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RELIABILITY OF BEAMS SUBJECTED TO TORSION DESIGNED USING STM

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This paper presents probabilistic assessment of load-bearing capacity and reliability for different STM of beams loaded with a torsional and bending moment. Three beams having different reinforcement arrangement obtained on the basis of STM but the same overall geometry and loading pattern were analysed. Stochastic modelling of this beams were performed in order to assess probabilistic load-bearing capacity. In the analysis, the random character of input data - concrete and steel was assumed. During the randomization of variables the Monte Carlo simulation with the reduce the number of simulations the Latin Hypercube Sampling (LHS) method was applied. The use of simulation methods allows for approximation of implicit response functions for complex in description and non-linear reinforced concrete structures. On the basis of the analyses and examples presented in the paper, it has been shown that the adoption of different ST models determines the different reliability of the analysed systems and elements.

Keywords: strut ad tie model, torsion, reinforced concrete beam, reliability

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1. INTRODUCTION

The use of strut-and-tie model (STM) for the design of reinforced concrete structures has a very long history and is practically inseparable from the history of reinforced concrete structures. The STM is a powerful tool for analysing the structures in D-regions where the Bernoulli hypothesis does not apply. The STM design concept assumes that the designed D-Region is sufficiently ductile (to allow the force redistribute after concrete cracked) based on the lower bound plasticity theorem. The lower bound theory of limit analysis states that: "A stress field that satisfies equilibrium and does not violate yield criteria at any point provides a lower-bound estimate of capacity of elastic-perfectly plastic materials; For this to be true, crushing of concrete (struts and nodes) does not occur prior to yielding of reinforcement (ties or stirrups)". So, lower bound plasticity theorem only satisfies "equilibrium and yield criteria", where the third requirement in solid mechanics framework "strain compatibility" does not have to be satisfied. An STM idealizes a complex force flow in the structures as a collection of compression members (struts), tension members (ties), and the intersection of such members (nodes).

Designing special structures and predicting their behaviour will increase the possibility of using them in confidence, and it will also increase the performance of the structure as a whole in safety and as well as economy. Many different types of techniques and algorithms have been proposed by dozens of researchers and the selection of the optimal model has been the subject of numerous scientific works published in recent years. The choice of the STM depending on the considered issue can be made by using truss analogies, the load path and knowledge of stress trajectories based on numerical models and topological optimization. The concepts of the STM are originally referred to truss analogy proposed more than one century ago by Ritter [1] and Mörsch [2]. In subsequent years strut-and-tie modelling techniques have been extensively investigated in comprehensive works by Thürlimann [3], Schlaich et al. [4], Adebar, Kuchma and Collins [5] and many others. Despite such a large number of studies, the standard recommendations [6, 7, 8] and the literature [9] do not provide rules to determine unambiguously the shape and direction of elements in the ST method, and the choice of the STM is usually made without reliability assessment of the obtained model. Designing safe structures should be the overriding objective, since the reliability of a structure is closely related to the ways of dealing with uncertainty and making decisions in the initial design phase.

In this paper, cantilever reinforced concrete beams with the same overall geometry and loading pattern loaded but having different reinforcement arrangements obtained on the basis of STM were analysed. Stochastic modelling of this beams were performed in order to assess probabilistic load-bearing capacity. The Monte Carlo simulation was applied during the randomization of variables. In order to reduce the number of simulations to an acceptable level the Latin Hypercube Sampling (LHS) method was selected.

2. REINFORCED CONCRETE BEAMS DESIGNED WITH THE STM

The subject of analysis was a cantilever reinforced concrete beam with real dimensions shown in Fig. 1.

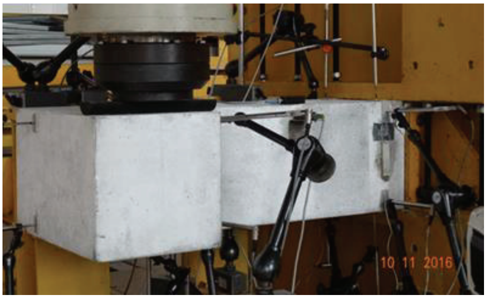
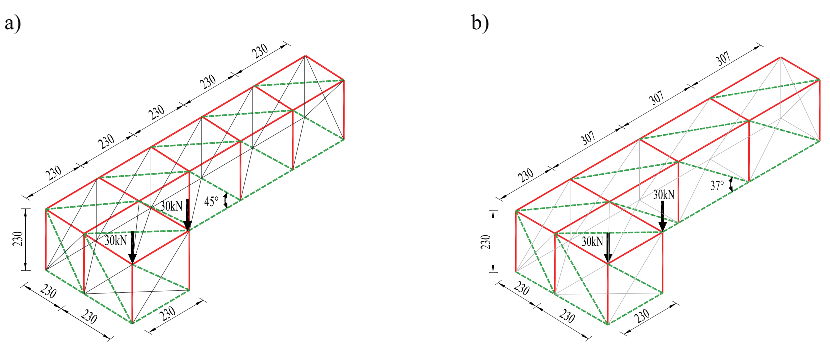


