. Fig. 6 contains chart $k(x)$ prepared for a new transition curve with $l_k = 135$ m (with the same α and R as in Fig. 5).

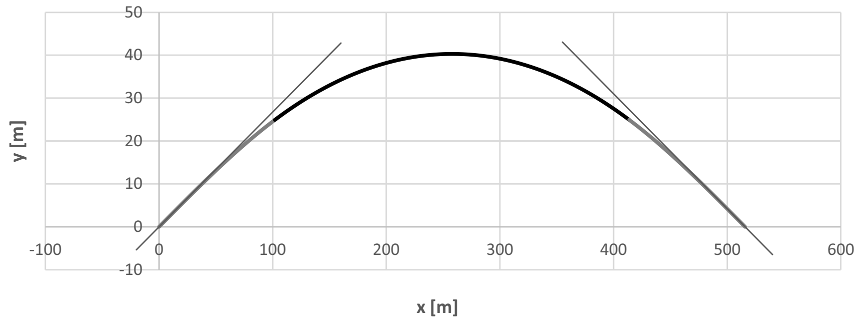

Fig. 4. Chart of the horizontal ordinates $y(x)$ in the analytical design method [11] for $\alpha = \pi/6$ rad, $R = 800$ m, clothoid $l_k = 105$ m (where the axes represent two different scales)

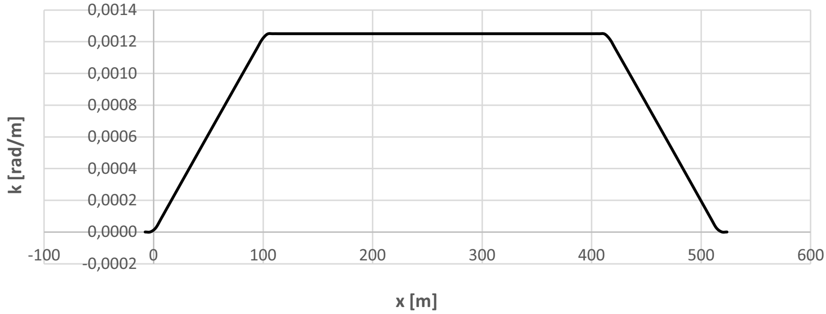

Fig. 5. Chart of the ordinates of curvature $k(x)$ determined by the method of changing the angles of inclination of the moving chord ($\alpha = \pi/6$ rad, $R = 800$ m, clothoid $l_k = 105$ m)

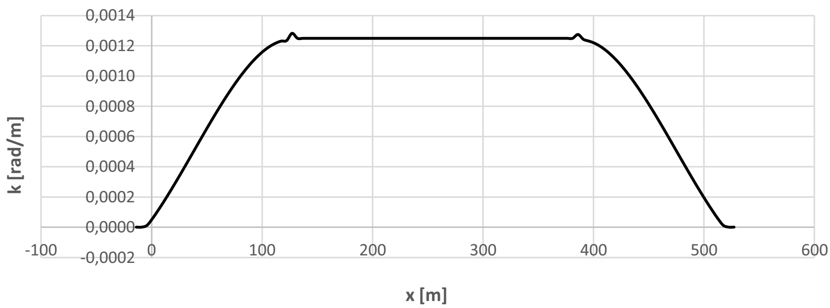

Fig. 6. Chart of the ordinates of curvature $k(x)$ determined by the method of changing the angles of inclination of the moving chord ($\alpha = \pi/6$ rad, $R = 800$ m, new curve $l_k = 135$ m)

The verification which was carried out, fully confirmed the correctness of the proposed method of determining the curvature of the track axis. Sample curvature charts in Figs. 5 and 6 show a sufficient compliance with the charts constituting the basis for obtaining the corresponding geometric solution. Along the whole circular arc the value of curvature is constant and for nominal $R = 800$ m it gives radius values equal to 799.9987 m for both cases presented.

As for the transition curves, a model (i.e. linear) curvature course for clothoid (Fig. 5) and curvature smoothed out in its final segment for the new transition curve (Fig. 6) were obtained. In the case of the new curve, there is locally some slight disturbance in the area of transition from the curve into the circular arc. This disturbance can be easily eased by shortening the measuring chord; this, however, does not seem to be expedient as it would not allow for finding the borderline between the transition curve and the circular arc (in the case of clothoid the border line is the point of bend of straight sections on the curvature chart).

5. THE SCOPE OF APPLICATION

So far our considerations have been of theoretical nature and concerned geometrical layouts of track axes with given characteristics. They mainly included the verification of a new method for determining track curvature. The practical aspect of these considerations may be fully revealed when geometrical characteristics of the track axis are not known and then their determination becomes the main goal. This situation occurs in Fig. 1.