Fig. 1. The view of the analysed cantilever beam

In this article, for the beam loaded with a torsional and bending moment, three Strut and Tie Models (STMs) shown in Fig.2 were compared.



c)

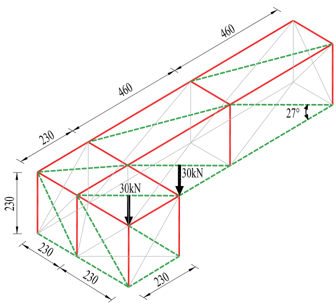


Fig. 2. STMs of cantilever reinforced concrete beams, a) model B1, b) model B2, c) model B3

For all the analysed beams (STM), the designed reinforcement was performed for the external load of 60 kN. On the basis of the value of forces in the tension bars of the truss, longitudinal reinforcement and stirrups were chosen and the conditions for anchoring the reinforcement in the nodes were checked. In the compression elements of the STMs, the condition of not exceeding the allowable stresses were checked. Struts can be assigned as prismatic shaped. The design strength for notional concrete struts were reduced in cracked compression zones according the standard EC 2 [21]. Depending on the type of nodes CCC, CCT or CTT, the design values for the compressive stresses for S- elements and the anchorage length of the reinforcement for T-elements were checked according to standard EC 2 [10].

The analysed cantilever beams with reinforcement obtained on the basis of the adopted STM are shown in Fig. 3.

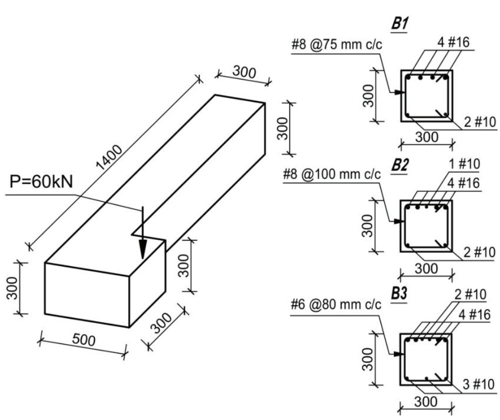


Fig. 3. Geometry and reinforcement of tested beams

3. EXPERIMENTAL RESEARCH OF BEAMS

Assessment of the reliability of beams under torsion was preceded by experimental tests, carried out on elements on a natural scale. The research was aimed at presenting the complex character of the torsion phenomenon in reinforced concrete constructions, and the obtained results were used to validate numerical models. Due to the statistical representativeness and reliability of the obtained results, each series contained four beams of the same parameters, i.e. concrete class and reinforcement. The results obtained from the measurements were statistically evaluated. The mean value and standard deviation of the selected measurement results, i.e. the maximum load, the deflection of point A and the angle of rotation of the side plane along line 2 are presented in Table 1.

Table 1. The results obtained from the measurements i.e. the maximum load, the deflection of point A and the angle of rotation

Measured values	No. beams	Mean value	Standard deviation
P [kN]	B1	121.0	4.7
	B2	109.1	7.0
	B3	97.4	8.5
u [mm]	B1	13.4	0.7
	B2	15.8	1.3
	B3	11.8	1.9
φ [deg/m]	B1	0.94	0.06
	B2	0.86	0.05
	B3	0.92	0.08

Localization of the elements i.e. point A and line 2 are shown in Fig. 4.

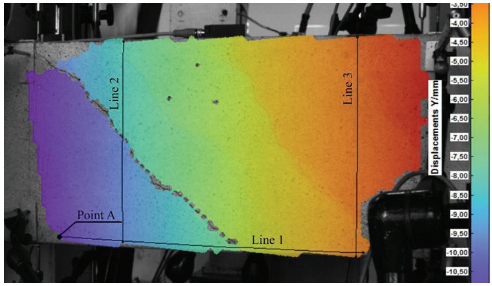


Fig. 4. Point A and linear elements defined on the beam surface

On the basis of the experiment results were noticed that the average breaking load, depending on the model, ranged from 121 kN for model 1 to 97 kN for model 3. Beams with reinforcement shaped on the basis of different ST models are also characterized by different deflection and angle of rotation of the lateral plane. There was also a different measure of scattering of the results obtained, in the case of breaking load and deflection, a relation between the value of standard deviation and the adopted ST model was visible. No significant differences were observed in the cracks image of the analysed beams, the example cracks of beams B1 and B3 are shown in Fig. 5.

a) b)

Fig. 5. The cracks image of the analysed beams, a) beam B1, b) beam B3

4. NUMERICAL MODELS OF BEAMS

In the next step, the numerical analysis of beams in ATENA 3D – Studio [11] were made. For non-linear analysis of reinforced concrete beams, it used concrete models describing the dependence $\sigma - \varepsilon$ in the complex state of stress with material parameters modified in accordance with the results of strength tests of the concrete used. The reinforcement was modelled discreetly using the reinforcement model built into the ATENA program to ensure adhesion between concrete and reinforcement. In ATENA program was used the three dimensional combined fracture–plastic material model for concrete. Tension is handled by a fracture model, based on the classical orthotropic smeared crack formulation and the crack band approach. It employs the Rankine failure criterion, exponential softening, and it can be used as a rotated or a fixed crack model. The plasticity model for concrete in compression is based on the Menétrey–Willam failure surface, the plastic volumetric strain as a hardening/softening parameter and a non-associated flow rule based on a nonlinear plastic potential function [12, 13]. Initially, the beams to be tested were supposed to be fully fixed, but during the tests, due to inaccurate beam design and surface irregularities in the adhesion of the planes, small displacements were observed at the junction between the beams and the steel members of the fixture. This was taken into account by using a conventional material in the numerical model, the "interface", the behaviour of which, under load, was supposed to reflect the minimal inaccuracies in the beam execution and the occurrence of displacements at the beam - steel elements junction. A detailed description of the numerical model of the analysed beams and its validation based on experimental studies have been presented in the publication [14].

A comparison of selected results for the beam B3 obtained from numerical simulations and from experimental data is shown in Figures 6 through 9. The location of selected points and lines, for which the detailed results are presented, is marked in Fig. 4. The curve load-deflection plot obtained from the FEA for point A is shown in Fig. 6. The curve agrees accurately with the experimental data. A comparison of displacements from numerical simulations and experimental tests of points along the lines 1, 2 and 3 is shown in Figures 7, 8 and 9. These are displacements obtained for the maximum measured forces. Fig. 7 presents vertical displacements from FEM simulations and an experimental test for the horizontal line 1. Fig. 8 shows horizontal displacement perpendicular to the plane of the beam, along the vertical line 2, caused by torsion. Fig. 9 presents horizontal displacement in the direction of the beam axis, along the vertical line 3, caused by the sliding out of (slipping off) the beam from the fixing. As shown in Figures 6 to 9, the displacement and load obtained from numerical simulations are close to the results obtained from experimental research.

Given the coefficients of variation of specific parameters, numerical simulations and experimental tests can be considered consistent.