When applying the proposed method of changing the angles of inclination of the moving chord, one basic condition must be met – it is necessary to know the coordinates of the points of a given section of the route in the Cartesian system. In most cases those values will be defined by taking measurements. In this situation, this method ideally corresponds to basic principles of mobile satellite measurements [8-10]. Those measurements provide track axis coordinates in the rectangular coordinate system. The number of coordinates is very large and they may be gathered in a very short time.

To determine the horizontal curvature at point *i,* whose coordinates Y_i and X_i are known, it is necessary to set the end of one (rear) chord and the beginning of the second chord facing forward at this point. The basic task will be to determine the coordinates of the starting point of the rear chord $Y_{i(B)}$ and $X_{i(B)}$, as well as the end point of the chord at the front $Y_{i(F)}$ i $X_{i(F)}$. This will help to determine the inclination of both chords using the following formulas:

$$s_{i(B)\div i} = \frac{X_i - X_{i(B)}}{Y_i - Y_{i(B)}} \tag{5.1}$$

$$s_{i\div i(F)} = \frac{X_{i(F)} - X_i}{Y_{i(F)} - Y_i} \tag{5.2}$$

and then determining the curvature using formula (3.1), wherein $\Delta\Theta_i = \Theta_{i\div i(F)} - \Theta_{i(B)\div i}$, $\Theta_{i(B)\div i} = \operatorname{atan} s_{i(B)\div i}$, $\Theta_{i\div i(F)} = \operatorname{atan} s_{i\div i(F)}$.

Mobile satellite measurements also create other, additional opportunities. When operating a measuring flat car with two satellite receivers installed on bogie kingpins, satellite antennas basically define the measuring chord; the method of changing the inclination angle of the chord for determining the curvature becomes even more effective.

6. CONCLUSIONS

The basis for determining the geometrical characteristics of the measured railway route should be its horizontal curvature. The fact that it is necessary to use angles of tangent inclination to the geometric system results from the definition of curvature. In a real railway track, recreated on the basis of measurements, defining the location of straight tangents appears to be very difficult. Therefore, the concept appeared not to use tangents but corresponding measuring chords when determining the curvature of the track. It was assumed that for the considered small sections of the track they are parallel to each other. In this way, the idea of curvature determination using the method of changing the angles of the chord was created.

The verification of the proposed method was carried out on a clearly defined elementary geometrical layout of tracks, consisting of a circular arc and two symmetrically set transition curves (of the same type and length), calculated according to the principles of the analytical design method [11]. A number of geometrical cases were considered, achieving sufficient compliance of the obtained curvature charts with the charts constituting the basis for obtaining the corresponding geometric solution. This concerned both sections of the circular arc as well as sections of transition curves.

The proposed method creates a great range of applications. The practical aspect of the presented considerations may be revealed when the geometrical characteristics of the track axis determined by measurements are not known and then the basic goal will be to find them out. In this situation, the discussed method ideally corresponds to mobile satellite measurements. These measurements define a great number of track axis coordinates in a rectangular coordinate system in a very short time.

REFERENCES

1. “British railway track design, construction and maintenance, 6th ed.”, London, UK: The Permanent Way Institution, 1993.
2. ”Railway applications—Track—Track alignment design parameters—Track gauges 1435 mm and wider—Part 1: Plain line. EN 13803-1”, Brussels, Belgium: CEN (European Committee for Standardization), 2010.