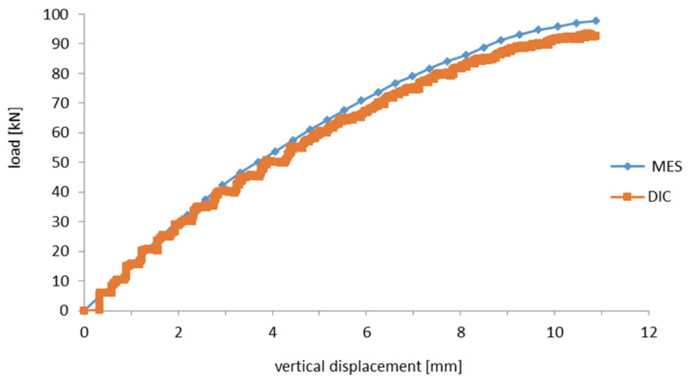


Fig. 6. Load – vertical displacements for the point A, FEM simulations and test

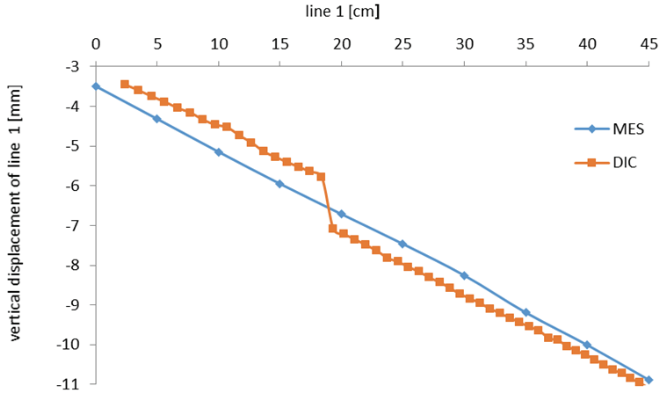


Fig. 7. Comparison of vertical displacements for the line 1, FEM simulations and test

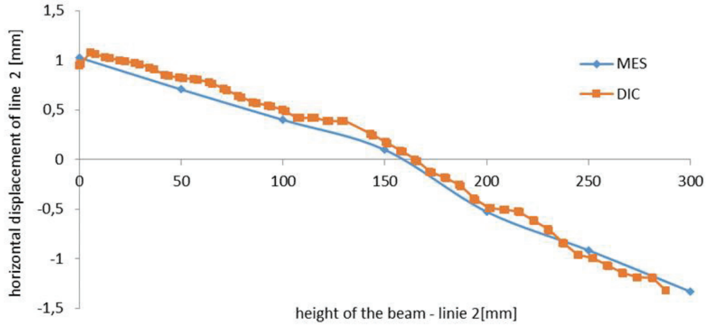


Fig. 8. Comparison of horizontal displacements for the line 2, FEM simulations and test

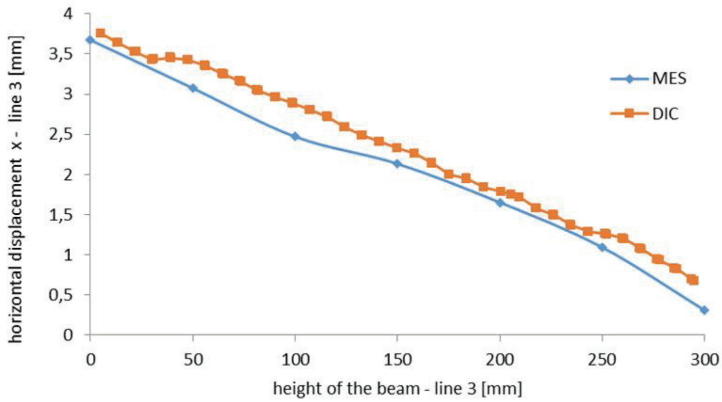


Fig. 9. Comparison of horizontal displacements for the line 3, FEM simulations and test

5. STOCHASTIC MODELLING OF BEAMS

The constructed and validated numerical models were used to estimate the reliability of the analysed beams. The objective was to study the impact of a type of STM and some input data on the bearing capacity and displacements of the beams. The reliability of beams were assessed using probabilistic method - FORM and fully probabilistic method Monte Carlo with the use of variance reduction techniques by Latin hypercube sampling (LHS). The basic feature of LHS is that the probability distribution functions for all random variables are divided into N_{Sim} equivalent intervals (N_{Sim} is a number of simulations); the values from the intervals are then used in the simulation process (random selection, middle of interval or mean value). This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function based on the inverse transformation of the distribution function. The representative parameters of variables are selected randomly, being based on random permutations of integers 1, 2, ..., j, N_{Sim}. Every interval of each variable must be used only once during the simulation – Fig.10. Being based on this precondition, a table of random permutations can be used conveniently, each row of such a table belongs to a specific simulation and the column corresponds to one of the input random variables [15].

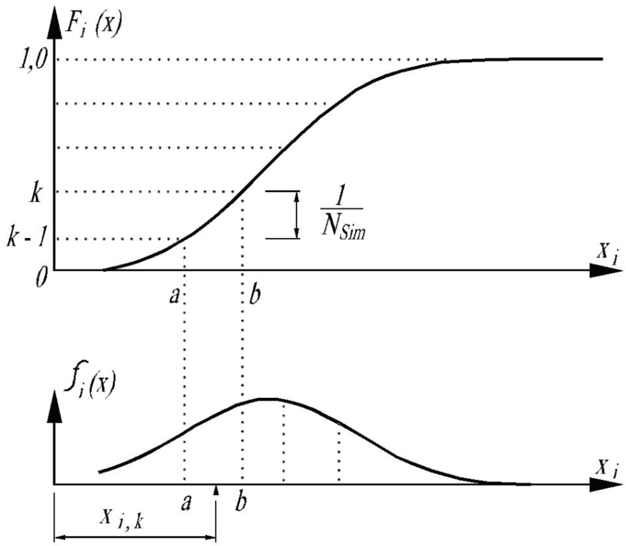


Fig. 10. Illustration of LHS - samples as the probabilistic means of intervals

The mean of each interval should be chosen as (4.1):

(4.1) $$x_{i,k} = \frac{\int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)dx}{\int_{y_{i,k-1}}^{y_{i,k}} f_i(x)dx} = N_i \cdot \int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)dx$$

where f_i is the probability density function of variable Xi and the integration limits are (4.2):

(4.2) $$y_{i,k} = F_i^{-1}\left(\frac{k}{N_i}\right)$$

The estimated mean value is achieved accurately and the variance of the sample set is much closer to the target one [16, 17].

In the analysis, the random character of input data - concrete and steel was assumed. Taking into account the calculation time and the accuracy of the results obtained, the number of samples in the LHS method was optimized. The optimization criterion was the approximate stabilization of the Cornell reliability index and the reliability index determined by the FORM method. The optimization was carried out for the B2 beam, analysing samples with 10, 30, 50 and 70 elements. Taking into account the results obtained, it was finally decided that for each of the beams, 30 simulations were performed with modified statistic parameters. The statistic parameters were described using the recommendations specified in JCSS [18], ISO [19]. The input values should be properly described, e.g. with a mean value, a coefficient of variation, or a type of distribution. The distribution and coefficient of variation - COV for the input variables of concrete and steel are shown in Table 2.

Table 2. Material properties in the stochastic modelling

Input	Mean value	Standard deviation	Distribution
E_c $[\frac{N}{mm^2}]$	34867	5230	Lognormal
f_t $[\frac{N}{mm^2}]$	4.4	0.79	Lognormal
f_c $[\frac{N}{mm^2}]$	64	4.86	Lognormal
f_y $[\frac{N}{mm^2}]$	545	27.25	Lognormal

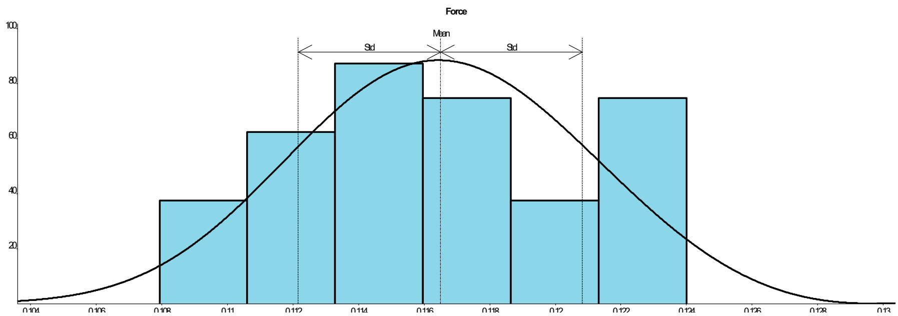
In the analysis, the correlation between parameters of concrete was assumed. The correlation matrix used for the concrete in the stochastic modelling was built based on the publication [20] and shows in Table 3.

Table 3. Correlation matrix of concrete

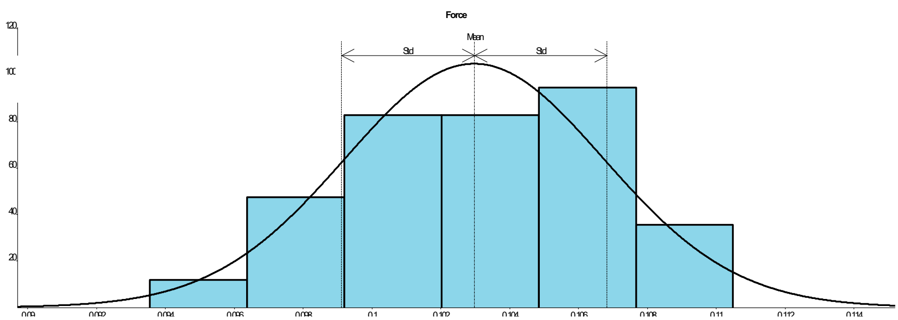
Input	E_c	f_t	f_c
E_c	1	0.9	-0.7
f_t	0.9	1	-0.8
f_c	-0.7	-0.8	1