3. ”883.2000 DB_REF-Festpunktfeld”, Berlin, Germany: DB (Deutsche Bahn) Netz AG, 2016.
4. ”Code of federal regulations title 49 transportation”, FRA (Federal Railroad Administration), Washington, DC: US Government Printing Office, 2008.
5. ”NR/L3/TRK/0030 NR_Reinstatement of Absolute Track Geometry (WCRL Routes), Iss. 1”, London, UK: NR (Network Rail), 2008.
6. ”Standard: Railway Surveying, Version 1.0. T HR TR 13000 ST”, Sydney, Australia: NSW (New South Wales) Government (Transport for NSW), 2016.
7. PKP (Polish State Railways). 2018. “Standardy Techniczne – Szczegółowe warunki techniczne dla modernizacji lub budowy linii kolejowych do predkości Vmax ≤ 200 km/h (dla taboru konwencjonalnego) / 250 km/h (dla taboru z wychylnym pudłem) – TOM I - DROGA SZYNOWA – Załącznik ST-T1_A6: Układy geometryczne torów”, Warsaw, Poland: PKP Polskie Linie Kolejowe, 2018.
8. W. Koc, C. Specht, ”Selected problems of determining the course of railway routes by use of GPS network solution”, Archives of Transport 23 (3): 303-320, 2011.
9. C. Specht, W. Koc, ”Mobile satellite measurements in designing and exploitation of rail roads”, Transportation Research Procedia 14: 625-634, 2016..
10. C. Specht, W. Koc, P. Chrostowski, J. Szmagliński, ”Accuracy assessment of mobile satellite measurements in relation to the geometrical layout of rail tracks”, Metrology and Measurement Systems 26 (2): 309-321, 2019.
11. W. Koc, ”Design of rail-track geometric systems by satellite measurement”, Journal of Transportation Engineering 138 (1): 114-122, 2012.
12. W. Koc, “Analytical method of modelling the geometric system of communication route”, Mathematical Problems in Engineering 2014: 679817.
13. A. Wilk, C. Specht, W. Koc, K. Karwowski, P. Chrostowski, J. Szmagliński, P. Dąbrowski, M. Specht, S. Judek, J. Skibicki, M. Skóra, S. Grulkowski, ”Research project BRIK: development of an innovative method for determining the precise trajectory of a railway vehicle”, Transportation Oveview – Przegląd Komunikacyjny 74 (7): 32-47, 2019.
14. W. Koc, “The analytical design method of railway route’s main directions intersection area.” Open Engineering 6 (1): 1-9, 2016.
15. L. Marx, “Satellitengestützte Gleisvermessung – auch beim Oberbau”, EI – Eisenbahningenieur 58 (6): 9-14, 2007.
16. C. Qijin, N. Xiaoji, Z. Lili, Z. Tisheng, X. Fuqin, L. Yi, L. Jingnan, “A railway track geometry measuring trolley system based on aided INS,”, Sensors 18 (2): 538, 2018.
17. Y. Naganuma, T. Yasukuni, T. Uematsu, “Development of an inertial track geometry measuring trolley and utilization of its high-precision data”, International Journal of Transport Development and Integration 3 (3): 271–285, 2019.
18. W. Koc, ”New transition curve adapted to railway operational requirements”, Journal of Surveying Engineering 145 (3): 04019009, 2019.
19. R. J. Grabowski, ”Kształtowanie geometryczne krzywych przejściowych w drogach kołowych, kolejowych i trasach wodnych”, Rozprawy Naukowe 38, Białystok: Wydawnictwa Politechniki Białostockiej, 1996.
20. A. Kobryń, ”Polynomial solutions of transition curves”, Journal of Surveying Engineering 137(3): 71-80, 2011.
21. A. Kobryń, “New solutions for general transition curves”, Journal of Surveying Engineering 140(1): 12-21, 2014.
22. A. Kobryń, “Universal solutions of transition curves”, Journal of Surveying Engineering 142(4): 04016010, 2016.
23. A. Kobryń, ”Transition curves for highway geometric design”, Springer International Publishing, Series: Springer Tracts on Transportation and Traffic, 14, 2017.

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Fig. 6. Chart of the ordinates of curvature $k(x)$ determined by the method of changing the angles of inclination of the moving chord ($\alpha = \pi/6$ rad, $R = 800$ m, new curve $l_k = 135$ m)

WYKAZ RYSUNKÓW I TABEL:

METODA WYZNACZANIA KRZYWIZNY POZIOMEJ W UKŁADZIE GEOMETRYCZNYM DROGI KOLEJOWEJ Z WYKORZYSTANIEM RUCHOMEJ CIĘCIWY

Słowa kluczowe: Droga kolejowa, krzywizna pozioma, wykorzystanie ruchomej cięciwy, weryfikacja nowej metody

PODSUMOWANIE:

W pracy podjęto kwestię wyznaczania krzywizny poziomej osi toru kolejowego w celu określenia nieznanych charakterystyk geometrycznych pomierzonej trasy (położenia odcinków prostych i łukowych, promieni łuków kołowych, długości krzywych przejściowych i in.). Problem ten nie został dotąd rozwiązany, a określanie charakterystyk geometrycznych odbywa się metodami przybliżonymi (np. z wykorzystaniem wykresu strzałek poziomych). Ponieważ wynikające z definicji krzywizny operowanie kątami nachylenia stycznej do układu geometrycznego jest w rzeczywistym, odtworzonym na drodze pomiarów torze kolejowym bardzo utrudnione, pojawiła się nowa koncepcja, żeby przy wyznaczaniu krzywizny toru nie operować stycznymi lecz odpowiadającymi cięciwami. W ten sposób powstała idea wyznaczania krzywizny metodą zmiany kątów nachylenia ruchomej cięciwy pomiarowej. Weryfikacja zaproponowanej metody, przeprowadzona na jednoznacznie zdefiniowanym elementarnym układzie geometrycznym torów, wykazała całkowitą zgodność uzyskanych wykresów krzywizny z wykresami stanowiącymi podstawę uzyskania odpowiadającego rozwiązania geometrycznego. Aby można było stosować omawianą metodę, niezbędna jest znajomość współrzędnych punktów danego rejonu trasy w kartezjańskim układzie współrzędnych.

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