The stochastic modelling was carried out using SARA [21] software application. For each of the beams, the results of the stochastic simulations were: estimations of the mean value, variance, coefficient of skewness, kurtosis and empirical cumulative probability density function estimated by empirical histogram structural response. This basic statistical assessment is visualized through the histograms. The histograms with estimate ultimate load for the beams B1, B2 and B3 are shown in Fig 11.

a) Histogram of ultimate load for the beam B1- the 3-Parameter Weibull Distribution



b) Histogram of ultimate load for the beam B1- Student’s t Distribution



c) Histogram of ultimate load for the beam B1- the 2-Parameter Weibull Distribution

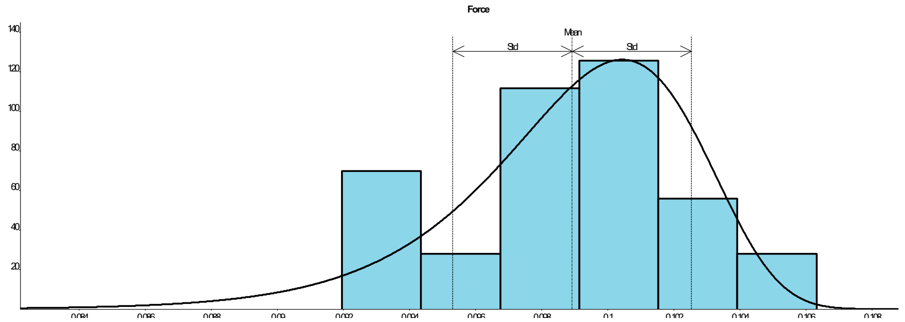


Fig. 11. The histograms with estimate ultimate load for the beams B1, B2 and B3

The mean value of the ultimate load- P, standard deviation, COV, coefficient of skewness, kurtosis, the characteristic value of the ultimate load P_k, the minimum value of load – P_{min} and the maximum value of load – P_{max} are compared in Tables 4. The P_k is a 5% fractile of the statistical distribution for P.

Table 4. Mean value, standard deviation, COV, skewness and kurtosis, upper and lower value of load bearing capacity of beams

Element	P[kN]	σ_P [kN]	ν[%]	skewness	kurtosis	P_k [kN]	P_{min}[kN]	P_{max}[kN]
B1	116.5	4.33	3.7	-0.0522	-0.846	109.3	107.9	124.0
B2	103.0	3.85	3.7	0.0503	0.095	96.6	93.5	110.5
B3	98.9	3.62	3.7	-0.1639	-0.543	92.2	91.9	106.3

The reliability analyses were carried out after statistical analyses. The limit state function -Z (margin of safety) was formulated (4.3):

$$Z = R - E \tag{4.3}$$

This function is a difference between resistance – R and load effect - E.

In presented analysed simulation methods were used for obtained implicit limit state functions - R. In simulation methods many random samples for each of the basic random variables in the problem are generated in a way that they can represent their true probability distributions and stochastic characteristics. Using each realisation of the random variables in the problem and analysing the problem deterministically will lead to a realisation of the problem itself. Each of these deterministic analyses is referred to as a simulation trial.

According to the original assumptions, the design load applied to the considered beams was 60 kN. For the load effect -E, probabilistic description by means of normal distribution with COV 0.15 was used. In presented analyses, the average value of load amounted to 45 kN.

In the next step, reliability analysis methods employing the Cornell´s reliability index – β_c and the FORM method-β_{FORM} were applied. The probability of failure, the Cornell's reliability index, the probability of failure obtained from the FORM method and the reliability index of the tested beams are presented in Table 5.

Table 5. Probability of failure and reliability index estimated by Cornella method and FORM method

Element	β_c	p_{f_C}	β_{FORM}	$p_{f_{FORM}}$
B1	8.38	$2.56 \cdot 10^{-17}$	9.61	$3.75 \cdot 10^{-22}$
B2	6.90	$2.54 \cdot 10^{-12}$	7.37	$8.77 \cdot 10^{-14}$
B3	7.32	$1.22 \cdot 10^{-13}$	6.63	$1.67 \cdot 10^{-11}$

Vertical displacements of the point A indicated in Fig and the angle of rotation of the side plane of the beam at line 2 (analogously to the experimental tests) were also statistically evaluated. The results of the statistical assessment of horizontal displacements and the angle of rotation of the side plane of the beam are presented in the Tables 6 and 7.

Table 6. Estimators of the deflection distribution function at point A

Element	u [mm]	σ_u [mm]	v[%]	β_c
B1	14.2	1.14	8.0	12.4
B2	12.6	1.02	8.1	12.3
B3	11.0	0.70	6.3	15.8

Table 7. Estimators of the side plane rotation function

Element	φ [deg/m]	σ_φ[deg/m]	β_C
B1	1.42	0.14	9.9
B2	1.39	0.21	6.8
B3	1.02	0.14	7.3

The presented results of the reliability analysis, confirmed a safe estimation of the load capacity in the STM. The most reliable, taking into account the bearing capacity, deflection and the angle of rotation of the side plane, is characterized by the B1 beam. Although the measured deflection of point A and the angle of rotation of the lateral plane are in this case the highest of the three analysed beams, the result obtained is characterized by the least coefficient of variation.

6. CONCLUSION

The paper presents the probabilistic assessment of load-bearing capacity and reliability for different STM of beams loaded with a torsional moment, bending moment as well as a shear force. Summing up the results of the analysis, the following detailed conclusions can be formulated:

- The highest values of the reliability index, taking into account the bearing capacity, deflection and angle of rotation of the side plane were obtained from the beam B1.
- Structures designed on the basis of different ST models are characterized by different load bearing capacity and different mass of reinforcement. The mean value of load bearing capacity ranged from 121 kN for the beam B1 to 97.4 kN for the beam B3.
- The result of the stochastic simulations shows that the coefficients of variation of load bearing capacity are different for the tested beams. The obtained coefficients of variation means from 4% for the beam B1 to 9% for the beam B3.
- Based on the analysis, it can be concluded that the STM with the compressed concrete struts, inclined at an angle of 45° to the axis of the bar is the most advantageous due to the bearing capacity and reliability. Therefore this STM should be recommended for element subjected to torsion and bending.

To sum up, it should be stated that the knowledge of the random nature of variable states, including the strength of materials and loads used, on the designed structure and the estimated probability of failure, allows for a better understanding of the behaviour of the tested structural system than the deterministic analysis, using even the most complex finite element model. The application of probabilistic methods allows for the selection of the most advantageous one with regard to the reliability of the ST model, and the presented examples and methods of reliability estimation may serve as an indication in the process of ST model optimization.

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LIST OF FIGURES AND TABLES:

NIEZAWODNOŚĆ BELEK SKRĘCANYCH PROJEKTOWANYCH METODĄ STRUT AND TIE

Słowa kluczowe: metoda ST, skręcanie , belki żelbetowe, niezawodność.

STRESZCZENIE

W pracy analizowano niezawodność wspornikowych belek żelbetowych. Obiektem badań były belki skręcane i zginane, w których zbrojenie zaprojektowano metodą ST , przedstawione na rys. 1.

a) b)

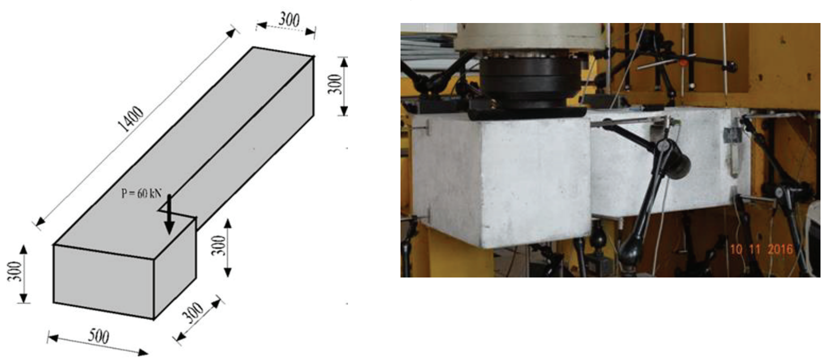

Fig. 1. Analizowana belka wspornikowa, a) wymiary, b) widok belki na stanowisku badawczym

Zbrojenie analizowanych belek uzyskano na podstawie trzech modeli Strut i Tie. Model 1 stanowiła kratownica przestrzenna, w której ściskane krzyżulce betonowe wydzielone rysami ukośnymi są na każdym boku pionowym i poziomym nachylone do osi pręta pod kątem 45^{o}. Pręty pionowe kratownicy to rozciągane zbrojenie w postaci strzemion. Pas górny i pas dolny kratownicy to odpowiednio zbrojenie rozciągane i ściskane pasy betonowe. W modelu 2 kratownicy przestrzennej, ściskane krzyżulce betonowe przyjęto nachylone do osi pręta pod kątem 37^{o}, natomiast w modelu 3 nachylone pod kątem 27^{o}.

Ocenę niezawodności belek skręcanych poprzedziły badania doświadczalne przeprowadzone na elementach w skali naturalnej. Badania miały na celu obserwację złożonego charakteru zjawiska skręcania w konstrukcjach żelbetowych, a uzyskane wyniki posłużyły do walidacji modeli numerycznych. Z uwagi na statystyczną reprezentatywność i wiarygodność otrzymanych wyników badań każda seria zawierała po 4 belki o takich samych parametrach tj. klasie betonu i zbrojeniu. Otrzymane z pomiarów wyniki podano ocenie statystycznej. Na podstawie przeprowadzonych badań doświadczalnych zauważono, że średnie obciążenie niszczące w zależności od modelu wynosiło od 121 kN dla modelu 1 do 97 kN dla modelu 3. Belki o zbrojeniu ukształtowanym na podstawie różnych modeli ST charakteryzuje też inne ugięcie oraz kąt obrotu płaszczyzny bocznej. W następnym kroku przeprowadzono analizę numeryczną belek w ATENA 3D - Studio. Do nieliniowej analizy belek żelbetowych wykorzystano modele betonu opisujące zależność $\sigma - \varepsilon$ w złożonym stanie naprężenia o parametrach materiałowych zmodyfikowany zgodnie z wynikami badań wytrzymałościowych zastosowanego betonu. Zbrojenie zamodelowano dyskretnie wykorzystując wbudowany w programie ATENA model zbrojenia zapewniający przyczepność między betonem i zbrojeniem.

Ocenę niezawodności belek przeprowadzono metodami probabilistycznymi – FORM oraz w pełni probabilistyczną Monte Carlo z wykorzystaniem technik redukcji wariancji hipersześcianem łacińskim. Metody FORM wykorzystują pełny opis probabilistyczny zmiennych losowych czyli średnią, odchylenie standardowe i informacje o funkcji gęstości. W modelu stochastycznym przyjęto, że wartości wejściowe opisane są średnią, odchyleniem standardowym i rodzajem rozkładu. Modelowanie stochastyczne przeprowadzono w programie SARA. Oszacowano nośność średnią, maksymalną i minimalną nośność uzyskaną z symulacji, odchylenie standardowe, współczynnik zmienności, współczynnik skośności oraz kurtozę dla wszystkich analizowanych belek. Na podstawie histogramów odpowiedzi i przybliżonej postaci funkcji gęstość prawdopodobieństwa nośności, uzyskanych z symulacji stochastycznej, wyznaczono wskaźnik niezawodności Cornella - βc oraz wskaźnik niezawodności metodą FORM- $\beta_{FORM.}$ W tabeli 1 zestawiono oszacowany wskaźnik niezawodności Cornella i odpowiadające mu prawdopodobieństwo zniszczenia oraz otrzymywane metodą FORM i SORM prawdopodobieństwo awarii i wskaźnik niezawodności.

Tabela 1. Prawdopodobieństwo awarii i wskaźnik niezawodności metodami FORM i Cornella

Element	β_c	p_{fc}	β_{FORM}	$p_{f_{FORM}}$
B1	8,38	$2{,}56\cdot10^{-17}$	9,61	$3{,}75\cdot10^{-22}$
B2	6,90	$2{,}54\cdot10^{-12}$	7,37	$8{,}77\cdot10^{-14}$
B3	7,32	$1{,}22\cdot10^{-13}$	6,63	$1{,}67\cdot10^{-11}$

W wyniku przeprowadzone oceny statystycznej zauważono wpływ przyjętego modelu ST na niezawodność belek. Przedstawione wyniki analizy niezawodności potwierdziły bezpieczne oszacowanie nośności metodą ST dla wszystkich badanych belek. Zauważono, że w przypadku belek B2 i B3 wskaźnik niezawodności jest niższy niż w przypadku belki B1.

Podsumowując, można stwierdzić, że zastosowanie metod probabilistycznych pozwala na wybranie najkorzystniejszej pod względem niezawodności modelu ST, a przedstawione przykłady i metody szacowania niezawodności mogą służyć jako wskazanie w procesie optymalizacji modelu ST stosowanych w konstrukcjach żelbetowych.

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